

# Bulk Modulus Measurement of Hydraulic Oil Based on Drop-hammer Calibration Device

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**Abstract**—Hydraulic oil bulk modulus is acquiescently set as constants in practical engineering application, so that the theoretical predicted pressure from drop-hammer calibration device will be much different from the measured one. In this situation, bulk modulus measurement of hydraulic oil in the basis of the drop-hammer calibration device is proposed. The practical model for bulk modulus of hydraulic oil was established according to the analysis of the cause to the data errors. With the help from the existing drop-hammer calibration device, "pressure - piston displacement" data pairs were got to work out the model parameters further, and the bulk modulus change rule was obtained. The comparison experiment between the theoretical predicted pressure using the new model and the measured value shows that the bulk modulus model being built is simple and practical. It indicates that the prediction accuracy is less than 2.0% when the pressure is within hydraulic oil's compression prosperities (under 500MPa); the accuracy is less than 3.5% in the pressure extrapolation area (500MPa or more). It meets the requirements of engineering application.

**Keywords-** *Hydraulic Oil; Bulk Modulus; Drop-hammer Calibration Device; Pressure Measurement*

## I. INTRODUCTION

Bulk modulus is an important characteristic of hydraulic oil, whose accuracy influences directly the hydraulic modeling, the theoretical prediction correctness. Usually the work pressure of hydraulic transmission and control system are under 50Mpa, here bulk modulus can be set as constants and will not change with pressure. But for some hydraulic system with extremely high pressure, it will get inaccurate predictions in theory using the constant volume modulus.

In recent years, many methods<sup>[1-2]</sup> for the measurements of the hydraulic oil bulk modulus have been put forward. Toshiyuki designed a device<sup>[3]</sup> for bulk modulus measurement, which can be used for practical parameter measurement. On the basis of volume modulus theory analysis, Wang jing considered the impact on oil bulk modulus with the factor: air, temperature and pressure, and then an online measuring device<sup>[4-6]</sup> for oil bulk modulus was designed according to the bulk modulus definition. Similarly, Christopher<sup>[7]</sup> proposed to use time-resolved laser technique to detect the bulk modulus of liquid. Cong Hengbin<sup>[8]</sup> got the correct modulus of oil and hose with his designed device. And also he gave the rule for

oil volume modulus changing with the pressure and hose modulus.

This paper chose the error analysis between the theoretical predicted pressure and the measured one in application as the entry point. The practical model of hydraulic oil bulk modulus is established; the change rule of bulk modulus is worked out by the existing drop-hammer dynamic pressure calibration device; and theoretical predictions and the practically measured results are compared to prove our method to be effective.

## II. WORK PRINCIPLE OF THE DROP-HAMMER CALIBRATION DEVICE

The drop-hammer calibration device's principle is shown in Figure 1.

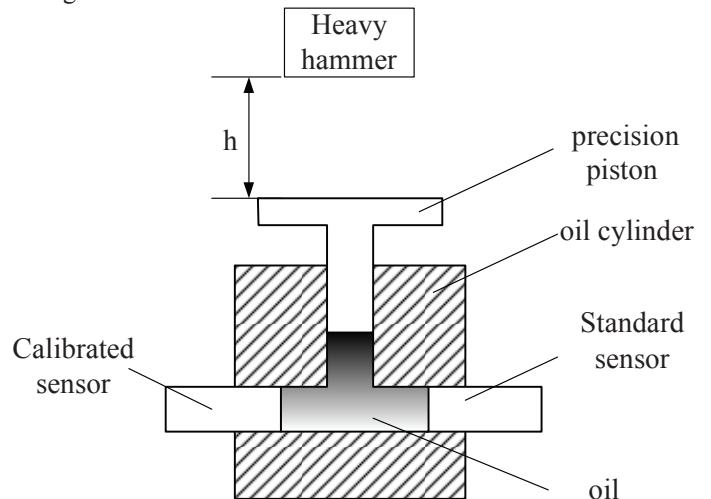


Figure 1. drop-hammer calibration device schemes

The hammer fall free along with the guiding system, and collide with the precise piston in oil cylinder. The piston compresses the oil in cylinder and produces pressure in the cylinder. When the kinetic energy of drop-hammer calibration device transmitted into the elastic energy of the hydraulic oil, the hammer and piston reach the maximum compression stroke and oil cylinder pressure get to be the maximum. Thereafter, because of hydraulic oil elastic recovery function, the piston is pushed upward until the hammer jump from the

pistons. Elastic energy transform back to heavy hammer potential energy. Finally the oil cylinder has formed an approximately half-sine pulse pressure.

The sine pulse can simulate most weapons chamber pressure curve, therefore the drop-hammer calibration device is usually used for the quasi-dynamic calibration of plastic pressure measuring element, the comparative calibration and the quasi-static absolute calibration of sensors.

It assumes that hammer with quality  $M$  fall from the height  $h$  to strike the piston with area  $S$  on the top of oil cylinder whose initial bulk is  $V_0$ . Then the hammer compresses the oil in cylinder and produce pressure pulse. Usually it is default to taken the bulk modulus as constant  $K_0$  in analysis and calculations. Therefore, we can regard the liquid oil in cylinder as a liquid spring with equal stiffness. Given the mass of piston can be ignore, and the collision between hammer and piston is taken as incompletely elastic collision, it will be a second-order mass-spring system. And the produced half-sine pulse can be described with peak  $P_m$  and pulse width  $\tau$  (seen in Figure 2), there is

$$P_m = \sqrt{\frac{2MgK_0}{V_0}} \cdot \sqrt{h} \quad (1)$$

$$\tau = \frac{\pi}{S} \sqrt{\frac{MV_0}{K_0}} \quad (2)$$

By equation (1) and (2), the corresponding half-sine pressure peak and pulse width can be made when choosing the proper quality of the drop-hammer components, the initial volume of oil cylinder producing pressure and the effective area of precision piston component.

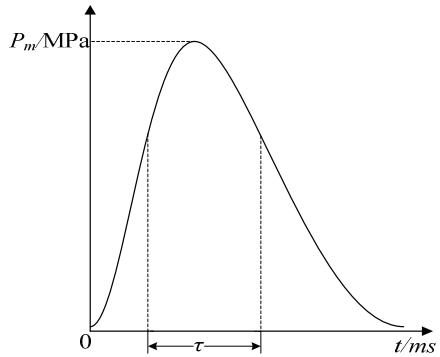


Figure 2. The pressure pulse in the cylinder

### III. THINKING ON CONSTANT VOLUME MODULUS

The compression ratio “ $k$ ” is ordinarily used to characterize the compressibility of hydraulic oil. Its physical meaning is the relative bulk change rate when pressure increasing one unit.

$$k = -\frac{\Delta V}{V_0(p - p_0)} \quad (3)$$

Where,  $\Delta V$  is the amount bulk changed,  $P$  denotes the pressure;  $V_0$  and  $P_0$  are the initial bulk and pressure respectively, the negative sign make  $k$  be positive. The reciprocal of compression ratio is called bulk modulus, denoted by  $K$ .

This section will shows that there is the error problem when the bulk modulus is defaulted as constant by referring to the engineering calibration data. As listed in Table 1,  $P_{ml}$ ,  $\tau_l$  are theoretical value of the pressure peak and the pulse width based on equation (1) and (2),  $P_{ms}$ ,  $\tau_s$  are the measured pressure peak and pulse width in the engineering calibration. The operating parameters of the drop-hammer calibration device are setting as follows:  $M = 6.13 \text{ kg}$ ,  $V_0 = 3.86 \text{ cm}^3$ ,  $S = 1.0 \text{ cm}^2$ , and the castor oil is used as the hydraulic oil,  $K_0 = 2.10 \times 10^9 \text{ Pa}$ . Data from Table 1 show that there are obvious differences between the measured value and the theoretical value: the measured peak  $P_{ms}$  is more than the theoretical prediction  $P_{ml}$ , and the difference increased when the drop height increasing; the measured pulse width  $\tau_s$  is less than the theoretical pulse width predictive value  $\tau_l$ , and the difference increased when the drop height increasing (the theoretical calculated pulse width should be unrelated to drop height).

TABLE I. COMPARISON BETWEEN THE MEASURED VALUE AND THE THEORETICAL VALUE

	Drop height $h/m$	0.50	1.00	1.50	2.00
peak	$P_{ml}/\times 10^6 \text{ MPa}$	180.9	255.9	313.4	361.9
	$P_{ms}/\times 10^6 \text{ MPa}$	222.1	330.0	415.7	497.0
Pulse width	$\tau_l/\text{ms}$	3.34	3.34	3.34	3.34
	$\tau_s/\text{ms}$	2.72	2.58	2.47	2.39

Fitting data  $\{h_i, P_{msi}\}$  in Table 1 with the power function, we get

$$P_{ms} = 331h^{0.58} \quad (4)$$

Where the power exponent of  $h$  is more than 0.5, according to equation (1) the prediction  $P_m$  should be proportion to the 0.5 power of  $h$ . After analyzing the parameters in equation (1),  $M$ ,  $V_0$  and  $S$  are the structure parameters of drop hammer calibration device, all of which are both measurable fixed value and unrelated to  $h$ . Therefore there can only be bulk modulus containing the factors related to  $h$ : the initial height of the drop hammer  $h$  must be the function of the relative compression  $\Delta V/V$  (also can be said the pressure  $P$ ), so the physical model of the hydraulic oil in oil tank should be amended to the fluid spring with variable stiffness, which gradually harden during the compression process. Based on the above analysis, it gets to be a reasonable explanation that as the initial height of the drop hammer increased, the pressure peak is more than expected, and the pulse width is less than expected.

#### IV. DYNAMIC MEASUREMENT OF BULK MODULUS

There is abnormal when measuring the peak pressure of the drop hammer calibration device experimentally, which shows the peak pressure contains the information of bulk modulus of hydraulic oil changing with compression. From another perspective, it can be a good idea constructing the measurement experiment scheme for the changing rule of hydraulic oil bulk modulus, which using the drop hammer calibration device for experimental facilities. This is the inevitable logic results of the dialectical thinking.

First get the model for bulk modulus: after the analysis of experimental data, it is assumable there is linear relationship between the bulk modulus and the relative amount of compression, which can be represented as follows:

$$K = K_0 + \beta \frac{\Delta V}{V} \quad (5)$$

And

$$\Delta V = Sx \quad (6)$$

So

$$p = K \frac{\Delta V}{V} = \frac{K_0 S}{V_0} x + \frac{\beta S^2}{V_0^2} x^2 \quad (7)$$

Where  $V_0$  is the initial cylinder bulk;  $S$  is the piston cross-sectional area;  $x$  denotes the displacement of the drop hammer (piston) after strike. After adjusting,

$$\frac{V_0}{Sx_m} p_m = K_0 + \frac{Sx_m}{V_0} \beta \quad (8)$$

According to the principle of combination measurement, the  $\{P_{mi}, x_{mi}\}$  of the drop hammer calibration device under each running state are measured after setting some drop height  $h_i$  ( $i > 2$ ), (seen in Table 2, working parameters of drop hammer calibration device are setting as follows:  $M = 6.13 \text{ kg}$ ,  $V_0 = 3.86 \text{ cm}^3$ ,  $S = 1.0 \text{ cm}^2$ ). After substituting them into (8), getting a set of over determined equations, measured values of  $K_0$  and  $\beta$  can be obtained using least squares method.

TABLE II. EXPERIMENTAL DATA MEASURING CASTOR OIL BULK MODULUS

$h/m$	0.50	1.00	1.50	2.00
$P_{ms}/\text{MPa}$	222.1	330.0	415.7	497.0
$x_m/\text{mm}$	2.97	4.07	4.88	5.55

Deal with the experimental data in Table 2 with above ideas, we get

$$K_0 = 2.24 \times 10^9 \text{ Pa}$$

$$\beta = 8.36 \times 10^9 \text{ Pa}$$

According to the classification of the data, the measured data from the experiment scheme are average adiabatic bulk modulus. Figure 3 shows the curve of corresponding pressure and the relative amount of compression.

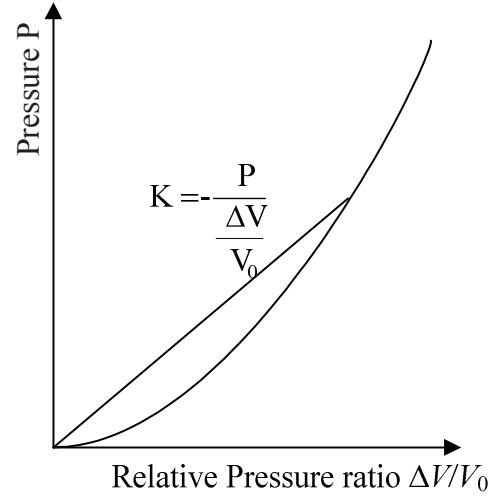


Figure 3. The relationship between pressure and relative amount of compression

#### V. VALIDITY VERIFYING EXPERIMENT FOR THE METHOD

In order to check the predicting accuracy of the ultrahigh pressure in the hydraulic system, following experiments are carried:

Considering the motion process when the drop-hammer impulse the piston in the drop hammer calibration device, according to the conservation of mechanical energy we can get:

$$\frac{1}{2} Mv^2 = Mgh - \int_0^x p S dx$$

Substituted into

$$p = \frac{K_0 S}{V_0} x + \frac{\beta S^2}{V_0^2} x^2$$

Then we get

$$\frac{2\beta S}{3K_0 V_0} x_m^3 + x_m^2 = \frac{2MghV_0}{K_0 S^2} \quad (9)$$

And

$$\tau = 2 \int_0^{x_m} \frac{dx}{\sqrt{2gh - \frac{K_0 S^2}{MV_0} \left( x^2 + \frac{2\beta S}{3K_0 V_0} x^3 \right)}} \quad (10)$$

According to equation (9), the value of  $x_m$  can be got using the iterative method. Then substituting  $x_m$  into equation (7) and equation (10), the theoretical predictive value of peak and pulse width can be calculated. Finally the prediction accuracy in theory is checked with the relative differences between the predictive value above and the measured pressure peak and pulse width in the calibration experiments.

Carrying on verification experiments using working parameters:  $M = 23.04 \text{ kg}$ ,  $V_0 = 3.56 \text{ cm}^3$ ,  $S = 1.0 \text{ cm}^2$ . The result is showed in table 3. It indicates that the prediction accuracy is less than 2.0% when the pressure is within hydraulic oil's compression prosperities (under 500MPa); the

accuracy is less than 3.5% in the pressure extrapolation area ( $500MPa$  or more).

TABLE III. THE RESULT OF VERIFICATION EXPERIMENTS

Drop height $h/m$		0.25	0.5	0.75
peak	$P_{ml}/\times 10^6 MPa$	329.8	497.7	635.0
	$P_{ms}/\times 10^6 MPa$	336.4	492.3	613.6
Pulse width	$\tau_l/ms$	4.86	4.58	4.40
	$\tau_s/ms$	4.85	4.62	4.49

## VI. CONCLUSION

In this paper, a dynamic measurement method for the bulk module of hydraulic oil is proposed. The bulk modulus model being built is simple and practical, and features such as short experiments period and convenient for operation are also showed in this measuring method. On contrast of the theoretical prediction value and the data in pressure calibration experiments, it indicates that this method for measuring bulk modulus is effective and the accuracy of the results meets the demands of engineering applications.

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