Digitally assisted coaxial bridge for automatic quantum Hall effect measurements at audio frequencies

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Abstract—This paper describes the principle of a new fully automatic digitally assisted coaxial bridge having a large bandwidth ranging from 100 Hz to 20 kHz. The bridge is characterized by making a 1:1 comparison between calculable ac resistors. The agreement between the calculated and the measured frequency dependence of the resistors is better than $10^{-7}$ over its entire bandwidth. Such a bridge is a perfect tool to start investigating the ac transport properties of graphene in the quantum Hall regime.

Index Terms—Metrology, quantum Hall effect, graphene, impedance standard, precision measurement, bridge circuit.

I. INTRODUCTION

The investigation of the Quantum Hall Effect (QHE) requires the use of coaxial ac bridges to compare the Quantum Hall Resistance (QHR) to calculable resistance standards at audio frequencies [1], [2], [3]. Such dedicated bridges are optimized to give the highest accuracy in impedance comparison [4]. However, this high accuracy is only achieved in a restricted frequency bandwidth (typically between 500 Hz and 5 kHz) and requires a tedious manual balancing of the bridge.

Only a few tentatives to automate ac coaxial bridges have been carried out using expensive automated inductive voltage dividers (IVDs) [5], [6].

In this paper, a new digitally assisted bridge [7] is presented. The precise voltage ratio is still given by a voltage transformer, however, all the balances required to precisely compare the impedances are automatically performed —over a larger bandwidth (100 Hz to 20 kHz)— by adjusting digital sources and detectors instead of IVDs and lock-in amplifiers.

II. BRIDGE DESCRIPTION AND DATA PROCESSING

Figure 1 shows the schematic of the new digitally assisted coaxial bridge for the comparison of four terminal-pair standards in a 1:10 ratio. The outers of the coaxial cables have been omitted for clarity. The bridge is formed by 1 ratio transformer $RT$; 5 signal generators $S$, $S_{top}$, $S_{bot}$, $S_{inj}$ and $S_K$; 6 digitizers $V_{ref}$, $V_{inj}$, $V_{top}$, $V_{bot}$, $V_{HP top}$ and $V_{LP bot}$; and 6 IDTs.

![Simplified schematic of the digitally assisted coaxial bridge for the comparison of four terminal-pair standards in a 1:10 ratio.](image-url)

The signal generators and the digitizers are either the analogue outputs (AO) or the analogue inputs (AI) of high-performance, high-accuracy analogue I/O devices commercially available (NI PXI 4461). Each channel has its own 24-bit converter, amplifier/attenuator and anti-aliasing filter. The maximum generation/sampling rate is 204.8 kSa/s.

The 6 IDTs are home-made transformers with 100 turns at the primary winding and either 100 turns or 1 turn for the secondary winding. A double electrostatic shield is placed between the primary and secondary windings to avoid any leakage current between the different parts of the electrical circuit. Coaxial chokes [8] are also implemented, one in each mesh of the bridge, to guarantee the current equalization and the immunity of the bridge to external interferences [9].

Each AO channel generates a single tone signal at the same frequency $f$. The relative phase and amplitude of each generator can be independently adjusted.

Each AI channel simultaneously samples $N$ values of the voltage at a sampling frequency $f_s$. The duration of the data set is therefore given by $N/f_s$ and contains $P$ periods of the measured signal. The amplitude, $A$, and the phase, $\phi$, of the fundamental component of each measured signal is then...
obtained from the Discrete Fourier Transform (DFT) of the data sets. To avoid spectral leakages and to guarantee the accuracy of the DFT calculation, \( N \) and \( P \) have to be integers and \( N \geq 2 \).

For each digitizer, the quantity of interest, \( D \), is finally calculated normalizing the measured voltage by the reference voltage measured by the digitizer \( V_{ref} \). For example, the voltage measured by the digitizer \( V_{inj} \) leads to the measured quantity \( D_{inj} \):

\[
D_{inj} = \frac{\hat{A}_{inj}}{A_{ref}} e^{j(\hat{\varphi}_{inj} - \varphi_{ref})} = A_{inj} + jB_{inj}
\]

where \( j = \sqrt{-1} \)

III. BALANCING PROCEDURE AND STATE EQUATION

The purpose of the balancing procedure is to realize the electrical conditions required by the four-terminal-pair definition of the impedance standards [10]. The procedure consists in the adjustment of the amplitude and phase of the four signal generators \( (S_{top}, S_{bot}, S_{inj} \) and \( S_K ) \) to simultaneously null the four measured quantities \( (D_{HP}, D_{LP}, D_{bot} \) and \( D_{top} ) \). The bridge has been designed to get the different balances almost independent from each other, making the balance procedure easier. The realization of the balances listed in Table I leads to the following state equation for a bridge ratio of 1:10:

\[
\frac{Z_{bot}}{Z_{top}} = \frac{1}{10} \left( 1 + \delta_{10} + \frac{D_{inj}}{100} \right)
\]

where \( \delta_{10} \) is the small deviation of the voltage ratio from the turn ratio of \( RT \). For a 1:1 ratio i.e. when the two standards have the same nominal value, the ratio error, \( \delta_1 \), can be determined repeating the measurement with the top and bottom standards inverted.

IV. FIRST RESULTS

As a first step in the bridge characterization, two calculable resistors [11] of the same nominal value \( (R_K - 90)/2 \) have been compared over the frequency range from 100 Hz to 20 kHz. Figure 2 shows the difference between the measured and the calculated frequency dependence of the resistance ratio. The agreement between the calculated and the measured values is remarkable i.e. smaller than 0.02 \( \mu \Omega/\Omega \) up to 10 kHz. At higher frequencies, a low residual quadratic dependence (see dotted line) is visible and deserves further investigation.

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\text{REFERENCES}
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