Geophysical Applicability of Atomic Clocks

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Using General Relativity to Determine the Geoid?

- **General Relativity predicts that space-time curves near massive objects. As a consequence clocks beat slower near Earth.**

- The geoid is the surface of constant gravitational potential that extends the mean sea level accounting for the effects of all subsurface density variations. **The tick rate of clocks is constant on the geoid.**

- If a clock is placed at mean sea level, i.e., the exact altitude of the geoid, a second clock could be positioned anywhere on the continent to determine its position relative to the geoid. A clock positioned one meter above the sea level will beat faster that one at sea level by about 40 seconds in the life-time of the universe. In 2010, Chou *et al.* measured the different tick rate of two optical atomic clocks that were displaced horizontally by 33 cm.

- **Ultra-precise atomic clocks measure geopotential differences at the location of the clock providing the most direct possible local measurement of the geoid on continents.** All other methods are indirect and rely on ill defined integrals of derivatives of the gravitational potential. For example, gravimeters typically measure only the vertical component of $\mathbf{g} = -\nabla U$ when $\mathbf{g}$ is a 3D vector.

Surveying the Interior of the Earth with Ultra-precise clocks?

- **Sources below the surface of the Earth with different densities with respect to their environment also affect the tick rate of clocks. A clock positioned over an iron ore ticks slower than one located over an empty cave.**

- geopotential measurements can be combined with other local geophysical measurements such as gravimetric measurements to reduce some of the degeneracy of the inverse problem. For the simple example of a buried sphere, one could find the distance to the object by simply dividing the ΔU and Δg_z measurements.

- In realistic situations the exact shape of the anomaly is never known. So, we would not be able to locate the density anomaly from the ratio of ΔU and Δg_z, but the geopotential and gravimetric measurements will still be valuable independent measurements with different sources of error.

- **Portable atomic clocks are sensitive to the local, small-scale internal structure of the Earth.**
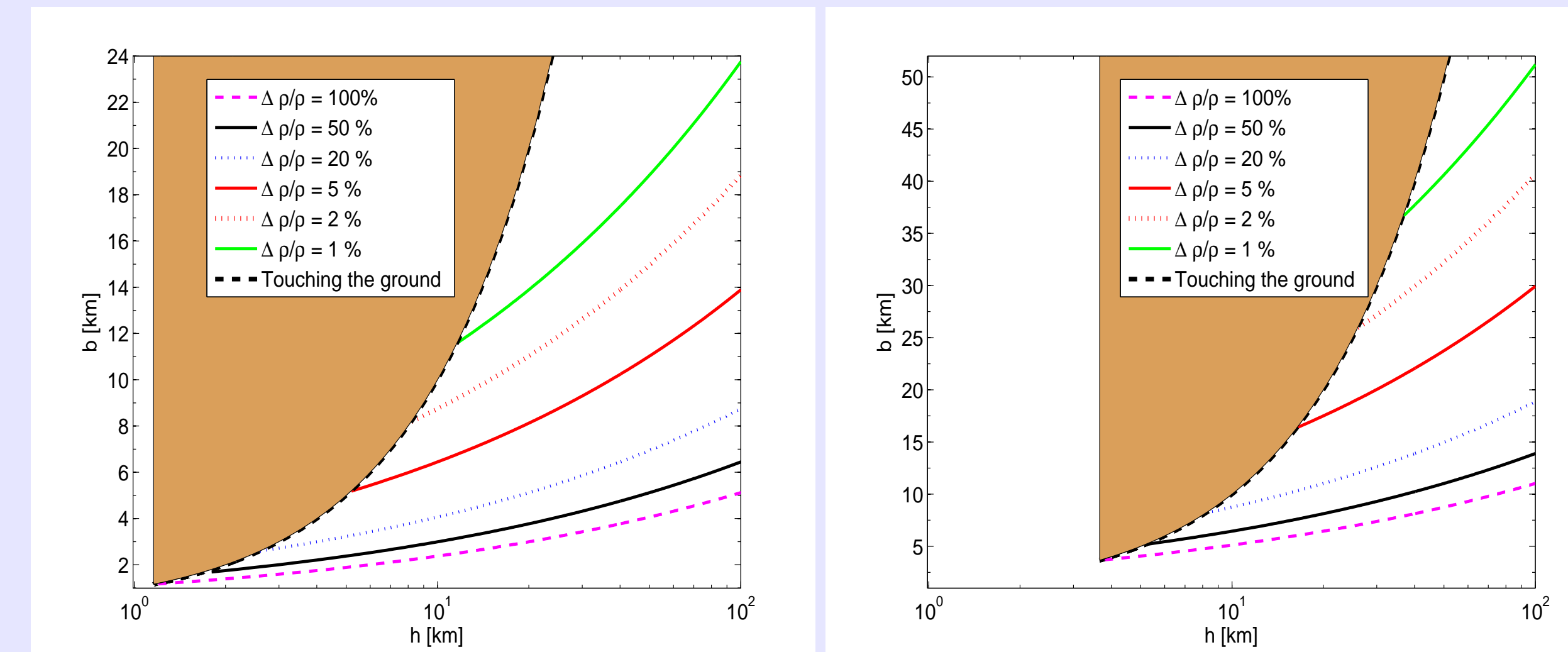
The Simplest Source Size Estimate?

Fig.1: $\Delta U_{\max} = 1$ m²/sec², $\Delta f/f = 10^{-17}$ $\Delta U_{\max} = 10$ m²/sec², $\Delta f/f = 10^{-16}$

For a buried sphere

$$\Delta U = \frac{4\pi G b^3 \Delta \rho}{3\sqrt{x^2 + h^2}}, \qquad (1)$$

where $\Delta\rho$ is the density contrast with the surrounding material, b is the radius of the sphere, x is the horizontal distance from the center of the sphere, and h is the depth to which the center of the sphere is buried. The radius of the smallest detectable sphere can be computed from eq. 1:

$$b = \left(\frac{3h\Delta U_{\max}}{4\pi G \Delta\rho}\right)^{1/3}. \qquad (2)$$

Fig. 1 shows b as a function of h for several density contrasts. For $\Delta f/f = 10^{-17}$ (sensitivity reached by optical atomic clocks in the laboratory) and $\Delta\rho/\rho = 20\%$, which corresponds to a bubble of magma that has reached the upper crust, the minimum detectable sphere has $b \sim 2.5$ km. A sphere with $b \sim 9$ km could be seen to a depth of $h \approx 100$ km. For $\Delta f/f = 10^{-16}$, the minimum radius is $b \sim 8$ km. A sphere of radius $b \approx 20$ km can be seen to a depth of $h \approx 100$ km. The related gravity anomaly in the z-direction is

$$\Delta g_z = \frac{4\pi G b^3 \Delta\rho h}{3\left(x^2 + h^2\right)^{3/2}}. \qquad (3)$$

The ratio of ΔU and Δg_z is a parabola centered around the depth h:

$$\frac{\Delta U}{\Delta g_z} = h + \frac{x^2}{h} \approx h \text{ for } x \ll h. \qquad (4)$$

The frequency inaccuracy of atomic clocks can be directly related to the geopotential difference for stationary observers

$$\frac{f(x_1)}{f(x_2)} = 1 - \frac{U(x_2) - U(x_1)}{c^2} \text{ or } \frac{\Delta f}{f} = \frac{\Delta U}{c^2}. \qquad (5)$$
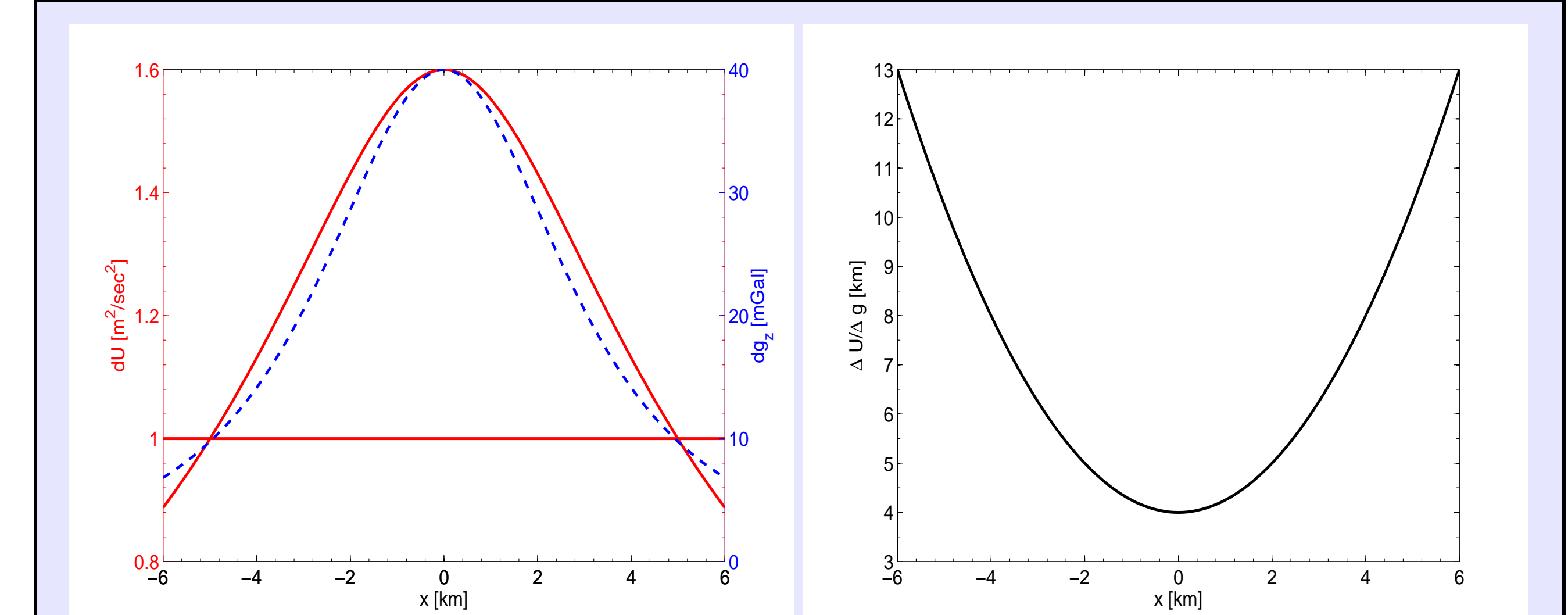
Combining Gravimetric and Geopotential Surveying?

Fig. 2: A sphere of magma of radius $b = 3.5$ km and $\Delta\rho/\rho = 20$ % is buried in the upper crust with its center at $h = 4$ km. (a) ΔU and Δg are plotted as a function of horizontal distance. The red line is $\Delta U = 1$ m²/sec². (b)$\Delta U/\Delta g_z$ is shown as a function of distance. The depth of 4 km can be easily deduced.

Conclusions

- We provide simple estimates that suggest that ultra precise clocks with $\Delta f/f = 10^{-17}$ (and perhaps $\Delta f/f = 10^{-16}$) could already be useful for local geophysical surveying.

- **On continents, ultraprecise clocks provide the most direct form of local geoid mapping that could add detail to satellite maps.** This procedure has been known for many years (Bjerhammar 1986), but clocks are only now starting to reach the required accuracy.

- **For ultraprecise clock surveying to be possible the clocks need to be portable and stable. They have to be synchronized via fiber optics cable or free space transmission.**

- **As early as 2014 or 2015, the "Atomic Clock Ensemble in Space" (ACES) will be placed on the International Space Station. Once ACES reaches a precision of $\Delta f/f = 10^{-16}$ it may be used as a reference clock in space provided that the transmission through the atmosphere is accurate enough.**

- local surveying with portable clocks combined with gravimetric measurements will lower the degeneracy of the inverse problem and improve our understanding of the interior of the Earth.

References
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