19NRM02 “RevStdLED”

Guideline on
Measurement Uncertainty
in
Photometry and Radiometry:

Distribution of correlation matrices

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Content

1 Introduction

When spectral data sets are used to determine spectrally integrated values (e.g., for photometry), the correlations between individual spectral measurements become important because, unlike intrinsic correlation (e.g., due to filters), these extrinsic correlations are not fixed and change depending on boundary conditions and on the setup used.

This guideline provides information on how to use correlation data to improve traceability of measured data. The guideline also includes information regarding boundary conditions with respect to Monte Carlo simulations to determine uncertainty. Publications JCGM 100 [1], JCGM 101 [2], and CIE 198 series [3], [4], [5], provide useful information and tools for uncertainty evaluation.

2 Terminology

There are many terms in use which describe various aspects regarding the quality of a measurement. In the following, the correct use of terms like "accuracy", "error", "tolerance", "reproducibility", "repeatability" and "uncertainty" are explained, based on the documents JCGM 200:2012 *"The International vocabulary of metrology – Basic and general concepts and associated terms" (VIM) 3rd edition"* [6], and CIE TN 009:2019 [7].

Accuracy	
The term accuracy describes the "closeness of agreement between a measured quantity value and a true quantity value of a measurand". In this concept, "accuracy" is the description of a general assessment and rating on the scale from very good to very poor. It is not a quantity and cannot be assigned a numerical quantity value	
Example	The accuracy of the measurement is very high

Precision	
Precision describes the " closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions". In metrology it is used to define repeatability and reproducibility and is numerically expressed by standard deviations or variances. It has to be distinguished from the term accuracy.	
Example	The reading has a precision relative standard deviation of σ = 0.01

Repeatability	
Measurement repeatability is the "measurement precision under a set of repeatability conditions of measurements". The conditions include the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time	
Example	The repeatability standard deviation is within 0.3 % of the measured value

Reproducibility	
Measurement reproducibility is the "measurement precision under reproducibility conditions of measurement". The conditions include different locations, operators, measuring systems, and replicate measurements on the same or similar objects. For instance measurement reproducibility is the closeness of agreement of measurements taken at different measurement setup using the same artefact (reproducibility artefact) or using comparable artefact at the same measurement setup (reproducibility of the setup).	
Example	The reproducibility standard deviation is within 0.2 % of the measured value

Error	
A measurement error is the "measured quantity value minus a reference quantity value". This concept assumes that an error can be expressed by an absolute value without uncertainty. However, an error is neither a mistake nor an uncertainty as it typically occurs not by intention and the true value of a measurement is typically not known.	
Example	Due to the offset not yet taken into account, the length measurement still has an error of 6 mm.

Uncertainty	
Uncertainty is a "non-negative parameter to characterize the dispersion of the quantity values assigned to a measurand, based on the information used". It includes all components that influence the measurement as a whole, i.e. systematic effects, statistical components and others. Hence, uncertainty is never attributed to a measurement device, an artefact or a measurement setup per se. It is based on a confidence level of a resulting probability distribution.	
Example	The measurement of the luminous flux of the lamp had an associated expanded uncertainty of U = 0,83 % with k = 2,15 according to a confidence level of 95 %.

Tolerance	
A tolerance interval is an interval of permissible values, i.e. it describes a zone of values which are acceptable for a particular purpose. Since each measured value has an associated uncertainty, it depends on the risk management whether the best estimate of the acceptable measured values according to their uncertainty may be inside or outside the tolerance interval.	
Example	Permissible temperature values within the temperature limits T = 25,0 °C ± 1,2 °C define the temperature tolerance interval: [23,8; 26,2] °C.

Correlation	
Correlations in the metrological sense are mutual relationships between two or more measured quantities. Two quantities are correlated if a common third quantity influences their respective values.	
Example	A fluctuation of the ambient temperature around a mean value can lead to a dependent fluctuation of the measured value of the spectroradiometer, which measures the illuminance by an LED source, as well as to a fluctuation of the luminous flux of the LED source to be measured itself.

Intrinsic Correlation	
An intrinsic correlation is a correlation that is attributed to a specified input quantity that exists by itself and does not depend on the measurement conditions or set-up.	
Example	The multivariate chromaticity coordinates (x, y) are intrinsically correlated by overlapping tristimulus functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$.

Extrinsic Correlation	
An extrinsic correlation is a correlation that depends on outside parameters or measurement set-up.	
Example	When calibrating the spectral irradiance of a detector, the spectral bandwidth and the associated slit function of the monochromator must be taken into account to evaluate the uncertainty. This quantity contributes to correlations with adjacent wavelengths depending on the settings.

Multivariate	
When performing uncertainty evaluation using Monte Carlo technique, a probability distribution is termed univariate when it relates to a single (scalar) random variable, and multivariate when it relates to a vector of random variables. A multivariate probability distribution is also described as a joint distribution.	
Example	Uncertainty of illuminance measurement performed with an illuminance meter calibrated in spectral responsivity includes the uncertainty of the illuminance meter calibration data that are correlated. A multivariate Gaussian distribution should be assigned to the spectral data.

3 Spectral irradiance responsivity of detector

At the National Metrology Institute (NMI) level, the traceability chain for calibrating the spectral irradiance responsivity of a reference detector is commonly linked to the cryogenic radiometer through three distinct routes:

- **Primary Route directly to Cryogenic Radiometer:** This route establishes a direct link between the reference detector and the cryogenic radiometer. The cryogenic radiometer is a primary measurement standard used for maintaining the accuracy and traceability of spectral irradiance measurements.

- **Secondary Route through Reference Trap Detectors:** In this route, calibration is achieved using reference trap detectors. These detectors are calibrated at another NMI with primary route capability, primarily based on detector characteristics. This secondary route offers an additional layer of traceability.

- **Secondary Route through Lamp Standards:** Here, calibration is based on lamp standards that are calibrated against a black body at an NMI. The black body's temperature is determined using filtered detectors calibrated against a cryogenic radiometer. This source-based approach provides a different path to traceability.

Both the cryogenic radiometer and trap detectors are commonly used in radiant power mode. This mode involves the light beam not completely filling the entrance window of the detector, allowing for accurate measurement of the radiant power of the beam. In the case of measurements against lamp standards, the detector is used in irradiance mode. In this mode, the light beam overfills the entrance window of the detector.

To effectively calibrate the irradiance responsivity of a filtered radiometer, such as a photometer, two technical requirements must be met:

- **Determination of Spectral Irradiance:** The spectral irradiance at the reference plane of the reference detector needs to be determined. This is achieved by limiting the effective area of the entrance window at the reference plane using an aperture with a known area. This transition from radiant power mode to irradiance mode is crucial for accurate calibration.

- **Distance and Uniformity Considerations:** The Device Under Test (DUT) must be positioned at the same distance from the source as the reference detector, with respect to its reference plane. This alignment ensures comparability. If the aperture of the DUT differs from that of the reference detector, factors related to the uniformity of the radiation field need to be taken into account.

This section discusses the calibration routes and technical requirements involved in establishing the spectral irradiance responsivity of detectors, ensuring accurate and traceable measurements across different methodologies.

3.1 Calibration facility

A robust calibration facility is essential to achieve accurate results. This facility needs to be capable of generating a uniform monochromatic beam precisely in the reference plane of the Device Under Test (DUT). The schematic representation of the monochromatic beam generation aspect of the calibration facility is illustrated in Figure 1. This section also covers the components involved, including a monochromator and optics for beam shaping.

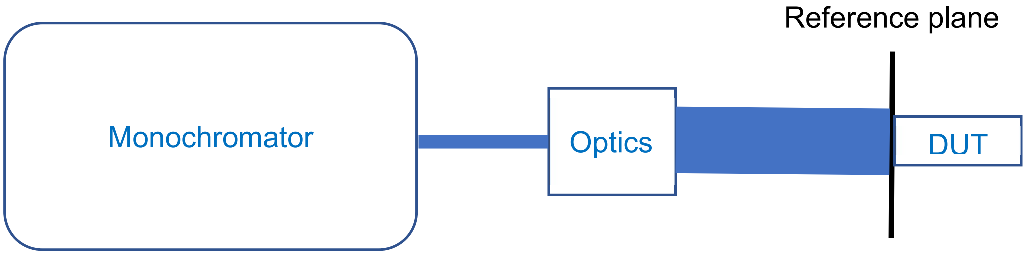

Figure 1: Schematic of a spectral irradiance responsivity calibration facility

The monochromator can be constructed using either of the following equipments

- **Tunable Laser:** In the modern context, tunable lasers covering a range from UV to Vis to IR are readily available in the market.

- **Wavelength Dispersive Device and Broadband Lamp:** An alternative approach involves coupling a wavelength dispersive device with a broadband lamp. To minimize stray light interference, employing a double-monochromator setup is recommended.

To ensure accurate comparison, a mechanical apparatus should be designed for the reference plane. This apparatus facilitates the alternating placement of the reference detector and the Device Under Test (DUT) in an identical position on the monochromatic beam.

Examples of calibration facilities at PTB (Physikalisch-Technische Bundesanstalt), utilizing a laser-based set-up for irradiance responsivity, and at LNE-CNAM (Laboratoire national de métrologie et d'essais - Conservatoire national des arts et métiers), employing a monochromator-based approach for power responsivity, are depicted in Figure 2 and Figure 3 respectively.

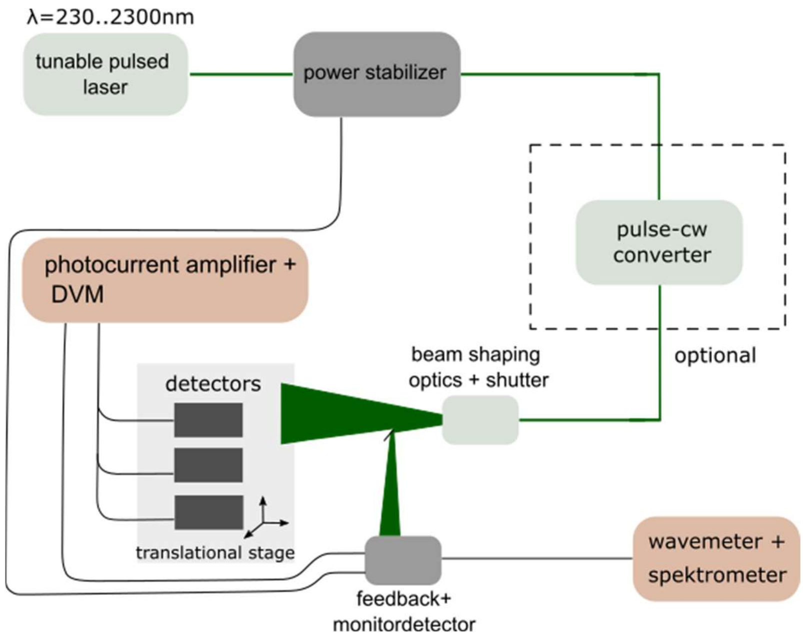

Figure 2: PTB Calibration Facility (Laser-Based for Irradiance Responsivity)

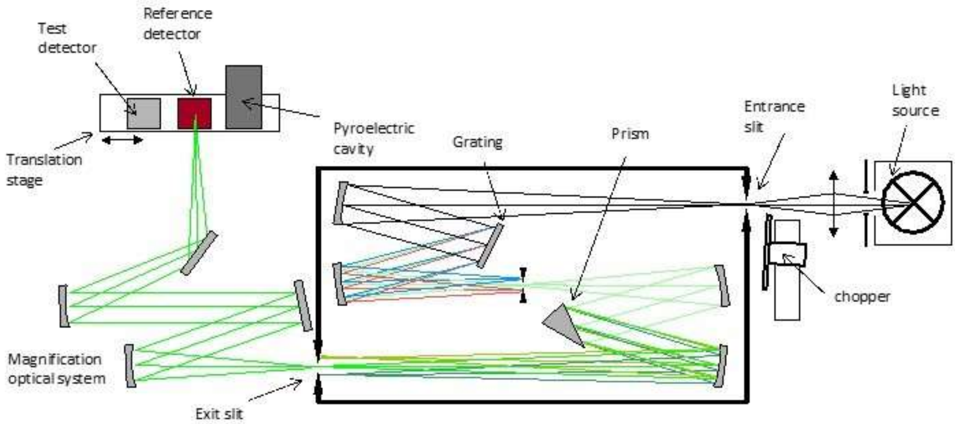

Figure 3: LNE-CNAM Calibration Facility (Monochromator-Based for Power Responsivity)

This section underscores the significance of an appropriately designed calibration facility capable of generating a uniform monochromatic beam for accurate and traceable measurements. It also shows real-world examples of such facilities at PTB and LNE-CNAM.

3.2 Calibration procedure

The Device Under Test (DUT) is affixed to a translational stage to facilitate precise positioning. To ensure the detector surface is perpendicular to the optical axis of the setup, a laser beam placed along the optical axis is used for alignment purposes.

Wavelength Interval and Regular Calibrations

For determining irradiance or power responsivity within a specific wavelength range, using a consistent wavelength grid for setting the wavelengths is advantageous. In the case of regular calibrations, such as unfiltered Si-detectors, a typical interval of 5 nm between wavelengths is used. However, for detectors with varied responsivity slopes, such as filtered detectors, and considering the monochromator system's bandwidth, different wavelength step settings might be necessary.

Reference Trap Detector Calibration

A calibrated reference trap detector is chosen and then positioned and aligned in the same manner as the DUT, using the translational stage. An aperture with a calibrated area, smaller than the effective area of the trap detector, is mounted onto the reference detector.

Measurement Principle and Substitution Technique

The measurement principle is based on the substitution technique, involving the use of a monochromatic beam for irradiating the reference detector and the DUT. This technique facilitates accurate comparison.

Incorporating a Monitor Detector

To correct for potential variations in beam power during measurements between the reference detector and the DUT, a monitor detector can be introduced into the setup.

This section emphasizes the intricacies of mounting, aligning, and calibrating the DUT. It also delves into the significance of wavelength intervals, the substitution technique, and the potential integration of a monitor detector to maintain measurement accuracy.

3.3 Measurement model

The measurement [8] is based on a substitutional method for transferring the known power responsivity of the reference detector to the irradiance responsivity of the DUT by adding an aperture with calibrated area to the reference detector. The resulting measurement equation is as follows:

$$s(\lambda) = \frac{U_{DUT}}{U_{ref}} \cdot \frac{R_{ref}}{R_{DUT}} \cdot \frac{Mon_{ref}}{Mon_{DUT}} s_{\Phi,ref}.(\lambda) \cdot A_{ref} \cdot c_{wl}(\lambda) \cdot c_{bw}(\lambda) \cdot c_{pol}(\lambda) \cdot c_{unif}(\lambda) \cdot c_{dist} \qquad 1$$

where

$s(\lambda)$	is the spectral irradiance responsivity of the DUT
$U_{\mathrm{DUT/ref}}$	are the voltage readings when measuring either the DUT or reference detector
$Mon_{\mathrm{ref/DUT}}$	are the simultaneous voltage readings of the monitor detector of the DUT or reference signal
$R_{\mathrm{ref/DUT}}$	are the calibrated resistances of the used photocurrent amplifier (gain setting)
$s_{\Phi,\mathrm{ref}}(\lambda)$	is the calibrated spectral power responsivity of the reference detector

A_{ref} is the calibrated area of the aperture in front of the reference detector

$c_{\mathrm{wl}}(\lambda)$ is the correction factor for the wavelength measurement

$c_{\mathrm{bw}}(\lambda)$ is the correction factor for bandwidth effects

$c_{\mathrm{pol}}(\lambda)$ is the correction factor for polarization dependency of the DUT

$c_{\mathrm{unif}}(\lambda)$ is the correction factor for non uniformity of DUT, reference detector and the used radiation

c_{dist} is the correction factor for distance offsets between DUT and reference

3.4 Measurement uncertainty and correlation calculation

3.4.1 Uncertainty components

According to the measurement equation the uncertainty components are the following:

Repeatability of the voltage readings of DUT.

Voltage readings (signal – dark signal) is the average of n measurements from which the standard deviation is determined. In our case, the uncertainty of the voltage reading is itself a combined uncertainty of the uncorrelated uncertainties of the readings and the calibrated feedback resistor.

Repeatability of the voltage readings of reference detector

Voltage readings (signal – dark signal) is the average of n measurements from which the standard deviation is determined. In our case, the uncertainty of the voltage reading is itself a combined uncertainty of the uncorrelated uncertainties of the readings and the calibrated feedback resistor.

Repeatability of the monitor detector readings

Voltage readings (signal – dark signal) is the average of n measurements from which the standard deviation is determined.

The voltage readings of the DUT and reference detector are fully correlated with the voltage readings of the respective monitor detectors. Therefore, in Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of the monitors as for the DUT and reference detector signals, respectively for a given wavelength.

Calibration of the aperture area of the reference detector (correlation)

This uncertainty $u_{\mathrm{ApertureArea}}$ is obtained from the calibration certificate of the aperture. The spectral data of the DUT are fully correlated.

Therefore, in Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of the aperture area for every wavelength if the spectral responsivity of the detector is to be specified for more than one wavelength.

Calibration of the photocurrent amplifier, i.e. R_{ref}, for reference detector (correlation)

This uncertainty $u_{Amp.Ref}$ is obtained from the calibration certificate of the current/voltage amplifier or feedback resistor R_{ref} associated to the reference detector. This uncertainty depends on the signal level. All measurements using the same amplifier setting are fully correlated with respect to systematic uncertainty components of the amplification setting.

In Monte Carlo simulations, one has to use the same randomly drawn numbers for simulation of the photocurrent amplification for equal amplifier or feedback resistor settings, respectively. The same set of random numbers must also be used for every wavelength if the spectral responsivity of the detector is to be specified for more than one wavelength.

Calibration of the photocurrent amplifier, i.e. R_{DUT}, for DUT (correlation)

This uncertainty $u_{Amp.DUT}$ is obtained from the calibration certificate of the current/voltage amplifier or feedback resistor R_{ref} associated to the DUT. This uncertainty depends on the signal level. All measurements using the same amplifier setting are fully correlated with respect to systematic uncertainty components of the amplification setting.

In Monte Carlo simulations, one has to use the same randomly drawn numbers for simulation of the photocurrent amplification for equal amplifier or feedback resistor settings, respectively. The same set of random numbers must also be used for every wavelength if the spectral responsivity of the detector is to be specified for more than one wavelength.

Calibration of the spectral irradiance responsivity of the reference detector (correlation)

This uncertainty $u_{Ref.Cal}$ is obtained from the calibration certificate of the reference detector. The calibration certificate may provide also the correlation matrix or covariance matrix for a range of wavelength to be covered.

Also if no correlation matrix is provided, it must be expected that the spectral data are correlated. The upper limit of the influence of partial correlation can be estimated using the base functions approach described in *[9]*.

Uncertainty due to wavelength measurement uncertainty (correlation)

An uncertainty $u(\lambda)$ in the wavelength measurement implies a measurement of the responsivity at a wavelength $\lambda_0+u(\lambda)$.

Taking into account the first order Taylor expansion of $s(\lambda)$ at λ_0,

$$s(\lambda) = s(\lambda_0) + s'(\lambda_0)\,(\lambda - \lambda_0) + o\big(\lambda - \lambda_0\big) \quad 2$$

the spectral responsivity at the wavelength $\lambda_0+u(\lambda)$ is:

$$s\big(\lambda_0 + u(\lambda)\big) = s(\lambda_0) + s'(\lambda_0)\,u(\lambda) \qquad 3$$

The error E_{rr} in the measured spectral responsivity $s(\lambda_0)$ is:

$$E_{rr} = \frac{s(\lambda_0) - s\big(\lambda_0 + u(\lambda)\big)}{s(\lambda_0)} = -\,\frac{s'(\lambda_0)}{s(\lambda_0)}\,u(\lambda) \qquad 4$$

The correction factor $C_{wl,DUT}$ on the measured signal U_{DUT} of the DUT is:

$$C_{wl,DUT} = 1 - \frac{s'(\lambda_0)}{s(\lambda_0)}\, u(\lambda) \quad 5$$

We have a similar correction factor $C_{wl,Ref}$ for the measured signal U_{ref} of the reference detector

$$C_{wl,ref} = 1 - \frac{s'_{ref}(\lambda_0)}{s_{ref}(\lambda_0)}\, u(\lambda) \qquad 6$$

In the measurement model the correction factors are applied on the measured signals and the correction factor C_{wl} on the spectral responsivity is given by:

$$C_{wl} = \frac{C_{wl,DUT}}{C_{wl,ref}} = \frac{1 - \frac{s'(\lambda_0)}{s(\lambda_0)} u(\lambda)}{1 - \frac{s'_{ref}(\lambda_0)}{s_{ref}(\lambda_0)} u(\lambda)} \qquad 7$$

As the correction factor C_{wl} is in the measurement model it is expected to be close to unity, its uncertainty $u(C_{wl})$ can be estimated by:

$$u(C_{wl}) = 1 - C_{wl} = 1 - \frac{1 - \frac{s'(\lambda_0)}{s(\lambda_0)} u(\lambda)}{1 - \frac{s'_{ref}(\lambda_0)}{s_{ref}(\lambda_0)} u(\lambda)} \qquad 8$$

where

$u(c_{\mathrm{wl}}(\lambda))$	is the contribution of the wavelength uncertainty for the spectral responsivity
$u(\lambda)$	is the uncertainty of the measured wavelength
$s'_{\mathrm{DUT/ref}}(\lambda)$	are the first derivatives of the spectral responsivities of DUT and reference detector
$s_{\mathrm{DUT/ref}}(\lambda)$	are the spectral responsivities of DUT and reference detector

The correlation of wavelength uncertainty has three contributions:

- Uncertainty due to the calibration of the wavelength scale. This contribution is partially correlated (see CIE 198-SP1.4:2011). For Monte Carlo simulations, one has to use the multivariate distribution with the correlation coefficient matrix defined in CIE 198-SP1.4.
- Uncertainty on the ambient temperature that leads to a fully correlated contribution to spectral data. For Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of the wavelength variation for every wavelength setting if the spectral responsivity of the detector is to be specified for more than one wavelength.
- Uncertainty due to a shift of the wavelength scale (especially for scanning instruments) which is fully positive or negative correlated depending on the slope of the spectral responsivity. For Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of the wavelength variation for every wavelength setting if the spectral responsivity of the detector is to be specified for more than one wavelength.

In Monte Carlo simulations one has to draw numbers for the wavelength uncertainty from multivariate distributions including partial correlations as mentioned above. As default, one may consider only including the first contribution from CIE 198-SP1.4, with a correlation coefficient of 0.3 for the first neighbouring wavelengths. Note that this contribution is different from the contribution of the MU of the spectral responsivity of the reference detector.

Uncertainty due to spectral bandwidth

Using a monochromator with equal slits widths for the entrance and exit ports gives an output beam power P(λ) with triangular spectral shape

$$P(\lambda) = 1 - \frac{|\lambda-\lambda_0|}{\Delta\lambda} = \begin{cases} 1 + \frac{\lambda-\lambda_0}{\Delta\lambda}, \ \lambda \leq \lambda_0 \\ 1 - \frac{\lambda-\lambda_0}{\Delta\lambda}, \ \lambda \geq \lambda_0 \end{cases} \quad 9$$

where

$\Delta\lambda$ FWHM (Full Width Half Maximum)

The signal measured with a detector at the output of a monochromator is proportional to the product $S(\lambda) \cdot \mathrm{P}(\lambda)$ of the spectral responsivity of the detector and the spectral power distribution of the beam integrated over the wavelength interval [λ_0 - Δλ; λ_0 + Δλ].

Taking into account the second order of the Taylor expansion at a wavelength λ_0 .

$$s(\lambda) = s(\lambda_0) + s'(\lambda_0)\,(\lambda-\lambda_0) + \frac{s''(\lambda_0)}{2}\,(\lambda-\lambda_0)^2 + o((\lambda-\lambda_0)^2) \quad 10$$

where

S(λ) Spectral responsivity

o(λ) Error (due to higher orders)

For wavelength λ < λ_0:

$$S(\lambda) \cdot\ P(\lambda) = S(\lambda_0) + \frac{S(\lambda_0)}{\Delta\lambda}(\lambda-\lambda_0) + S'(\lambda_0)(\lambda-\lambda_0) + \frac{S'(\lambda_0)}{\Delta\lambda}(\lambda-\lambda_0)^2 + \frac{S''(\lambda_0)}{2}(\lambda-\lambda_0)^2 + \frac{S''(\lambda_0)}{2\,\Delta\lambda}(\lambda-\lambda_0)^3 \quad 11$$

and

$$\int_{\lambda 0-\Delta\lambda}^{\lambda 0} S(\lambda) \cdot\ P(\lambda)\, d\lambda \ = S(\lambda_0)\,\Delta\lambda - \frac{S(\lambda_0)}{2}\,\Delta\lambda - \frac{S'(\lambda_0)}{2}\Delta\lambda^2 + \frac{S'(\lambda_0)}{3}\Delta\lambda^2 + \frac{S''(\lambda_0)}{6}\Delta\lambda^3 - \frac{S''(\lambda_0)}{8}\Delta\lambda^3 = \frac{S(\lambda_0)}{2}\,\Delta\lambda - \frac{S'(\lambda_0)}{6}\Delta\lambda^2 + \frac{S''(\lambda_0)}{24}\Delta\lambda^3 \quad 12$$

For wavelength λ > λ_0:

$$S(\lambda)\cdot\ P(\lambda) = S(\lambda_0) - \frac{S(\lambda_0)}{\Delta\lambda}(\lambda-\lambda_0) + S'(\lambda_0)(\lambda-\lambda_0) - \frac{S'(\lambda_0)}{\Delta\lambda}(\lambda-\lambda_0)^2 + \frac{S''(\lambda_0)}{2}(\lambda-\lambda_0)^2 - \frac{S''(\lambda_0)}{2\,\Delta\lambda}(\lambda-\lambda_0)^3 \quad 13$$

and

$$\int_{\lambda 0\,-\,\Delta\lambda}^{\lambda 0} S(\lambda)\cdot\ P(\lambda)\,d\lambda \ = S(\lambda_0)\,\Delta\lambda - \frac{S(\lambda_0)}{2}\,\Delta\lambda + \frac{S'(\lambda_0)}{2}\Delta\lambda^2 - \frac{S'(\lambda_0)}{3}\Delta\lambda^2 + \frac{S''(\lambda_0)}{6}\Delta\lambda^3 - \frac{S''(\lambda_0)}{8}\Delta\lambda^3 = \frac{S(\lambda_0)}{2}\,\Delta\lambda + \frac{S'(\lambda_0)}{6}\Delta\lambda^2 + \frac{S''(\lambda_0)}{24}\Delta\lambda^3 \quad 14$$

Therefore the measured signal U is

$$U = \int_{\lambda 0\,-\,\Delta\lambda}^{\lambda 0+\,\Delta\lambda} S(\lambda)\cdot\ P(\lambda)\,d\lambda \ = S(\lambda_0)\,\Delta\lambda + \frac{S''(\lambda_0)}{12}\Delta\lambda^3 \quad 15$$

The true signal U_0 is

$$U_0 = S(\lambda_0)\,\Delta\lambda \qquad 16$$

Therefore the correction factor $corr_{bwl}$ is given by:

$$corr_{bwl} = \frac{U_0 - U}{U_0} = 1 - \frac{S''(\lambda_0)}{12}\frac{\Delta\lambda^2}{S(\lambda_0)} \quad 17$$

This correction factor applies for the reference detector and the DUT

$$c_{bw}(\lambda) = \frac{1-\Delta\lambda^2.\frac{1}{12}.s''_{REF}(\lambda)\Big/s_{REF}(\lambda)}{1-\Delta\lambda^2.\frac{1}{12}.s''_{DUT}(\lambda)\Big/s_{DUT}(\lambda)} \quad 18$$

where

$\Delta\lambda$ is the FWHM bandwidth of the spectral measurement setup

$s''_x(\lambda)$ is the second derivative of the DUT or reference detector responsivity

Propagating the uncertainty of the beam spectral bandwidth measurement u(Δλ) yields the uncertainty of the correction factor $u(c_{bw}(\lambda))$ for the spectral responsivity..

$$u\big(c_{bw}(\lambda)\big) = \frac{2\cdot\big(b(\lambda)-a(\lambda)\big)\cdot\Delta\lambda\cdot u(\Delta\lambda)}{\big(1-\Delta\lambda^2\cdot b(\lambda)\big)^2} \quad 19$$

where:

$$a(\lambda) = \frac{1}{12}\cdot s''_{REF}(\lambda)\Big/s_{REF}(\lambda) \quad 20$$

$$b(\lambda) = \frac{1}{12}\cdot s''_{DUT}(\lambda)\Big/s_{DUT}(\lambda) \quad 21$$

For Monte Carlo simulation one has to consider two cases:

- The wavelengths that are generated with the same combination of grating and slit width of the monochromator. The bandwidth corrections are fully correlated. The off-diagonal elements of the correlation coefficients matrix are equal to "1".
- For wavelengths that are generated with combinations that differ from the others by the grating or slit width, the corresponding bandwidth corrections are uncorrelated. The off-diagonal elements of the correlation coefficients matrix are equal to "0".

Uncertainty due to polarization dependency of the DUT (no correlation)

The maximum change in the detector responsivity due to polarization Δs_{pol} presented in (Schneider 2018) can be directly used for the correction factor:

$$c_{pol}(\lambda) = 1 + \Delta s_{pol} \cdot P_{pol}(\lambda) \cdot sin(2\pi \cdot \varphi) \quad 22$$

where

$P_{\mathrm{pol}}(\lambda)$ is the degree of linear polarization of the incident radiant flux

φ is the polarization angle

The calculation of the uncertainty of $c_{pol}(\lambda)$ is straightforward by propagating the uncertainties of the contributing parameters. For most trap detectors or single photodiode detectors (with photodiodes perpendicular to the optical axis) the effects of polarization are not significant compared to the signal-to-noise ratio of the measurement and can thus be neglected. Nevertheless, this needs to be checked for.

Uncertainty due to DUT, reference detector and the beam uniformities (no correlation)

The correction factor for non-uniformities of the detectors and the non-uniformity of incident radiation is calculated using the surface integral:

$$c_{unif}(\lambda) = \left[\oiint_A s_{rel,DUT}(x,y,\lambda)dxdy / \oiint_A s_{rel,DUT}(x,y,\lambda) \cdot E_{rel}(x,y,\lambda)dxdy\right] \cdot \left[\oiint_A s_{rel,ref}(x,y,\lambda) \cdot E_{rel}(x,y,\lambda)dxdy / \oiint_A s_{rel,ref}(x,y,\lambda)dxdy\right] \quad 23$$

where

$s_{\mathrm{rel,DUT/REF}}(x,y,\lambda)$ are the locally resolved relative spectral responsivity distributions of DUT and REF detector

$E_{\mathrm{rel}}(x,y,\lambda)$ is the locally resolved relative spectral irradiance distribution in the measurement plane. If scanning double-monochromators are used, the uniformity of the beam can be drastically improved if the monochromators are connected in subtractive mode instead of additive mode.

The uncertainty of $c_{\text{unif}}(\lambda)$ is calculated by propagating the uncertainties of the irradiance distribution and responsivity distributions of DUT and reference detector.

In Monte Carlo simulations, for each wavelength one has to generate different set of random numbers for $s_{rel,DUT}(x,y,\lambda)$, $s_{rel,REF}(x,y,\lambda)$ and $E_{rel}(x,y,\lambda)$ from the measurement of the spatial uniformities of detectors and beam irradiance.

NOTE 1: Beam uniformity may also cause correlations due to the uncertainty of positioning the reference detector and the DUT in the measurement beam. Also possible wavelength dependence of the uniformity may cause correlation, as it is impractical to measure the uniformity at each measurement wavelength.

Uncertainty due to distance error (correlation)

The correction factor for distance offsets is set to unity because the detectors are measured in the same nominal distance to the source of the radiation field. However, the deviation from ideal alignment and uncertainties of determining the detectors reference plane contribute to the uncertainty according to:

$$u(c_{dist}) = \sqrt{\left(\frac{u(d_{DUT})}{d_0}\right)^2 + \left(\frac{u(d_{REF})}{d_0}\right)^2} \qquad 24$$

where

$d_{\text{DUT/REF}}$ are the distances of the DUT and ref from the source

d_0 ______is the nominal distance between detector and source

In Monte Carlo simulations, one has to use the same set of random numbers for every wavelength if the spectral responsivity of the detector is to be specified for more than one wavelength.

3.4.2 Uncertainty evaluation

The combined measurement uncertainty is calculated using Monte Carlo technique (GUM supplement 1). For each input quantity in the measurement model random values are calculated based on the uncertainty estimated for the input quantity and according to the rules mentioned above.

The measured voltages for DUT and reference detector are converted to Student-T-Distributions based on the number of readings done for each wavelength. The random number generation should be performed for each wavelength according to the rules given in 3.4.1. The amplifiers resistances are converted to normal distributions as stated in the calibration certificates.

The correction factors for bandwidth, uniformity and polarization effects (non-correlated quantities) are calculated based on normal distributions.

The uncertainties due to the aperture area of the reference detector, the distances to the source and the wavelength scale shift due to temperature variation etc. are quantities fully correlated with respect to all other wavelengths. To carry out Monte Carlo Simulation, the uncertainty contributions are determined using multivariate Gaussian distributions, i.e. the spectral values are expected to be column-vector quantities to which respective correlation matrices are applied. In this way, the calculation is performed once and applied to all wavelengths.

Other uncertainty components that need to be evaluated taking into account the correlation between the wavelengths, are:

- Uncertainty due to the wavelength scale calibration. This is partially correlated and is evaluated using multivariate Gaussian distribution.
- Uncertainty due to the calibration of the spectral responsivity of the reference detector. This is converted to multivariate Gaussian distribution if the correlation matrix is available or to normal distribution if the correlation matrix is not provided

For the amplifiers resistance calibration correlation should be taken into account and multivariate Gaussian distribution should be applied. However it may not be necessary to determine the uncertainty contribution for each wavelengths. As mentioned in 3.4.1, the same random number generation can be applied for wavelengths for which the same amplifier resistance is used.

The uncertainty components of the DUT spectral irradiance responsivity $s(\lambda)$, measurement uncertainty and the information that define the PDF parameters are summarized in Table 1.

Table 1: $s(\lambda)$ uncertainty components

Type of variable	Uncertainty components	PDF type	Correlation	Comments	Effect on Monte Carlo simulation
Column vector of spectral data points	Signal readings DUT, reference detector and monitor detector	Gaussian multivariate	Partially correlated	Spectrally dependant	From the measurement data for a given wavelength standard deviations and covariance are determined. A covariance matrix can be built and a joint PDF can be determined
Column of amplifier settings for spectral ranges	Calibration of current amplifier	Gaussian Multivariate	Fully correlated for equal settings (simple correlation matrix)	Partially spectrally dependant	If for a given wavelength a same gain is used for at least two signals from DUT, REF or monitor detectors then correlation should be taken into account for those signals

Column vector of spectral data points	Spectral irradiance responsivity of reference detector, $s_{\phi,\mathrm{REF}}$	Normal	Partially correlated; correlation matrix if provided)	Spectrally dependant	If a NxN correlation matrix or covariance matrix is provided together with the N spectral data points, the multivariate drawing for the spectral input values has to be based on the provided covariance matrix - e.g. by Cholesky decomposition or adequate software tools. As a boundary condition, the correlation matrix must be positive semidefinite. If this is not the case, the matrix must be "cleaned".
Scalar	Aperture area of the reference detector, A_{REF}	Normal	Fully correlated	Non-spectrally dependant	This PDF is generated once and applied to all wavelengths
Column vector for spectral data points	Calibration of the wavelength scale, c_{wl}	Multivariate	Partially correlated (CIE 198-SP1.4:2011)	Spectrally dependant	Determination of the correlation coefficients and the associated correlation matrix.
Column vector for spectral data points	Wavelength scale temperature dependent, $c_{\mathrm{wl},\ T}$	Multivariate	Fully correlated	Spectrally dependant	Random numbers can be generated once and applied to all wavelengths taking into account the uncertainty associated to each wavelengths
Column vector for spectral data points	Source spectral bandwidth, c_{bw}	Multivariate	Fully correlated	Partially spectrally dependant	Correlation to be taken into account for groups of wavelengths with the same spectral bandwidth setting
Scalar	Distance, c_{dist}	Multivariate	Fully correlated	Non-spectrally dependant	This PDF is generated once and applied to all wavelengths

3.4.3 Covariance matrix

Although in principle the uncertainty considerations taking correlations into account can also be carried out using classical GUM procedures, the necessary determination of the measurement equation for multivariate demanding quantities is getting very sophisticated. Here, the approach of determining the combined measurement uncertainty via Monte Carlo simulations provides significant simplifications. Furthermore, in modern computer languages like Matlab or Python libraries are available which already provides program-packages to properly generate random numbers according to required distributions and to draw samples for correlated multivariate quantities. It is mandatory that proper random number generation is provided and validated (§9.1.4, Note §C1.2, JCGM 101:2008).

Monte Carlo technique relies on a set of random trials based on the random number generation. Sufficient number N of trials should be implemented to get a reliable value for the uncertainty: one million trials is recommended (Note §7.2.1, JCGM 101:2008).

For one particular wavelength λ_i and each uncertainty component a matrix column of N values is generated and the sorted order should not be changed. The values corresponding to a line "k" of all matrices are combined in the measurement model. This generates as output a matrix column of N values for each spectral data of the DUT.

$$s(\lambda_i) = \begin{pmatrix} s(\lambda_i, 1) \\ \vdots \\ s(\lambda_i, N) \end{pmatrix} \qquad 25$$

From this matrix column we can determine the estimated value of $s(\lambda_i)$ which is the average of the N values of the matrix and the variance of the average $s^2(\overline{s(\lambda_i)})$

$$\overline{s(\lambda_i)} = \frac{\sum_{k=1}^{N} s(\lambda_i,k)}{N} \qquad 26$$

$$s^2\left(\overline{s(\lambda_i)}\right) = \frac{\sum_{k=1}^{N}\left(s(\lambda_i,k)-\overline{s(\lambda_i)}\right)^2}{N\cdot(N-1)} \qquad 27$$

We can also determine the covariance between the spectral data $s(s(\lambda_i), s(\lambda_j))$

$$s\left(s(\lambda_i), s(\lambda_j)\right) = \frac{\sum_{k=1}^{N}\left(s(\lambda_i,k)-\overline{s(\lambda_i)}\right)\cdot\left(s(\lambda_j,k)-\overline{s(\lambda_j)}\right)}{N\cdot(N-1)} \qquad 28$$

And the correlation coefficient, which is a dimensionless quantity.

$$r\left(s(\lambda_i), s(\lambda_j)\right) = \frac{s\left(s(\lambda_i), s(\lambda_j)\right)}{s(s(\lambda_i)) \cdot s(s(\lambda_j))} \quad 29$$

Therefore

$$r\left(s(\lambda_i), s(\lambda_i)\right) = 1 \quad 30$$

$$-1 < r\left(s(\lambda_i), s(\lambda_j)\right) < 1 \quad 31$$

These results allow the construction of the correlation matrix.

$$\begin{pmatrix} 1 & r\left(s(\lambda_1), s(\lambda_2)\right) & r\left(s(\lambda_1), s(\lambda_3)\right) & r\left(s(\lambda_1), s(\lambda_4)\right) & r\left(s(\lambda_1), s(\lambda_5)\right) \\ r\left(s(\lambda_2), s(\lambda_1)\right) & 1 & r\left(s(\lambda_2), s(\lambda_3)\right) & r\left(s(\lambda_2), s(\lambda_4)\right) & r\left(s(\lambda_2), s(\lambda_5)\right) \\ r\left(s(\lambda_3), s(\lambda_1)\right) & r\left(s(\lambda_3), s(\lambda_2)\right) & 1 & r\left(s(\lambda_3), s(\lambda_4)\right) & r\left(s(\lambda_3), s(\lambda_5)\right) \\ r\left(s(\lambda_4), s(\lambda_1)\right) & r\left(s(\lambda_4), s(\lambda_2)\right) & r\left(s(\lambda_4), s(\lambda_3)\right) & 1 & r\left(s(\lambda_4), s(\lambda_5)\right) \\ r\left(s(\lambda_5), s(\lambda_1)\right) & r\left(s(\lambda_5), s(\lambda_2)\right) & r\left(s(\lambda_5), s(\lambda_3)\right) & r\left(s(\lambda_5), s(\lambda_4)\right) & 1 \end{pmatrix} 32$$

However

$$r\left(s(\lambda_i), s(\lambda_j)\right) = r\left(s(\lambda_j), s(\lambda_i)\right) 33$$

Therefore the correlation matrix has a symmetry and reduces to:

$$\begin{pmatrix} 1 & r\left(s(\lambda_2), s(\lambda_1)\right) & r\left(s(\lambda_3), s(\lambda_1)\right) & r\left(s(\lambda_4), s(\lambda_1)\right) & r\left(s(\lambda_5), s(\lambda_1)\right) \\ r\left(s(\lambda_2), s(\lambda_1)\right) & 1 & r\left(s(\lambda_3), s(\lambda_2)\right) & r\left(s(\lambda_4), s(\lambda_2)\right) & r\left(s(\lambda_5), s(\lambda_2)\right) \\ r\left(s(\lambda_3), s(\lambda_1)\right) & r\left(s(\lambda_3), s(\lambda_2)\right) & 1 & r\left(s(\lambda_4), s(\lambda_3)\right) & r\left(s(\lambda_5), s(\lambda_3)\right) \\ r\left(s(\lambda_4), s(\lambda_1)\right) & r\left(s(\lambda_4), s(\lambda_2)\right) & r\left(s(\lambda_4), s(\lambda_3)\right) & 1 & r\left(s(\lambda_5), s(\lambda_4)\right) \\ r\left(s(\lambda_5), s(\lambda_1)\right) & r\left(s(\lambda_5), s(\lambda_2)\right) & r\left(s(\lambda_5), s(\lambda_3)\right) & r\left(s(\lambda_5), s(\lambda_4)\right) & 1 \end{pmatrix} 34$$

Similarly the covariance matrix can be set.

4 Spectral irradiance of sources

In many NMIs the primary spectral irradiance scale is realized, maintained and disseminated using a high temperature blackbody radiator of type BB3200pg . The main parameter of a black body, the temperature, has to be determined very accurately. Usually broadband-filter detectors are well established for the detector-based determination of the so-called radiometric temperature .

The spectral irradiance at the reference plane of the spectroradiometer is calculated according to Planck's law using the geometric parameters and the measured radiometric temperature of the blackbody.

4.1 Calibration facility

Spectral irradiance of sources are measured with a facility that is based on a monochromator. It is recommended to use a double monochromator particularly in the UV range. A schematic of a calibration set-up is shown on Figure 4 .

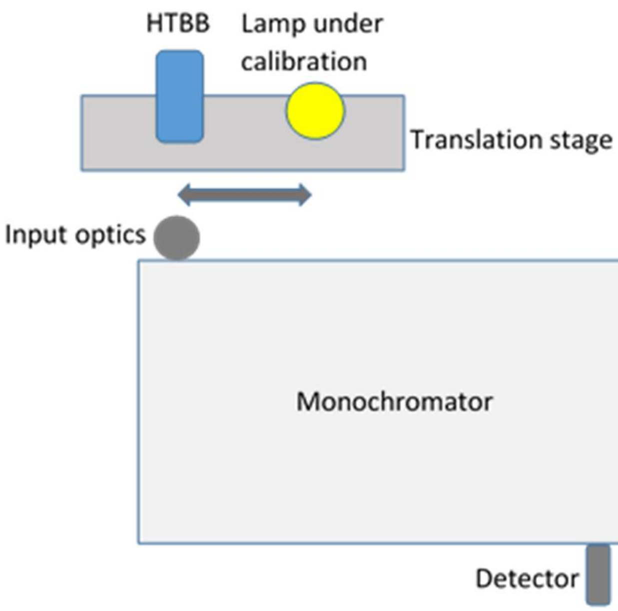

Figure 4: Schematic for a source spectral irradiance calibration set-up

Examples of facilities at PTB and LNE are shown on Figure 5 and Figure 6 respectively.

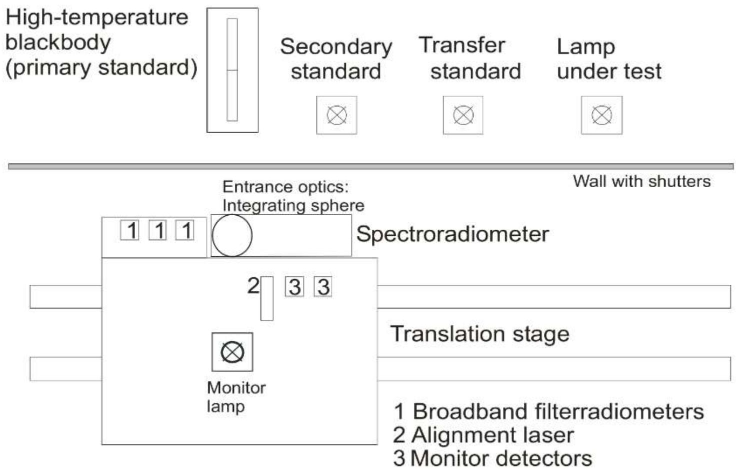

Figure 5 : PTB set-up

The PTB spectroradiometer for spectral irradiance calibrations (Figure 5) consists of an integrating sphere as entrance optics, an Acton-Research Spectra-Pro^{TM}-500 double monochromator system with triple grating turrets and three detectors to cover the spectral range from 250 nm to 2500 nm [8]. The grating turrets are equipped with three gratings with 1200 l/mm for the spectral range from 250 nm up to 1100 nm and 300 l/mm for the infrared spectral range above 1100 nm. The spectral bandwidth of the system is controlled by 8 motorized slits and varies for different spectral regions.

The entrance port of the integrating sphere is formed by a precise aperture with 11 mm diameter which defines the reference plane for spectral irradiance measurements. The detectors at the exit ports of the monochromators are a photomultiplier tube (PMT) for the spectral range from 250 nm to 670 nm, a Si-Photodiode (680 nm to 1100 nm) and an extended InGaAs-detector (1200 nm to 2500 nm). A built-in monitor lamp is used to monitor the stability of the spectroradiometer-system during extended measurement campaigns. The system is placed on a translation stage to allow the quasi-simultaneous measurement of a group of lamps with respect to the blackbody radiation.

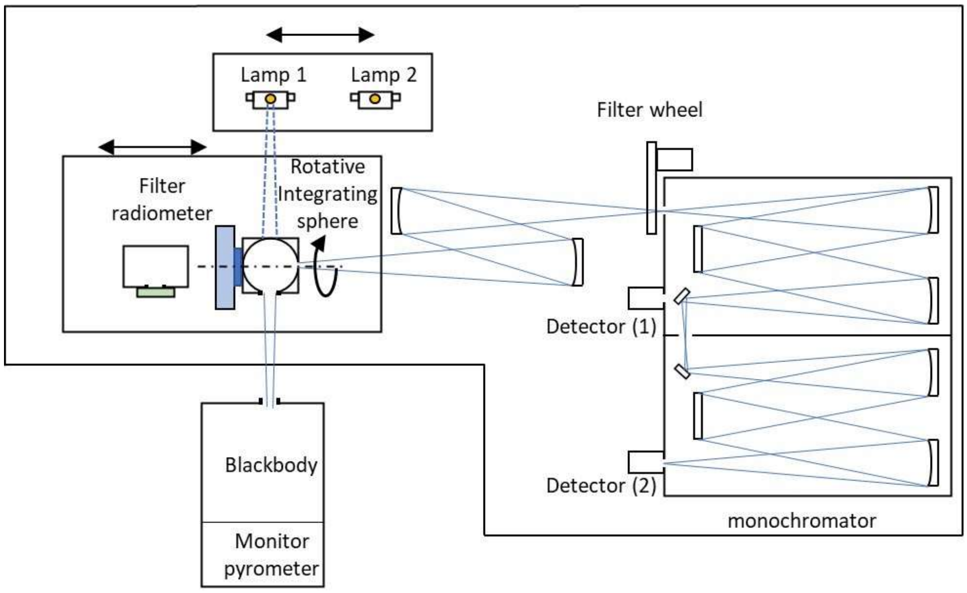

Figure 6 : LNE-CNAM set-up

The spectroradiometer of the LNE-CNAM is described in Figure 6. The High Temperature Black Body (HTBB) is used as an irradiance reference for the calibration of transfer sources. For this purpose, a spectroradiometric system have been placed between the HTBB and transfer lamps (Lamp1 and Lamp2). The integrating sphere that receives the flux from the sources has an entrance port equipped with a circular calibrated aperture and an output port located at 90° with a rectangular slit of 2 mm x 15 mm. Two Ø 150 mm spherical mirrors focus the image of the sphere exit slit onto the entrance slit of a double monochromator.

The filter radiometer and the integrating sphere are placed on the same automated translation table. This allows the blackbody temperature to be automatically measured during the measurement cycle. The integrating sphere is mounted on a rotation stage that allows a 180° rotation around the axis centered on the exit slit. This allows the collection of either the flux from the HTBB, or the flux of the transfer lamps. The top and bottom of the exit slit are reversed but its position is the same. The flux is then guided by the 2 mirrors from the sphere to the input slit of the monochromator. This makes it possible to measure the spectral distribution of sources from 250 nm to 2500 nm.

The monochromator is a Jobin Yvon HRD1 which have been retrofitted with a high precision motorized translation stage for automatic selection of the wavelength. Three pairs of gratings are used to cover the wavelength range from 250 nm to 2500 nm.

Four detectors are used:

- Photomultiplier tube (PM R456 Hamamatsu) for the spectral range from 250 nm to 700 nm
- Silicium photodiode (S6337 Hamamatsu) for the spectral range from 475 nm to 1100 nm
- InGaAs photodiode (G6126 Hamamatsu) for the spectral range from 1000 nm to 1600 nm
- Extended InGaAs photodiode (J23TE2-66C-R03M-2.6 Teledyn) for the spectral range from 900 nm to 2500 nm

The PM detector is mounted in front of the second exit slit of the monochromator (double monochromator configuration). Other detectors are mounted to first exit slit (single monochromator configuration). The extended InGaAs detector is Thermoelectrically Cooled.

4.2 Measurement procedure

The basic set-up as described in 4.1 is used for a standard lamp calibration and uncertainty evaluation. The measurement principle is based on a comparison between a reference source (black body) and a standard lamp using a monochromator for spectral distribution measurement. The calibration of the standard lamp is performed in three steps:

- Step 1: measurement of the black body temperature
- Step 2: determination of the spectral distribution of the black body
- Step 3: calibration of the standard lamp compared to the black body

4.2.1 Measurement of the temperature of the blackbody

The first step is to measure the radiometric temperature T_{BB} of the blackbody. This is performed using a filter radiometer which signal $U_{FB}(T_{BB})$ is directly related to the temperature of the blackbody according to :

$$U_{FB}(T_{BB}) = \epsilon \cdot V_{iU} \cdot \frac{A_{BB}}{d_{FD}^2} \int s(\lambda) \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \frac{1}{exp\left(\frac{c2}{n \cdot \lambda \cdot T_{BB}}\right) - 1} d\lambda \quad 35$$

Where,

$U_{FB}(T_{BB})$	Photosignal of the broadband-filter radiometer measured in Volts
ε	Effective emissivity of the blackbody radiator
V_{iU}	Gain of the electrical measurements
A_{BB}	Size of the blackbody opening aperture
d_{FD}	Distance of the filter detector to the blackbody aperture
$s(\lambda)$	Spectral responsivity distribution of the filter radiometer (A.m².W^{-1})
λ	Wavelength
T_{BB}	Radiometric temperature of the blackbody radiator
c_1,c_2	Planck constants
n	Refractive index of air

Equation $U_{FB}(T_{BB}) = \epsilon \cdot V_{iU} \cdot \frac{A_{BB}}{d_{FD}^2} \int s(\lambda) \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \frac{1}{exp\left(\frac{c2}{n \cdot \lambda \cdot T_{BB}}\right)-1} d\lambda$ 35 can be rewritten as:

$$U_{FB}(T_{BB}) = K \int s(\lambda)\, L(\lambda, T_{BB})\, d\lambda \quad 36$$

Where $L(\lambda, T_{BB})$ is the spectral radiance of the blackbody radiator.

Generally, $UFB(T_{BB}) = \epsilon \cdot V_{iU} \cdot \frac{A_{BB}}{d_{FD}^2} \int \quad s(\lambda) \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \frac{1}{exp\left(\frac{c2}{n \cdot \lambda \cdot T_{BB}}\right)-1} d\lambda$ 35, is solved numerically by iteratively varying T_{BB} until the calculated signal, right side of equation $U_{FB}(T_{BB}) = \epsilon \cdot V_{iU} \cdot \frac{A_{BB}}{d_{FD}^2} \int s(\lambda) \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \frac{1}{exp\left(\frac{c2}{n \cdot \lambda \cdot T_{BB}}\right)-1} d\lambda$ 35, is equal to the measured signal $U_{FB}(T_{BB})$ [10] . Methods such as the bisection rule can be used to achieve this, but the most efficient method is to use the Newton-Raphson algorithm, based on an initial estimate T_0 . The algorithm then proceeds by forming successively better estimates, T_i , for i = 1, 2, 3, …, using the equation

$$T_{i+1} = T_i + \frac{U_{FB}(T_{BB}) - K \int s_{rel}(\lambda)\, L(\lambda, T_i)\, d\lambda}{\frac{c_2}{T_i^2} K \int s_{rel}(\lambda) \frac{L(\lambda, T_i)}{n\lambda[1 - exp(-c_2/(n\lambda T_i))]} d\lambda} \quad 37$$

Convergence to better than 0.1 mK is usually achieved in fewer than 5–10 iterations, depending on how close the initial guess, T_0 , is to the true temperature.

4.2.2 Determination of the blackbody spectral irradiance

Knowing the temperature T_{BB} of the blackbody the spectral *irradiance* $E(\lambda, T_{BB})$ at the input port of the integrating sphere is given by:

$$E(\lambda, T_{BB}) = \epsilon \cdot \frac{A_{BB}}{{d_{diff}}^2} \cdot \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \cdot \frac{1}{exp\left(\frac{c_2}{n \cdot \lambda \cdot T_{BB})}\right)-1} \qquad 38$$

With

ε	Effective emissivity of the blackbody radiator
A_{BB}	Size of the blackbody opening aperture
d_{diff}	Distance of the input port of the integrating sphere to the blackbody aperture
λ	Wavelength
T_{BB}	Radiometric temperature of the blackbody radiator
c_1, c_2	Planck constants
n	*R*efractive index of air

4.2.3 Measurement of the spectral irradiance of the standard lamp

For each wavelength setting of the monochromator, the signal delivered by a detector set on the monochromator when irradiated by the standard lamp is compared to the signal delivered by the same detector when irradiated by the black body.

Then the standard lamp spectral irradiance $E_{ST}(\lambda)$ is given by:

$$E_{ST}(\lambda) = E(\lambda, T_{BB}) \cdot \frac{V_{ST}}{V_{BB}} \cdot \frac{G_{BB}}{G_{ST}} \qquad 39$$

Where

$E(\lambda, T_{BB})$ Calculated spectral irradiance of the blackbody-radiator

V_{ST}, V_{BB} Measured photosignal with the standard lamp or the blackbody

G_{ST}, G_{BB} Amplifier gains for standard lamp or blackbody

4.3 Uncertainty evaluation

4.3.1 Uncertainty of the blackbody temperature

The model for the blackbody temperature measurement with a broadband filter radiometer is:

$$U_{FB}(T_{BB}) = \epsilon \cdot (V_{iU} + \delta V_{iU}) \cdot \cos\alpha_1 \cdot \cos\alpha_2 \cdot \frac{A_{BB} + \delta A_{BB}}{d_{FD}^2} \int \left(s(\lambda) + \delta s(\lambda)\right) \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \frac{1}{exp\left(\frac{c2}{n \cdot \lambda \cdot T_{BB}}\right) - 1} d\lambda \qquad 40$$

With:

$U_{FB}(T_{BB})$ Photosignal of the broadband-filter radiometer measured in Volts

ε Effective emissivity of the blackbody radiator

V_{iU}, δV_{iU} Gain of the electrical measurements and its drift

$\cos\alpha_1$, $\cos\alpha_2$ Misalignment of filter radiometer to the optical axis of the black body

A_{BB}, δA_{BB} Size of the blackbody opening aperture and its drift

d_{FD} Distance of the filter radiometer to the blackbody aperture

s, δs Absolute spectral responsivity of the filter radiometer and its drift

λ Wavelength used for calculation

T_{BB} Radiometric temperature of the blackbody radiator

c_1,c_2 Planck constants

n Refractive index of air

For uncertainty evaluation the different input quantities are uncorrelated except for the calculation of the integral for which the filter radiometer spectral responsivity data are correlated and multivariate PDF must be use taken into account the correlation matrix given with the calibration certificate of the filter radiometer (eventually the data from the simulation performed by the calibration laboratory).

Repeatability of the filter radiometer signal should be evaluated in order to determine the associated uncertainty. Then for Monte Carlo simulation a Student's t PDF is assigned to this uncertainty component.

The **emissivity** ϵ is determined with a standard uncertainty $u(\varepsilon)$ associated with a Gaussain PDF. For Monte Carlo simulations, one has to use a set of randomly drawn numbers for simulation of the emissivity.

The uncertainty of the **amplifier gain** $u(V_{iU})$ is obtained from the calibration certificate of the gain associated with a Gaussian PDF. For Monte Carlo simulations, one has to use a set of randomly drawn numbers

For the uncertainty of **the drift of the amplifier** $u(\delta V_{iU})$ a Uniform PDF is assigned. For Monte Carlo simulations, one has to use a set of randomly drawn numbers for $u(\delta V_{iU})$

For the uncertainty of the **alignments of the blackbody** $u(\cos\alpha_1)$ **and the filter radiometer** $u(\cos\alpha_2)$ Gaussian PDF are assigned. For Monte Carlo simulations, one has to use a set of randomly drawn numbers separately for $u(\cos\alpha_1)$ and $u(\cos\alpha_2)$.

The uncertainty of the **aperture area of the blackbody** $u(A_{BB})$ is obtained from the calibration certificate of the aperture associated with a Gaussian PDF. For Monte Carlo simulations, one has to use a set of randomly drawn numbers for simulation of the aperture area.

For the uncertainty of **the aperture contamination during operation of the blackbody** $u(\delta A_{BB})$ Uniform PDF is assigned. For Monte Carlo simulations, one has to use a set of randomly drawn numbers for $u(\delta A_{BB})$.

The uncertainty $u(d_{FD})$ of the **d**istance between the filter radiometer and the blackbody aperture measurement is obtained from the calibration certificate of the distance gauge associated with a Gaussian PDF. For Monte Carlo simulations, one has to use a set of randomly drawn numbers for simulation of the distance measurement.

The uncertainty of the **filter radiometer spectral responsivity** $u(s(\lambda))$ is obtained from the calibration certificate along with the covariance matrix and eventually the data of the Monte Carlo simulation performed by the calibration laboratory. The spectral input are correlated therefore a multivariate Gaussian PDF must be taken into account.

For the uncertainty of **the drift of the filter radiometer spectral responsivity u(δs(λ))** Uniform PDF are assigned to the drift of each spectral value that are considered uncorrelated. For Monte Carlo simulations, one has to use a set of randomly drawn numbers for each spectral data.

A summary of the uncertainty components contributions for the blackbody temperature measurement is shown in Table 2.

Table 2: Uncertainty components for the measurement of the blackbody temperature

Type of variable	Uncertainty components	PDF type	Correlation	Comments	Effect on Monte Carlo simulation
Scalar	Filter radiometer signal readings	Student's t	No		One set of random values
Scalar	Emissivity	Gaussian	No		One set of random values
Scalar	Amplifier gain	Gaussian	No		One set of random values
Scalar	Drift of amplifier gain	Uniform	No		One set of random values
Scalar	Alignment of the blackbody $u(\cos\alpha_1)$	Gaussian	No		One set of random values
Scalar	Alignment of the filter radiometer $u(\cos\alpha_2)$	Gaussian	No		One set of random values
Scalar	Aperture area	Gaussian	No		One set of random values
Scalar	Aperture contamination	Uniform	No		One set of random values
Scalar	Distance	Gaussian	No		One set of random values
Column vector	Filter radiometer spectral responsivity	Gaussian multivariate	Yes	Integral of spectral data	If a NxN correlation matrix or covariance matrix is provided together with the N spectral data points, the multivariate drawing for the spectral input values has to be based on the provided covariance matrix
Column vector	Drift of filter radiometer spectral responsivity	Uniform	No	Integral of spectral data	One set of random values for each wavelength

4.3.2 Uncertainty of the blackbody spectral irradiance

With the temperature of the blackbody-radiator known, its spectral irradiance at the sphere's entrance port of the measurement setup can be calculated as follows:

$$E(\lambda, T_{BB}) = \epsilon \cdot \cos\alpha_1 \cdot \cos\alpha_2 \cdot \frac{A_{BB} + \delta A_{BB}}{d^2} \cdot \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \cdot \frac{1}{exp\left(\frac{c_2}{n \cdot \lambda \cdot (T_{BB} + \delta T_{BB} + \Delta T_{BB})}\right) - 1} \quad 41$$

The quantities used in this equation are

$E(\lambda, T_{BB})$	Calculated spectral irradiance of the blackbody-radiator
ϵ	Emissivity of the blackbody radiator
$\cos\alpha_1$, $\cos\alpha_2$	Alignment of the integrating sphere opening to the optical axis of the black body
A_{BB}, δA_{BB}	Size and its drift of the black body opening aperture
d	Distance of the integrating sphere opening to the black body aperture
λ	Calculated wavelength
T_{BB}	Radiometric temperature of the blackbody radiator
δT_{BB}	Correction for blackbody temperature drift during measurement
ΔT_{BB}	Correction for blackbody nonuniformity
c_1, c_2	Planck constants
n	Refractive index of air

The input quantities have the following uncertainty contributions and correlations for the calculated spectral irradiance:

In equation $U_{FB}(T_{BB}) = \epsilon \cdot (V_{iU} + \delta V_{iU}) \cdot \cos\alpha_1 \cdot \cos\alpha_2 \cdot \frac{A_{BB} + \delta A_{BB}}{d_{FD}^2} \int (s(\lambda) + \delta s(\lambda)) \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \frac{1}{exp\left(\frac{c2}{n \cdot \lambda \cdot T_{BB}}\right) - 1} d\lambda$

40 the blackbody temperature is correlated with the emissivity, the aperture and the aperture drift due to contamination. However the emissivity, the aperture and the aperture drift due to contamination are not correlated Therefore a joint multivariate Gaussian PDF should be considered along with a 4 x 4 covariance matrix or correlation coefficient matrix. For the correlation coefficient matrix the values of the elements are:

- The diagonal elements are equal to "1"
- $r(\epsilon, A_{BB}) = r(\epsilon, \delta A_{BB}) = 0$
- $r(T_{BB}, \epsilon)$, $r(T_{BB}, A_{BB})$ and $r(T_{BB}, \delta A_{BB})$ can be evaluated from the curve of equation $U_{FB}(T_{BB}) = \epsilon \cdot V_{iU} \cdot \frac{A_{BB}}{d_{FD}^2} \int s(\lambda) \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5} \frac{1}{exp\left(\frac{c2}{n \cdot \lambda \cdot T_{BB}}\right) - 1} d\lambda$35 that determines the blackbody temperature T_{BB} as a function of the filter radiometer signal $U_{FB}(T_{BB})$. A change $\Delta\epsilon$ in ϵ, ΔA_{BB} in A_{BB} or $\Delta\delta A_{BB}$ in δA_{BB} produces a change in $U_{FB}(T_{BB})$ and therefore a change in T_{BB} ($\Delta T_{BB}(\epsilon)$, $\Delta T_{BB}(A_{BB})$ or $\Delta T_{BB}(\delta A_{BB})$) , and then the correlation coefficients are approximated by:

$$r(T_{BB},\epsilon))\approx\frac{u(T_{BB})\cdot\Delta\epsilon}{u(\epsilon)\cdot\Delta T_{BB}(\epsilon)}$$
$$r(T_{BB},A_{BB})\approx\frac{u(T_{BB})\cdot\Delta A_{BB}}{u(A_{BB})\cdot\Delta T_{BB}(A_{BB})} \qquad 42$$
$$r(T_{BB},\delta A_{BB}))\approx\frac{u(T_{BB})\cdot\Delta\delta A_{BB}}{u(\delta A_{BB})\cdot\Delta T_{BB}(\delta A_{BB})}$$

During the calibration of a standard lamp, although the blackbody temperature is determined separately for each wavelength setting, a slight **blackbody temperature drift δT_{BB}** might occur. It is considered to be zero with an uncertainty u(δT_{BB}) uncorrelated for all wavelengths. Therefore, in Monte Carlo simulations, one has to use different sets of randomly drawn numbers for simulation of blackbody temperature drift δT_{BB} for each wavelength of the spectral irradiance of the blackbody.

The spectral radiance of the blackbody comes with a slight non-uniformity. The effect can be expressed by a temperature non-uniformity over the blackbody emission area and thus the **correction for the blackbody non-uniformity ΔT_{BB}** is the average temperature correction. It is considered to be zero with an uncertainty of 0.15 K fully correlated for all wavelengths. Therefore, in Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of blackbody temperature drift δT_{BB} for every wavelengths of the spectral irradiance of the blackbody.

The two components **cos α_1 and cos α_2 of the alignment of the blackbody and the entrance optics** are considered uncorrelated and with a standard uncertainty u(cos α_1) and u(cos α_2) . However each component is fully correlated for all wavelengths. Therefore, in Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of **cos α_1** for every wavelength of the spectral irradiance of the blackbody and to use the same set of randomly drawn numbers for simulation of **cos α_2** for every wavelength of the spectral irradiance of the blackbody. A Gaussian PDF is assigned to these uncertainty components.

The uncertainty for the **distance between blackbody opening and entrance optics *d*** is split into two parts:

- Uncertainty u(d_{cal}) for the gauge calibration obtained from the calibration certificate
- Uncertainty u(d_{meas}) for the measurement of the distance with the gauge

These two contributions are uncorrelated and should be added before using the measurement model. Gaussian PDF are assigned to both uncertainties. Therefore, in Monte Carlo simulations, one has to use a set of randomly drawn numbers for simulation of the d_{cal} and d_{meas} and applied for every wavelength.

The **wavelength *λ*** has no associated uncertainty because it is used as a nominal calculation parameter.

A summary of the uncertainty components contributions for the blackbody spectral irradiance calculation is shown in Table 3.

Table 3: Uncertainty evaluation of the blackbody spectral irradiance

Type of variable	Uncertainty components	PDF type	Correlation	Comments	Effect on Monte Carlo simulation
Column vector	Blackbody temperature, Emissivity, Aperture size, Aperture contamination	Gaussian multivariate	Correlated	Non-spectrally dependant	The multivariate drawing for the spectral output values has to be based on the provided 4x4 covariance matrix
Scalar	Blackbody temperature drift	Gaussian	No	Spectrally dependant	Random values are generated once for and applied to all spectral data
Scalar	Alignment of blackbody X direction	Gaussian	Fully correlated	Non-spectrally dependant	Random values are generated once for and applied to all spectral data
Scalar	Alignment of blackbody Y direction	Gaussian	Fully correlated	Non-spectrally dependant	Random values are generated once for and applied to all spectral data
Scalar	Distance of the blackbody to the entrance of integrating sphere	Gaussian	Fully correlated	Non-spectrally dependant	Random values are generated once for and applied to all spectral data

Using Monte Carlo simulation for the evaluation of the uncertainty allows to determine the covariance matrix of the spectral irradiance of the blackbody according to 3.4.3.

4.3.3 Uncertainty of the spectral irradiance of the standard lamp

The blackbody and the lamps under test, are measured in nearly identical optical configurations of the system in different successive measurement cycles covering each the whole wavelength range. It is recommended to have on the set-up means to evaluate the stability of the blackbody emission during each measurement cycle (monitor lamp for PTB set-up, monitor pyrometer for Cnam set-up).

The spectral irradiance of the standard lamp $E_{ST}(\lambda)$ is determined according to the following measurement model:

$$E_{ST}(\lambda) = E(\lambda, T_{BB}) \cdot \frac{(V_{ST}+\delta V_{ST})}{(V_{BB}+\delta V_{BB})} \cdot \frac{(G_{BB}+\delta G_{BB})}{(G_{ST}+\delta G_{ST})} \cdot C(d_{ST}) \cdot C_{wl}(\lambda) \cdot C_{bw}(\lambda) \qquad 43$$

$E_{\lambda,\mathrm{BB}}$	Calculated spectral irradiance of the blackbody-radiator
V_{ST}, V_{BB}	Measured photosignal with the standard lamp or the blackbody
δV_{ST}, δV_{BB}	Uncertainty on photosignal measurement due to DVM drift
G_{ST}, G_{BB}	Amplifier gains for standard lamp or blackbody
δG_{ST}, δG_{BB}	Uncertainty on amplifier gain drift
$C(d_{ST})$	Standard lamp distance error
$C_{wl}(\lambda)$	Wavelength calibration error
$C_{bw}(\lambda)$	Spectral bandwidth error

The input quantities have the following uncertainty contributions and correlations for the standard lamp spectral irradiance.

Spectral irradiance of the Blackbody

This uncertainty is evaluated in 4.3.2. A Gaussian multivariate PDF is associated to this uncertainty. A covariance matrix is provided as well as the Monte Carlo simulations data for each spectral irradiance of the blackbody. These data can be used directly for uncertainty calculation for the spectral irradiance of the standard lamp

Repeatability of the voltage readings of standard lamp.

Voltage readings (signal – dark signal) is the average of n measurements from which the standard deviation is determined. A student's t PDF is assigned to this uncertainty. In Monte Carlo simulation random numbers are generated for each wavelength.

Repeatability of the voltage readings of blackbody radiator

Voltage readings (signal – dark signal) is the average of n measurements from which the standard deviation is determined. A student's t PDF is assigned to this uncertainty. In Monte Carlo simulation random numbers are generated for each wavelength.

Calibration of the Digital Voltmeter

This uncertainty is obtained from the calibration certificate of the Digital voltmeter for the voltage range that corresponds to the output of the photocurrent amplifier. A Gaussian PDF is assigned to this uncertainty. Photosignals for the standard lamp and the blackbody are fully correlated. In Monte Carlo simulation a set of random numbers are generated once and applied for all wavelengths and all voltage readings.

Drift of the calibration of the Digital Voltmeter

This uncertainty is obtained from the analysis of the successive calibration certificates of the Digital voltmeter over time. A Uniform PDF is assigned to this uncertainty. The drift is accounted for the photosignals of the standard lamp and the blackbody that are fully correlated. In Monte Carlo simulation a set of random numbers are generated once and applied for all wavelengths and all voltage readings.

Calibration of the photocurrent amplifier G_{ST} for the standard lamp (correlation)

This uncertainty $u(G_{ST})$ is obtained from the calibration certificate of the current/voltage amplifier associated to the blackbody radiator. This uncertainty depends on the signal level. All measurements using the same amplifier setting are fully correlated.

In Monte Carlo simulations, one has to use the same randomly drawn numbers for simulation of the photocurrent amplification for equal amplifier settings. Different randomly drawn numbers should be generated for different amplifier settings.

Drift of the gain of the photocurrent amplifier for standard lamp

This uncertainty is obtained from the analysis of the successive calibration certificates of the current amplifier over time. A Gaussian PDF is assigned to this uncertainty. The drift is accounted for the photosignals of the standard lamp for all wavelengths that are measured with the same gain setting that are fully correlated. In Monte Carlo simulation the same set of random numbers are generated once and applied to all wavelengths that are measured with the same gain setting.

Calibration of the photocurrent amplifier G_{BB} for the blackbody radiator (correlation)

This uncertainty $u(G_{BB})$ is obtained from the calibration certificate of the current/voltage amplifier associated to the blackbody radiator. This uncertainty depends on the signal level. All measurements using the same amplifier setting are fully correlated.

In Monte Carlo simulations, one has to use the same randomly drawn numbers for simulation of the photocurrent amplification for equal amplifier settings. Different randomly drawn numbers should be generated for different amplifier settings.

Drift of the gain of the photocurrent amplifier for blackbody radiator

This uncertainty is obtained from the analysis of the successive calibration certificates of the current amplifier over time. A Gaussian PDF is assigned to this uncertainty. The drift is accounted for the photosignals of the blackbody radiator for all wavelengths that are measured with the same gain setting that are fully correlated. In Monte Carlo simulation the same set of random numbers are generated once and applied to all wavelengths that are measured with the same gain setting.

Uncertainty due to wavelength measurement uncertainty (correlation)

This uncertainty is evaluated similarly as the uncertainty evaluation for the spectral responsivity of a detector in part 3.4.1.

As the correction factor C_{wl} is in the measurement model it is expected to be close to unity, its uncertainty $u(C_{wl})$ can be estimated by:

$$u(C_{wl}) = 1 - C_{wl} = 1 - \frac{1 - \frac{E'_{ST}(\lambda)}{E_{ST}(\lambda)} u(\lambda)}{1 - \frac{E'(\lambda, T_{BB})}{E(\lambda, T_{BB})} u(\lambda)} \quad 44$$

where

$u(c_{wl}(\lambda))$	Contribution of the wavelength uncertainty for the spectral irradiance
$u(\lambda)$	Uncertainty of the measured wavelength
$E'_{ST}(\lambda)$	First derivative of the spectral irradiance of standard lamp
$E'(\lambda, T_{BB})$	First derivative of the spectral irradiance of the blackbody
$E_{ST}(\lambda)$	Spectral irradiance of standard lamp
$E(\lambda, T_{BB})$	Spectral irradiance of the blackbody

The correlation of wavelength uncertainty has three contributions:

- Uncertainty due to the calibration of the wavelength scale. This contribution is partially correlated (see CIE 198-SP1.4:2011). For Monte Carlo simulations, one has to use the multivariate distribution with the correlation coefficient matrix defined in CIE 198-SP1.4.
- Uncertainty on the ambient temperature that leads to a fully correlated contribution to spectral data. For Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of the wavelength variation for every wavelength setting if the spectral responsivity of the detector is to be specified for more than one wavelength.
- Uncertainty due to a shift of the wavelength scale (especially for scanning instruments) which is fully positive or negative correlated depending on the slope of the spectral responsivity. For Monte Carlo simulations, one has to use the same set of randomly drawn numbers for simulation of the wavelength variation for every wavelength setting if the spectral responsivity of the detector is to be specified for more than one wavelength.

Uncertainty due to spectral bandwidth

This uncertainty is evaluated similarly as the uncertainty evaluation for the spectral responsivity of a detector in part 3.4.1.

$$u\left(c_{bw}(\lambda)\right) = \frac{2 \cdot \left(b(\lambda) - a(\lambda)\right) \cdot \Delta\lambda \cdot u(\Delta\lambda)}{\left(1 - \Delta\lambda^2 \cdot b(\lambda)\right)^2} \quad 45$$

where:

$$a(\lambda) = \frac{1}{12} \cdot E''(\lambda, T_{BB}) \Big/ E(\lambda, T_{BB}) \quad 46$$

$$b(\lambda) = \frac{1}{12} \cdot E''_{ST}(\lambda) \Big/ E_{ST}(\lambda) \quad 47$$

For Monte Carlo simulation one has to consider two cases:

- The wavelengths that are generated with the same combination of grating and slit width of the monochromator. The bandwidth corrections are fully correlated. The off-diagonal elements of the correlation coefficients matrix are equal to "1".
- For wavelengths that are generated with combinations that differ from the others by the grating or slit width, the corresponding bandwidth corrections are uncorrelated. The off-diagonal elements of the correlation coefficients matrix are equal to "0".

Uncertainty due to distance error (correlation)

The spectral irradiance of the standard lamp is measured at a given distance. The correction factor for distance offsets is set to unity with an uncertainty $u(C(d_{ST}))$.

In Monte Carlo simulations, one has to use the same set of random numbers for all wavelengths of the spectral irradiance of the standard lamp.

A summary of the uncertainty components contributions of the standard lamp spectral irradiance measurement is shown in Table 4.

Table 4: Uncertainty evaluation of the spectral irradiance of the standard lamp

Input data	**Uncertainty components**	**PDF type**	**Correlation**	**Comments**	**Effect on Monte Carlo simulation**
Covariance matrix or data from simulation	Blackbody spectral irradiance	Multivariate	Covariance matrix	Spectrally dependant	Gaussian Multivariate based on the provided correlation coefficient matrix or use of the simulation data for the blackbody uncertainty evaluation
Matrix column	Signal readings of black body	Student t	No	Spectrally dependant	Random values generated for each wavelength
Matrix column	Signal readings of standard lamp	Student t	No	Spectrally dependant	Random values generated for each wavelength
Column of amplifier settings	Blackbody amplifier gain calibration	Gaussian Multivariate	Fully correlated for equal settings (simple correlation matrix)	Partially spectrally dependant	If for a given wavelength a same gain is used for blackbody then correlation should be taken into account for those signals
Column of amplifier drift for gain settings	Drift of amplifier gain for blackbody	Gaussian Multivariate	Fully correlated for equal settings (simple correlation matrix)	Partially spectrally dependant	If for a given wavelength a same gain is used for the blackbody then correlation should be taken into account for those signals
Column of amplifier settings	Standard lamp gain calibration	Gaussian Multivariate	Fully correlated for equal settings (simple correlation matrix)	Partially spectrally dependant	If for a given wavelength a same gain is used for the standard lamp then correlation should be taken into account for those signals
Column of amplifier drift for gain settings	Drift of amplifier gain for blackbody	Gaussian Multivariate	Fully correlated for equal settings (simple correlation matrix)	Partially spectrally dependant	If for a given wavelength a same gain is used for the standard lamp then correlation should be taken into account for those signals

Column vector for spectral data points	Calibration of the wavelength scale, c_{wl}	Multivariate	Partially correlated (CIE 198-SP1.4:2011)	Spectrally dependant	Determination of the correlation coefficients and the associated correlation matrix.
Column vector for spectral data points	Wavelength scale temperature dependent, $c_{wl,\ T}$	Multivariate	Fully correlated	Spectrally dependant	Random numbers can be generated once and applied to all wavelengths taking into account the uncertainty associated to each wavelengths
Column vector for spectral data points	Source spectral bandwidth, c_{bw}	Multivariate	Fully correlated	Partially spectrally dependant	Correlation to be taken into account for groups of wavelengths with the same spectral bandwidth setting
Scalar	Distance, c_{dist}	Multivariate	Fully correlated	Non-spectrally dependant	This PDF is generated once and applied to all wavelengths

Using Monte Carlo simulation for the evaluation of the uncertainty on the spectral irradiance of the standard lamp allows to determine covariance matrix and/or correlation matrix using the same data analysis as described in paragraph 3.4.3.

NOTE 2: The data generated for the uncertainty evaluation of the black body temperature (4.3.1) and the spectral irradiance of the black body (4.3.2) can be used for the uncertainty evaluation of the standard lamp irradiance calibration.

NOTE 3: Spectral irradiance data are often interpolated with functions such as cubic splines, polynomials or modified Planck's radiation law. This introduces correlation in the resulting spectral irradiance. Severity of correlations is directly proportional to the data interval of the original data. The most extreme case is the use of filter radiometers to perform the calibration, where data intervals can be tens of nanometers. Correlation analysis of interpolation is described e.g. in [11].

5 References

[1] JCGM 100:2008, https://www.bipm.org/en/publications/guides/gum.html

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[3] CIE 198:2011 Determination of Measurement Uncertainties in Photometry

[4] CIE 198-SP1.(1-4):2011 Determination of Measurement Uncertainties in Photometry - Supplement 1: Modules and Examples for the Determination of Measurement Uncertainties (4 Parts)

[5] CIE 198-SP2:2018 Determination of Measurement Uncertainties in Photometry Supplement 2: Spectral measurements and derivative quantities

[6] JCGM 200:2012, https://www.bipm.org/en/publications/guides/gum.html

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[8] GUIDELINE_A2.2.1 MINIMUM SETUP SPECIFICATIONS, EMPIR project 19NRM02 RevStdLED

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[11] "Effect of correlations in fitting spectral irradiance data", S Nevas, E Ikonen, P Kärhä, and T Kübarsepp, Metrologia, Volume 41, Number 4, 246 (2004).