Good Practice Guide
for
Setting up an Uncertainty Budget for the Measurement of Luminance Distributions

Part 2

Measurement Uncertainty Contributions Originating From the Scene

This project has received funding from the EMPIR programme co-financed by the Participating States and from the European Union's Horizon 2020 research and innovation programme.

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States

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Scope

In the following, based on a simple model of a photometric measurement, it is shown how corrections or characterisation can be performed and added to the model and how the measurement uncertainty of the resulting quantity can be determined.

Reference is made to the notation and models described in the normative document (CIE 198:2011, 2011), hereafter called *CIE198*, and its supplements (CIE198-SP1.3:2011, 2010; CIE198-SP1.1:2011, 2011; CIE198-SP1.2:2011, 2011; CIE198-SP1.4:2011, 2011; CIE 198-SP2:2018, 2018) hereafter referred to as *CIE198-SP1* and *CIE198-SP2*.

The knowledge of the documents (JCGM 100:2008, 2008; JCGM 101:2011, 2011; JCGM 102:2011, 2011) and (JCGM106:2020, 2020) hereafter called *GUM*, *GUMS1*, *GUMS2* and *GUMS6* and the *CIE198*, incl. supplements, is assumed in broad outline.

The general approach based on Monte Carlo Simulations (MCS, *GUMS1*) is used to determine the measurement uncertainty so that no further explanations (partial derivatives, etc.), as used in the original *GUM,* are necessary here after the modelling and determining the model parameters.

Introduction

Measurement uncertainties play a key role in establishing comparability and metrological traceability of measurement results. Stating measurement uncertainties along with measurement results is therefore not only considered good practice but is most often a normative requirement of many measurement or application standards.

This good practice guide was written as part of the EMPIR project 19NRM01 RevStdLED to assist users in photometry in setting up measurement uncertainty calculations for their applications. The document focuses on practical guidance for situations where the available information about the measuring instrument is used and, if necessary, further estimations by an additional application-relevant characterisation are performed to allow parametrization of the measurement process, where the instrument is part of the measurement setup, to determine the measurement uncertainties. This is often the case for users of commercial measurement instruments, where only limited information about the inner workings of the instruments, especially the kind of applied corrections for significant internal and external influences, is available from the manufacturer and the user can at most perform a limited number of simple characterisation measurements. In such situations, the methods and models described in many other existing documents, especially from the CIE 198 series, can be difficult to apply, which largely motivated the writing of this document.

The objective of this document is to provide practical guidance for the estimation of measurement uncertainties for photometric measurements using largely effective models based on typical information available from calibration-/test certificates, data sheets and simple characterisation methods. The document's focus is on measurements with **I**maging **L**uminance **M**easurement **D**evices (ILMDs), but much of the provided information can be readily applied also to other photometric measurements. The target audience are users performing their measurements with calibrated commercial measurement instruments using the readings of their instruments mostly "as is", that is without applying extensive characterisations to implement corrections beyond that already internally provided by the manufacturers and covered by an uncertainty considering the critical contributions, cf. Part 1 of this GPG. Characterisation effort is limited to finding residual deviations of the corrected signals that might have application-relevant effects.

The provided information will also be useful for users working at calibration labs or NMIs. Methods to determine corrections to measurement results and their associated uncertainties are not covered in full detail by this document (cf. Part 1 of this GPG), as this would in many cases require to go beyond the described the effective models and to acquire more detailed information about the details of the processes inside a measurement instrument leading to the indicated measurement result (i.e. reading from the ILMD) and the luminance distribution to be measured (i.e. the lamp). Applying corrections that go beyond these require to parametrize a quite complex measurement model, i.e. as defined by an equivalent circuit of the pixel and signal processing block diagram. This is especially also true where corrections implemented by the manufacturer (e.g. a look up table rather than a parametrized low order function) cannot be bypassed to fully cover the parameter range or perform reverse engineering. Consequently, also the process of instrument adjustment, calibration and estimation of the associated uncertainties will not be covered in full detail.

It should also be noted that the required effort to determine the uncertainty also depends on the required quality of the measurement, i.e. not every contribution has to be taken into account if only very low requirements are placed on the uncertainty. Critical contributions from the ILMD itself and their correlation between different measurements are identified in Part 1 of this GPG.

This document (Part 2 of the GPG) describes and promotes measurement uncertainty evaluations by Monte Carlo methods. This approach is not only considered the most general and conceptually rigorous one, but, with nowadays freely available software tools, it is often also more easily implemented than a standard GUM calculation. Still, the document assumes that the reader is familiar with the basic concepts of measurement uncertainty and the standard GUM methods for measurement uncertainty evaluation.

This document is organised as follows:

Chapter 1 introduces the terminology and notation conventions usually used in measurement uncertainty calculations and photometric measurements. A very short introduction on the general concept of measurement uncertainty and standard methods for its evaluation is given.

Chapter 2 introduces a general model of evaluation that is the starting point for measurement uncertainty calculation. Based on this model and the outset of this document the workflow and governing principles of estimating measurement uncertainties is described.

Chapter 3 provides general guidance for identifying potential uncertainty contributions, setting up a corresponding uncertainty budget and establishing a hierarchy of uncertainty contribution that allows to select significant contributions that will have to be characterized in detail.

Chapter 4 introduces describes how the standard measurement uncertainty can be derived from the model of evaluation and the individual contributions from Chapter 4 using Monte Carlo methods.

1 General Notation

The notation of the quantities and models follows the *CIE198* as closely as possible. Where possible, direct reference is also made to the corresponding chapters.

According to *CIE198,* we can model a photometric measurement of a number of $k = 1 \dots N$ physical output quantities Y_k represented by a number of $l = 1 \dots n$ output values $y_{k,l}$ based on the measurement of $i = 1 \dots M$ physical input quantities X_i provided by j $= 1 \dots m$ input values $x_{i,j}$ using a general model:

$$(Y_1, \dots, Y_N) = F(X, \dots, X_M) \tag{1}$$

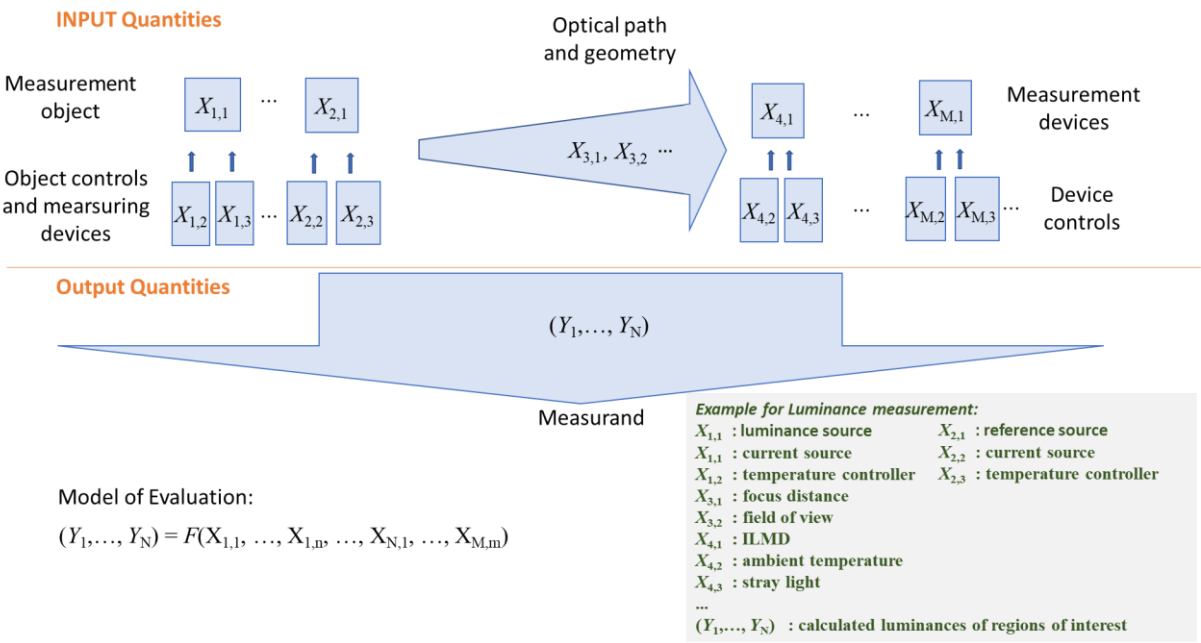


Figure 1: General modelling of the measurement by a model of evaluation

It is also possible to describe this modelling in more general terms by using vector the notation $\boldsymbol{X} = (X_1, \dots, X_M)$ and $\boldsymbol{Y} = (Y_1, \dots, Y_N)$. In this case, input and output quantities are described by $\boldsymbol{F}(\boldsymbol{X}, \boldsymbol{Y}) = \boldsymbol{0}$.

To measure a model input quantity X_i we typically collect at first multiple readings $x_{i,j}^{'}$ of the input quantity. Based on the multiple readings and possibly sometimes also on the basis of internal adjustment factors, we calculate the input quantity X_i'. However, if e.g. the environmental condition of the measurement was different from the absolute calibration condition, we have to correct X_i' to get the input quantity X_i we need in the model of evaluation for the measurement. Therefore, the quantity X_i denotes the expected input quantity.

$$X_i = c_i \cdot X_i^{'} \tag{2}$$

The use of the quantities with and without prime is used as a general concept in this document:

Primed quantities always represent the value of a quantity/indication **without correction**, as read (for measuring instruments) or as realised (for standards, i.e. under the currently given conditions). Quantities **without prime** represent **corrected** measured values (for measuring instruments) or reference

values at calibration/reference conditions (for standards or DUTs). The details are explained and applied step by step below.

According to GUM, the true value of a measured quantity is never known. The maximum knowledge we may get about a quantity, instead, is a distribution of measurement values, e.g. $x_{i,j}$, around an arithmetic mean value, $\overline{x_i}$, where the distribution of measurement values is characterized by the standard deviation, $s(x_{i,j})$, of measured values. A different set of measurement data will typically result in a different arithmetic mean value. Therefore, the associated uncertainty is defined as the experimental standard deviation of the mean value, $u(x_i) = s(\overline{x_i})$.

Also, in case of a measurement simulation, we start with the known uncertainty of an input quantity and its arithmetic mean value. Within the framework of the Monte Carlo simulation of a measurement, we generate individual measured values that deviate from the mean value by random amounts that lie within the measurement uncertainty interval of the measurand. However, if we simulate a measurement, we are able to draw (i.e. to generate) as much simulated random measurement values, x^{r}, as we want. Therefore, the arithmetic mean value of a real measurement turns into an expectation value of a probability distribution, $\mathcal{G}\big(\mu, \sigma(x)\big)$ of possible (measurement) values with an expectation value, $\mu \mathrel{\widehat{=}} \overline{x_i}$, and a standard deviation, $\sigma(x) \mathrel{\widehat{=}} u(x_i)$. The important difference between real measurements and simulated ones is, that the shape of the distribution of a real measurement is given by the distribution of the measured values while the distribution in a Monte Carlo simulation needs to be stated in advance based on the knowledge of the behaviour of the measurement process of the respective input quantity.

Further general notations used in this document:

T_{a}	ambient temperature
T_{aR}	reference ambient temperature (nominal value) • The subscript "R"[1] will always be used to state reference conditions / nominal values. • Nominal values have "A zero" uncertainty. (CIE198-SP1, 1.4).
$\Delta T_{\mathrm{a}} = T_{\mathrm{a}} - T_{\mathrm{aR}}$	temperature difference • All differences are stated as the difference of the current value minus the reference value.
$\alpha_{\mathrm{T},X}$	temperature coefficient of the quantity X
$\alpha_{\mathrm{T},X,\mathrm{rel}}$	relative temperature coefficient of the quantity X
$\mathcal{N}\big(x, u(x)\big)$	Normal distribution with expectation value x and standard deviation $u(x)$ A random number for the MCS will be named $x^{\mathrm{r}} \sim \mathcal{N}\big(x, u(x)\big)$
$\mathcal{U}(x,\Delta\mathrm{x})$	Uniform distribution over the interval $[x - \Delta x, x + \Delta x]$ A random number for the MCS will be named $x^{\mathrm{r}} \sim \mathcal{U}(x, \Delta x)$

2 Model evaluation

Measurement task:

An LMD/ILMD is calibrated with a luminance standard having a spectral distribution (SD) similar to CIE standard illuminant A. In the second step, the luminance of another luminance source, e.g. with the SD of a phosphor-type white LED, is determined. In this respect, the ILMD is an indicating device that needs to be linear and spectrally matched. (Type of calibration: Lamp calibrates lamp)

[1] This is different to *CIE198*, where the subscript "0" was used. The subscript "0" is used for the dark signal in this document.

Following an idea by Georg Sauter (Sauter, 2012), published with examples and details in (Krüger *et al.*, 2014), the measurement uncertainty budget of a measurement can be summarised very clearly in the following way and then refined step by step.

Consistent to Part 1 of the GPG, this document is written from a "luminance/photometric signal" in contrast to a "count/raw signal" perspective. This is a so-called "Black Box" approach for the complete device with respect to its indicated output quantity, meaning that we use measurement instruments with proper internal adjustment and model only minor deviations and imperfections rather than the underlaying signal processing which is covered by the estimate for critical uncertainty contributions. On the other hand, this approach means that we can usually not use equivalent models motivated by the physical implementation, but we only have to model the observations by mathematical functions that are approximating the effective characteristic.

The starting point is the model equation (noted here without the restriction of generality for luminance or luminance distributions measured with LMD's (luminance measuring devices) or ILMD's), respectively:

$$\frac{L_{\mathrm{Z}}^{'}}{L_{\mathrm{C}}^{'}} = \frac{Y_{\mathrm{Z}}}{Y_{\mathrm{C}}} \tag{3}$$

$$L'_Z = L'_{\mathrm{C}} \cdot \frac{Y_{\mathrm{Z}}}{Y_{\mathrm{C}}}$$

$$L_Z = \frac{c_{L,Z}}{c_{L,C}} L_{\mathrm{C}} \cdot \frac{c_{\mathrm{Y,Z}} Y'_{\mathrm{Z}}}{c_{\mathrm{Y,C}} Y'_{\mathrm{C}}}$$

Where

L_{Z}	luminance (distribution) of the light source to be measured (DUT)
L'_{C}	indicated luminance (distribution) of the luminance standard for calibration under the given condition
L_{C}	Luminance (distribution) of the luminance standard (Calibration certificate)
Y_{Z}	Corrected measured quantity for the luminance (distribution) of the DUT
Y_{C}	Corrected measured quantity for the luminance (distribution) of the luminance standard
Y'_{Z}	indicated quantity for the luminance (distribution) of the DUT
Y'_{C}	indicated quantity for the luminance (distribution) of the luminance standard
$c_{\mathrm{L,Z}}, c_{\mathrm{L,C}}, c_{\mathrm{Y,Z}}, c_{\mathrm{Y,C}}$	Correction factors provided by the manufacturer or derived from characterising measurements

The approach is explained step by step in deriving the individual model components. This model equation describes the absolute calibration of the measuring device and the measurement as a whole. It should also be noted that in nearly all cases, the measured values of the ILMDs are already provided as luminance values (readings). However, the physical measurement process with ILMDs is typically a counting process of collected photons and extensive signal processing which residual errors and related critical uncertainty contributions are considered in Part 1 of this GPG. This becomes especially important if linearity properties and their contributions to the MU are discussed. However, count values are often hidden in the manufacturer's software and not directly accessible by the user. How to deal with linearity will be shown in Chapter 4.7. The correction factors determined and provided by the manufacturer are typically implemented in the software of the measurement devices. In this case, the factors appear in the model of the user as unity with a given uncertainty, which may still depend on the application.

Remark: The modelling could also be done by describing the physical processes in a measuring device/light source by an equivalent circuit and a block diagram. However, this is increasingly problematic (also for the manufacturers) because not enough information about the pixel sensor is provided for this, and the measurement systems are becoming more complex. The BlackBox approach is probably the only method of describing current measuring systems and light sources. But even with this BlackBox approach, it is possible to model different levels so that, for example, the manufacturer can access further internal data that is no longer made available to the user in his modelling.

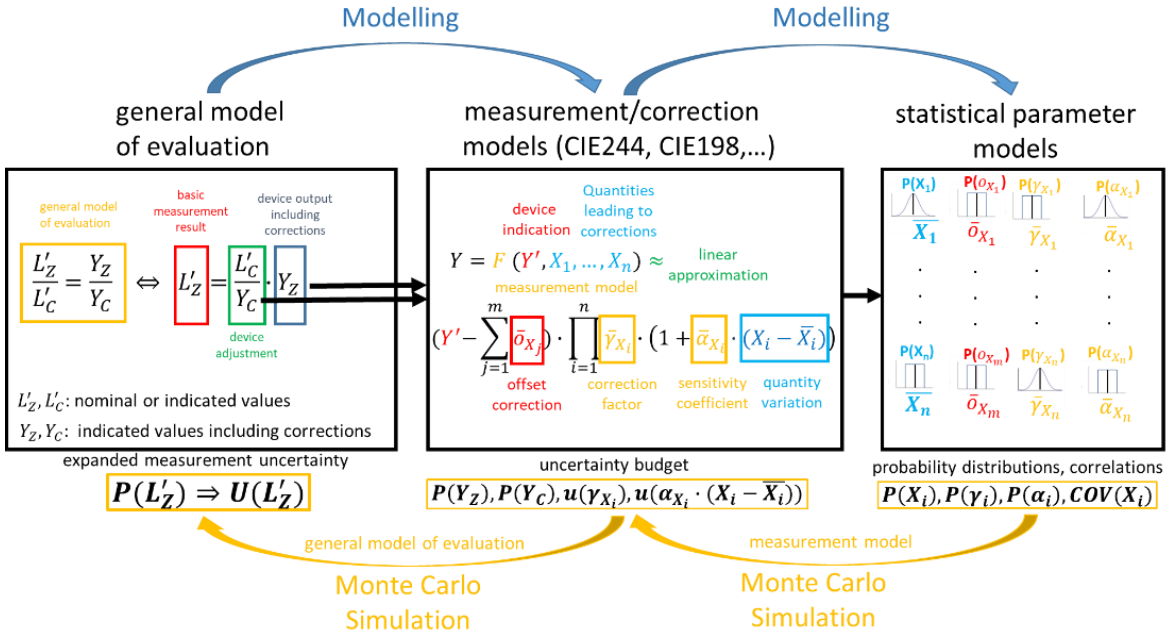


Figure 2: Detailed description of the modelling

2.1 Modelling DUT (source)

The following model can be used as a general model for measuring the light source (DUT). Whereby the Luminance L_Z under defined boundary conditions is to be determined from the measured luminance $L_Z^{'}$ under the current measurement conditions by multiplication with a correction factor.

$$L_Z = L_Z^{'}\, c_{L,Z} \tag{4}$$

This also applies analogously to the other variables from equation (3).

The correction factor can be broken down in detail step by step.

$$c_{L,Z} = \left(1 - \alpha_{T,Z}\Delta T_Z - \alpha_{\vartheta,Z}\Delta\vartheta_Z - \alpha_{J,Z}\Delta J_Z - \gamma_{U,Z} - \gamma_{t,Z} \dots\right) \tag{5}$$

The relative sensitivity coefficients are described with α, where the first letter of the index describes the variable to be changed, and the second letter represents the object (e.g. Z for the DUT, C for the calibration light source, etc.). The sensitivity is valid for the measurement quantity we observe, which is the luminance in this example. A sensitivity is connected to a second parameter, describing the change of the influencing variable. Example $\alpha_{T,Z}\Delta T_Z$: In this case $\alpha_{T,Z}$ is the sensitivity in 1/K describing the relative luminance change for every degree change of the reference temperature. And the ΔT_Z is describing that temperature change (or a possible range of temperatures with a random variable).

The values in the models depicted with γ represent model parameters without an additional scaling like temporal noise or uniformity.

- T_Z The temperature of the DUT (depending on the measurement, the ambient temperature T_a or the temperature at a specific point $T_{p,\mathrm{Z}}$ on the enclosure is relevant here).
- $\Delta\mathrm{T}_\mathrm{Z}$ Difference to the reference Temperature $\Delta T_\mathrm{Z} = T_\mathrm{Z} - T_\mathrm{ZR}$
- $\alpha_{T,\mathrm{Z}}$ Relative sensitivity for the luminance change with respect to the temperature change of the DUT
- $\alpha_{\vartheta,\mathrm{Z}}$ Relative sensitivity for luminance change with respect to angular dependence of the luminance
- $\Delta\vartheta_\mathrm{Z}$ Angular difference to the normal view
- $\gamma_{U,\mathrm{Z}}$ Non-uniformity of the DUT surface (e.g. if a different spot size with respect to the calibration condition is measured; zero degree)
- $\gamma_{t,\mathrm{Z}}$ Stability of the DUT after burn-in (remaining instability)
- $\alpha_{J,\mathrm{Z}}$ Sensitivity regarding the current setting of the device (e.g. if an external source is needed)
- ΔJ_Z Difference to the reference current $\Delta J_\mathrm{Z} = J_\mathrm{Z} - J_\mathrm{ZR}$, (e.g. if an external source is needed)

Further influencing variables can be added here depending on their relevance. In the very first step, one can start with $\mathrm{c}_{\mathrm{L,Z}} = 1$ and just estimate its uncertainty by a value covering typical characteristics.

The term noise is used here in a very general way. First of all, of course, for the description of quantities whose repeated observation allows to infer mean value, standard deviation and possibly the probability distribution function (GUM Type A). But then also for quantities about which e.g. limits are known from other sources or other information is available (GUM Type B).

For the modelling to be done here via MCS, however, the difference in the treatment during the modelling is not relevant, so that here both types of quantities are modelled equivalently. In the first case, normal distributed random variables are modelled. In the second case, one often meets with equally distributed random variables, e.g. if only the range is known. Furthermore, it does not matter for the modelling whether there is really a random process behind it or whether it is a systematic deviation/variation.

2.2 Modelling the luminance standard source

The luminance standard can generally be described similarly. However, more properties are usually known for this light source, so the modelling can be somewhat more complex:

$$\begin{aligned} L_\mathrm{C} &= L'_\mathrm{C} \cdot c_{L,C} \\ c_{\mathrm{L,C}} &= \left(1 - \alpha_{T,\mathrm{C}}\Delta T_\mathrm{C} - \alpha_{\vartheta,\mathrm{Z}}\Delta\vartheta_\mathrm{Z} - \alpha_{p,\mathrm{C}}\Delta p_\mathrm{C} - \gamma_{U,\mathrm{C}} - \gamma_{t,\mathrm{C}} \ldots\right) \end{aligned} \tag{6}$$

- $\alpha_{\mathrm{p,C}}$ Relative ageing coefficient of the luminance value
- Δp_C Accumulated time of operation since the last calibration (together with the ageing coefficient $\alpha_{\mathrm{p,C}}$, the ageing of the calibration source luminance value caused by its operation can be corrected here).

2.3 Modelling of the measurement

The determination of the luminance values Y_Z and Y_C (the actual measurements) can be described similarly, whereby the corrections and especially their uncertainties are to be applied and respected depending on the measurement technique used for the application.

$$\begin{aligned} Y_Z &= Y_Z^{'}\ c_{\mathrm{Y,Z}} \\ Y_C &= Y_C^{'}\ c_{\mathrm{Y,C}} \end{aligned} \tag{7}$$

The correction factors can be modelled as in (5) and (6) with extension based on focus, linearity and size of source properties.

$$c_{\mathrm{Y,Z}} = \left(1 - \alpha_{T,\mathrm{C}}\Delta T_{\mathrm{C}} - \gamma_{U,\mathrm{Y}} - \gamma_{t,\mathrm{C}} - Focus - Linearity - Size\ of\ Source - \ldots\right)$$

$$c_{\mathrm{Y,C}} = \left(1 - \alpha_{T,\mathrm{C}}\Delta T_{\mathrm{C}} - \gamma_{U,\mathrm{Y}} - \gamma_{t,\mathrm{C}} - Focus - Linearity - Size\ of\ Source - \cdots\right)$$

Since the measurement of the DUT and the reference light source is carried out with the same *measuring* instruments and usually under the same or at least very similar conditions, some influences of the measuring instrument and the actual measurement conditions (if measured concurrently) may cancel each other out. Therefore, during numerical modelling, special care must be taken to ensure that logically related measured values (e.g. ambient temperatures) are modelled with the same random variables to represent these correlations adequately.

2.4 Modelling interaction

In addition to modelling the measurement and the corrections described in the previous sections, the interaction between the measurement system and the measured object must also be described. Here the spectral matching and the influence of stray light are essential to be considered, whereby "stray light" is usually modelled as an offset (not viewed further here as covered in Part 1 of the GPG).

As described by (Krüger *et al.*, 2022), the spectral mismatch correction factor $F\left(\mathrm{S_C}(\lambda), \mathrm{S_Z}(\lambda)\right)$ for a photometer calibrated with a relative spectral distribution (SD) $\mathrm{S_C}(\lambda)$ measuring a different relative SD of the DUT $\mathrm{S_Z}(\lambda)$ can be written as:

$$F_{\mathrm{C,Z}} = F\left(S_{\mathrm{C}}(\lambda), S_{\mathrm{Z}}(\lambda)\right) = \frac{\int_{360\ \mathrm{nm}}^{830\ \mathrm{nm}} S_{\mathrm{Z}}(\lambda)V(\lambda)d\lambda}{\int_{\lambda_{min}}^{\lambda_{max}} S_{\mathrm{Z}}(\lambda)s_{\mathrm{rel,C}}(\lambda)d\lambda} \tag{8}$$

Attention: Compared to other equations (e.g. in (ISO/CIE 19476:2014, 2014)) the normalised spectral responsivity $s_{\mathrm{rel,C}}(\lambda)$ here is calculated from the relative spectral responsivity $s_{\mathrm{rel}}(\lambda)$ using a weighting with the relative SD of the calibration light source to make the evaluation much easier.

$$s_{\mathrm{rel,C}}(\lambda) = \frac{\int_{360\ \mathrm{nm}}^{830\ \mathrm{nm}} S_{\mathrm{C}}(\lambda)V(\lambda)\mathrm{d}\lambda}{\int_{\lambda_{\mathrm{min}}}^{\lambda_{\mathrm{max}}} S_{\mathrm{C}}(\lambda)s_{\mathrm{rel}}(\lambda)\mathrm{d}\lambda} s_{\mathrm{rel}}(\lambda) \tag{9}$$

2.5 Summary

The individual modelling steps of the previous sections can then be summarised as follows:

$$L_{\mathrm{Z}} = \mathrm{c} \cdot L_{\mathrm{C}} \cdot \frac{Y'_{\mathrm{Z}}}{Y'_{\mathrm{C}}} \quad \text{with} \quad \mathrm{c} = \frac{c_{\mathrm{L,Z}}}{c_{\mathrm{L,C}}}\frac{c_{\mathrm{Y,Z}}}{c_{\mathrm{Y,C}}}F_{\mathrm{C,Z}} \tag{10}$$

This means one gets the luminance L_Z of the DUT using the luminance of the calibration source L_C (certificate of calibration), the measurement values (readings) during the calibration/adjustment Y_C' and the measurement values Y_Z' at the time of the DUT measurement, applying a couple of corrections.

Using a different notation introducing an adjustment factor[2] k_Y:

$$k_Y = \frac{L_C}{Y'_C} \cdot \frac{1}{c_{L,C} \cdot c_{Y,C}} \tag{11}$$

$$L_z = c_S \cdot k_Y \cdot Y'_Z \quad with \quad c_S = c_{L,Z} c_{Y,Z} F_{C,Z} \tag{12}$$

In this case, the calibration/adjustment of the measurement device can be separated from the DUT measurement. However, one should consider the correlations (e.g. using the same thermometer) which make it reasonable to set up a joint model for calibration and measurement whenever possible.

However, all correction factors and adjustment factors as well as the reading of the ILMD comes with an uncertainty, which need to be determined to finally calculate the combined uncertainty of the measurement process.

Summary of overall model

$$L_Z = \frac{c_{L,Z}}{c_{L,C}} \frac{c_{Y,Z}}{c_{Y,C}} F_{C,Z} \cdot L_C \cdot \frac{Y'_Z}{Y'_C}$$

with

$$c_{L,Z} = \left(1 - \alpha_{T,Z}\Delta T_Z - \alpha_{\vartheta,Z}\Delta\vartheta_Z - \alpha_{J,Z}\Delta J_Z - \gamma_{U,Z} - \gamma_{t,Z} \dots\right)$$

$$c_{L,C} = \left(1 - \alpha_{T,C}\Delta T_C - \alpha_{\vartheta,Z}\Delta\vartheta_Z - \alpha_{p,C}\Delta p_C - \gamma_{U,C} - \gamma_{t,C} \dots\right)$$

$$c_{Y,Z} = \left(1 - \alpha_{T,C}\Delta T_C - \gamma_{U,Y} - \gamma_{t,C} - Focus - Linearity - Size\ of\ Source - \dots\right)$$

$$c_{Y,C} = \left(1 - \alpha_{T,C}\Delta T_C - \gamma_{U,Y} - \gamma_{t,C} - Focus - Linearity - Size\ of\ Source - \cdots\right)$$

$$F_{C,Z} = (1 - k\, f_1')$$

3 Collecting information

In the following chapters, we will collect the information we need for the modelling and to derive the components making up the correction factors $c_{L,C}, c_{Y,C}$ *and* $c_{Y,Z}, c_{Y,Z}$ of the measuring device and the sources step by step.

The information we need for the modelling we can get from (the order does not represent an assessment of importance):

- Calibration sheets and the calibration history (e.g. of a luminance standard)
- Literature as well as specifications (e.g. as issued by the manufacturer)
- Characterisation of measurement devices used to perform the measurement
- Quality indices of luminance meter used
- Measurements

[2] Attention: The adjustment factor is sometimes also defined in the reciprocal version. The version used in this document facilitates its application in the correction, which is then really a factor.

According to (ISO/CIE 19476:2014, 2014; CIE244:2021, 2021), several quality indices are defined for LMDs and ILMDs. These quality indices have been developed to assign characteristic values to photometers including LMDs and ILMDs to allow users to identify the capabilities of such devices under specific measurement conditions. Some of these quality indices can be used to estimate the uncertainty contribution, but they cannot be used for correction. In the following, this will be mentioned for every possible contribution.

3.1 Noise and stabilisation

For the description of the remaining instability γ_t (see equations 5 and 6), a distinction must be made between effects belonging to the light sources and those belonging to the measuring devices.

3.1.1 Light source including power supply

A light source has a slight permanent drift or instability after a specific burn-in time. The burn-in time was therefore defined in CIES025 4.4.1.:

"Specific requirement: The DUT shall be operated for at least 30 min and it is considered as stable if the relative difference of maximum and minimum readings of light output and electrical power observed over the last 15 minutes is less than 0,5 % of the minimum reading. If the DUT is pre-burned, it does not need to be operated for 30 min, and it is considered stable if the readings of the last 15 min meet above requirement."

However, no distinction is made here between noise (random instability after reaching a stable operating point) and a small remaining drift.

It is therefore recommended to examine the light to be used in detail. For luminance standards, this should be self-evident. With unknown DUTs, the situation is different.

In this guide, a relative instability factor is used to model the residual instability $\gamma_{t,\mathrm{C}}$ or $\gamma_{t,\mathrm{Z}}$. To handle it in the context of MCS, a random variable $\gamma_{t,\mathrm{C}}^{\mathrm{r}} \sim \mathcal{N}\left(0, \sigma_{t,\mathrm{C}}\right)$ is introduced with an expectation value of zero and the standard deviation $\sigma_{t,\mathrm{C}}$.

3.1.2 Measurement device

The situation is similar for the measuring instruments. Here, too, a specific warm-up time must be waited for before measurements can be started. But even then, you do not get the same measured value for every measure, but different measured values that fluctuate around an average value.

Care must be taken to ensure that the scatter of the measured values is sufficiently large and that one is not limited by the quantisation noise (Sripad and Snyder, 1977). In the following, the quantisation step is denoted as Δ_{ADC} in units of the evaluated Quantity $[Y]$.

Another point is to pay attention to temporal light modulation. Here, the frequency bandwidth settings in the measuring devices or the selected integration times may have to be adjusted accordingly. This can be checked by recording a series of measured values in close succession and examining them for mean value, standard deviation and (low/high frequency) fluctuations.

3.1.2.1 LMD

For an LMD, N measured values Y_i are to be recorded and the mean value $\bar{Y}$ and experimental standard deviation of the mean value $u(\bar{Y}) = \sigma(Y)/\sqrt{N}$ are included in the further evaluation.

Hence, in our MCS, we model the measurement value Y as the realization of a random number $Y^{\mathrm{r}} \sim \mathcal{N}\left(\bar{Y}, \sigma(Y)/\sqrt{N}\right)$, i.e. as a normal distributed number around the mean value with the uncertainty (i.e. the experimental standard deviation of the mean) used as a parameter to describe the width of the distribution.

Attention: If the noise is too low ($\sigma < \Delta_{\mathrm{ADC}}/3$), an additional contribution for the consideration of the quantization resolution has to be included ($u(\bar{Y}) = \Delta_{\mathrm{ADC}}/3$). The measurement value Y in the MCS is then the realization of a random number $Y^{\mathrm{r}} \sim \mathcal{N}(\bar{Y}, \Delta_{\mathrm{ADC}}/3)$ without an influence of the number of measurements. An alternative solution is to model with $\mathrm{Y}^{\mathrm{r}} \sim \mathcal{U}(\bar{Y}, \Delta_{\mathrm{ADC}}/2)$ in this case. The quantisation resolution Δ_{ADC} must be determined experimentally or provided by the manufacturer.

3.1.2.2 ILMD

For ILMDs, what has been said for LMDs applies analogously. The quantization noise does not usually play a role here, as the quantization resolution is usually sufficiently good. In addition to the number of measurements N, the size of the evaluation region M (macro pixel, containing M physical camera pixel) also plays a role. One, therefore, averages over $N * M$ values so that the experimental standard deviation of the mean value is reduced accordingly by the factor $1/\sqrt{N * M}$. However, it should be checked here whether an enlargement of the measurement region or an increase in the number of images taken also leads to a reduction in the experimental standard deviation of the mean value or whether correlations prevent this. For $N * M > 1000$, a noise reduction can usually no longer be achieved in practice.

3.1.2.2.1 Difference Image Method

The user of an ILMD usually finds it difficult to access the physical model parameters. These are usually also changed by the correction algorithms of the manufacturer, so it is better to make investigations with the luminance images themselves.

Here, one can work in the same way as with LMDs with short-time standard deviations, i.e. one takes a series of measured values and analyses the temporal standard deviations of the values for a defined time range.

With ILMDs, however, the difference image method can also be used. Here, two images are acquired quickly after each other, and the difference image is determined. The temporal noise σ_t of the ILMD measurement data can now be determined from the spatial noise σ_o in the difference image, where $\sigma_t = \sigma_o/\sqrt{2}$. The $\sqrt{2}$ comes from the difference image. The difference image represents a random number based on the difference of two random numbers with the standard deviation σ_t. Therefore, the difference image itself has a standard deviation of $\sqrt{2}\sigma_t$.

Using this approach, we will get for the signal to be modelled by MCS: $Y^{\mathrm{r}} \sim \mathcal{N}\left(\bar{Y}, \sigma_o/\sqrt{2}\right)$

3.1.2.2.2 Photon Transfer Method

The standard deviation can be estimated by measurements or based on the physical properties of the sensor (assuming that the camera electronics itself is not the limiting factor). This modelling is usually done by the photon transfer method (PTM, (Janesick, Klaasen and Elliott, 1985)) and described in detail in (EMVA, 2016).

A very short summary for a physical model using the signal Y as direct ADC counts: In an image sensor, the incident photons are converted into electrons, which can be read out in very different ways. In principle, however, due to the Poisson distribution of shot noise generated by the electron flow, one obtains the relationship that the variance of the signal shot noise σ_{Y}^2 corresponds to the mean value $\bar{Y}$ of the signal shot noise. From this relationship, the model parameters system transmission factor, k_{sys}, and dark signal noise, σ_0, can then be derived. A further refinement of this modelling can be found in (EMVA, 2016), hereafter denoted as *EMVA1288*.

This results in the following for the modelling of the signal noise for a single pixel $M = 1$ in a single capture $N = 1$:

$$\sigma^2(Y) = k_{\mathrm{sys}} Y + \sigma_0^2 \tag{13}$$

Using multiple image captures and or larger regions, the noise can be reduced accordingly.

$$\sigma(\bar{Y}) = \sqrt{\frac{k_{\mathrm{sys}} Y + \sigma_0^2}{M * N}}$$

The signal for the MCS can be modelled $Y^{\mathrm{r}} \sim \mathcal{N}\left(\bar{Y}, \sqrt{\frac{k_{\mathrm{sys}} Y + \sigma_0^2}{M * N}}\right)$

Attention: This is only true for the ADC counts of the conversion process. For the luminance readings, which the user usually only has access to, several other effects also play a role (e.g. dark signal, defect pixel, and shading correction) so that the relationship mentioned above only represents the lower noise limit.

3.2 Adjustment factor

Using an adjusted instrument, we have an adjustment factor $\mathrm{k_Y}$ and its uncertainty. That means we can use the luminance reading of our measurement device $Y_{\mathrm{C}}^{'}$ and apply the adjustment factor:

$$L'_{\mathrm{C}} = k_{\mathrm{Y}} Y'_{\mathrm{C}} = \mathrm{Y_C} \tag{14}$$

In our MCS, we model the adjustment factor as the realization of a random number $k_{\mathrm{Y}}^{\mathrm{r}} \sim \mathcal{N}\left(k_{\mathrm{Y}}, u(k_{\mathrm{Y}})\right)$

There are several possibilities for getting information about the adjustment factor:

- Using a adjusted instrument with a calibration sheet, one can assume that one can use $k_{\mathrm{Y}}^{\mathrm{r}} \sim \mathcal{N}(1, U(k_{\mathrm{Y}})/2)$. That means we assume the factor is one, and we use half of the expanded measurement uncertainty stated in the calibration sheet of the luminance meter as standard deviation.
- Using a known adjustment factor (usable if one adjusts in the in-house laboratory). In this case, one can use the adjustment factor for the luminance meter as measured and its uncertainty $k_{\mathrm{Y}}^{\mathrm{r}} \sim \mathcal{N}\left(k_{\mathrm{Y}}, u(k_{\mathrm{Y}})\right)$.
  Attention: Do not apply the factor twice in this case!
- Using the information from the characteristic value initial adjustment index f_{adj}. $k_{\mathrm{Y}}^{\mathrm{r}} \sim \mathcal{N}\left(1, \sqrt{f_{\mathrm{adj}}^2 + u^2\left(f_{\mathrm{adj}}\right)}\right)$.

Using the characteristic value:

The initial adjustment index, f_{adj}, (typical or individual value from the luminance meter) (ISO/CIE 19476:2014, 2014) is defined as (with the notation of this document):

$$f_{\mathrm{adj}} = \left|\frac{Y}{Y_0} - 1\right| = \left|\frac{Y_{\mathrm{C}}}{L'_{\mathrm{C}}} - 1\right| \tag{15}$$

From the practical point of view, the value itself should be zero after the initial adjustment process. But the uncertainty $\mathrm{u}\left(f_{\mathrm{adj}}\right)$ has to be stated, too and can be used in a measurement budget.

3.3 Temperature Dependence

Influence of ambient or device temperature change. Here we can use the same procedures for light sources and measurement devices. The explanation will be done with the luminance of a light source.

$$\mathrm{L} = c_\mathrm{T}\mathrm{L}' \tag{16}$$

The temperature dependence can be modelled generally according to[3]

$$\mathrm{X} = c_\mathrm{T}\mathrm{X}' = (1 - \alpha_\mathrm{T}\Delta T_\mathrm{a})\mathrm{X}' \tag{17}$$

The relative temperature coefficient α_T and its uncertainty should be estimated by a linear regression model (see 4.1.1).

1. In the case that the data for α_T and ΔT_a are only typically known and may not be corrected, c_T must be set to 1, and the typical information for α_T and ΔT_a must be integrated into the MU of c_T.

   $$c_\mathrm{T} = \mathcal{U}\left(1, \left|\alpha_\mathrm{T}\Delta T_\mathrm{a,max}\right|\right) \tag{18}$$

   Where $\Delta T_\mathrm{a,max}$ is the maximal temperature deviation, e.g. based on a tolerance interval ($\Delta T_\mathrm{a,max} = 1.2\ K$ in *CIES025*, see 4.2.2).

2. One source of information, in this case, is the quality index $f_\mathrm{6,T}$. This index states the absolute value of 10 times the relative temperature coefficient. Therefore, one gets:

   $$c_\mathrm{T} = \mathcal{U}\left(1, \left|\frac{1}{10} f_\mathrm{6,T}\Delta T_\mathrm{a,max}\right|\right) \tag{19}$$

3. In the case that α_T and ΔT_a are known (with expectation value and MU), one can generate the random numbers for $\alpha_\mathrm{T}^\mathrm{r} = \mathcal{N}\left(\alpha_\mathrm{T}, u(\alpha_\mathrm{T})\right)$ and $\Delta T_\mathrm{a}^\mathrm{r} = \mathcal{N}\left(\Delta T_\mathrm{a}, u(\Delta T_\mathrm{a})\right)$ and use (17) for the MCS.

Using the characteristic value:

The quality index, $f_\mathrm{6,T}$, describing the temperature dependence of the photometer is defined as:

$$f_\mathrm{6,T} = \left|\frac{Y(T_2) - Y(T_1)}{Y(T_\mathrm{R})}\frac{\Delta T}{T_2 - T_1}\right| \tag{20}$$

With $T_2 = 40°\mathrm{C}$, $T_1 = 5°\mathrm{C}$, $T_\mathrm{R} = 20°\mathrm{C}$ and $\Delta T = 10°\mathrm{C}$. This means $f_\mathrm{6,T}$ represents the absolute value of 10 times the relative temperature coefficient α_T.

3.4 Ageing information

Information about the ageing of sources and detectors can only be obtained through many years of experience and the evaluation of calibration certificates (for example, see 4.5). Some indications can also be found in the literature.

- The ageing of light sources is usually described as a function of the operating hours.
- The ageing of detectors is usually described as a function of a lifetime (time since the last calibration).
- Controlled standards, common for luminance standards, can be an exception here, as the ageing of the detector is usually the dominant variable.

If no information is available from your calibration certificates, you can ask the manufacturer or use the typical values from the literature.

Example values from literature:

[3] The sign in this equation is different from CIE198-SP1:1.4.

- *CIE198-SP1.2* 2.8 (ageing of a luminous intensity standard lamp depending on the operation time) → 0.0007 1/h
- *CIE198-SP1.2* 2.9 (ageing of a luminance meter depending on the time since the last calibration) → 0.002 1/year

In addition, ageing is typically spectrally nonuniform. This may affect the calibration of spectroradiometers and light sources for spectral radiance/irradiance.

3.5 Spectral mismatch

A photometer should be matched to the $V(\lambda)$-function. If this match is not ideal and the SD of the DUT differs from the SD of the light source used for calibration a correction may be necessary or the correction is set to one and the possible spectral mismatch correction factor is used to determine the measurement uncertainty of the spectral mismatch.

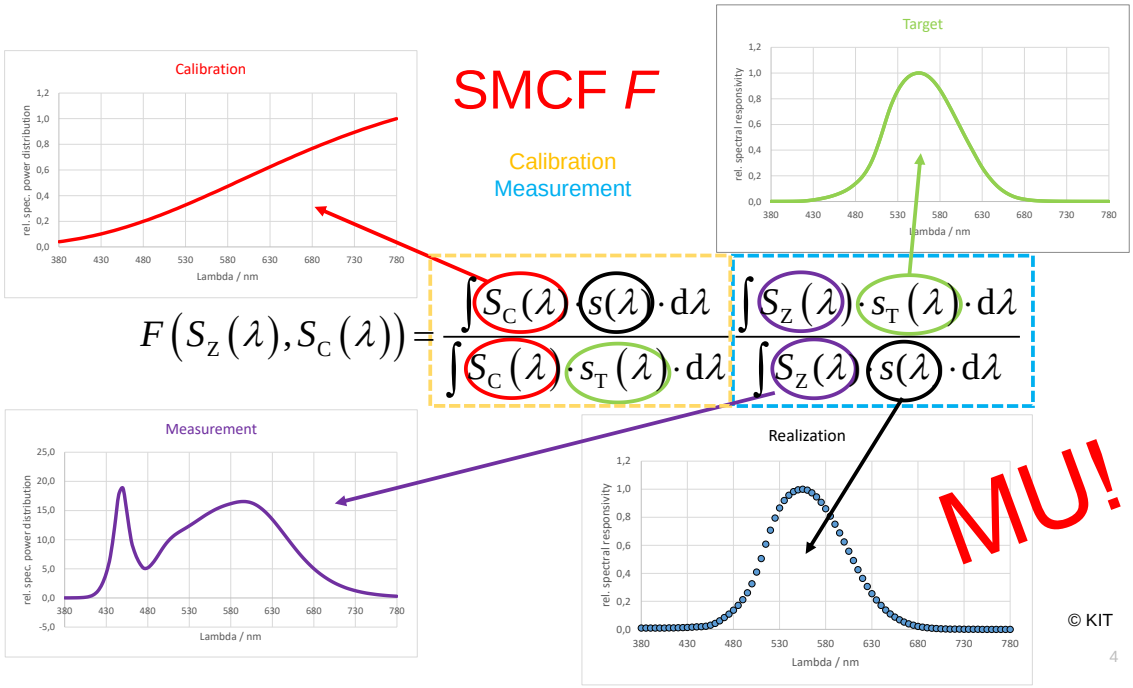


Figure 3: Calculation of the Spectral Mismatch Correction Factor (SMCF)

We have to deal with the calibration and the measurement state, and we have to describe the MU of integrated quantities, which was investigated in this project for the conference paper (Krüger *et al.*, 2023), which will be submitted to the peer review journal "Lighting Research & Technology".

A more general way to calculate the SMCF:

$$F\left(S_Z(\lambda_Z),S_C(\lambda_C)\right)=\frac{F_{C,N}}{F_{C,D}}\frac{F_{M,N}}{F_{M,D}}$$

$$F_{C,N}=\int S_C(\lambda_C)s(\lambda_s)d\lambda$$

$$F_{C,D}=\int S_C(\lambda_C)s_T(\lambda)d\lambda$$

$$F_{M,N}=\int S_Z(\lambda_Z)s_T(\lambda_s)d\lambda$$

$$F_{M,D}=\int S_Z(\lambda_Z)s(\lambda_s)d\lambda$$

$$F\left(S_{\mathrm{Z}}(\lambda_{\mathrm{Z}}), S_{\mathrm{C}}(\lambda_{\mathrm{C}})\right) = \frac{\int S_{\mathrm{C}}(\lambda_{\mathrm{C}}) s(\lambda_{\mathrm{s}}) \mathrm{d}\lambda}{\underbrace{\int S_{\mathrm{C}}(\lambda_{\mathrm{C}}) s_{\mathrm{T}}(\lambda) \mathrm{d}\lambda}_{\text{Calibration}}} \frac{\int S_{\mathrm{Z}}(\lambda_{\mathrm{Z}}) s_{\mathrm{T}}(\lambda) \mathrm{d}\lambda}{\underbrace{\int S_{\mathrm{Z}}(\lambda_{\mathrm{Z}}) s(\lambda_{\mathrm{s}}) \mathrm{d}\lambda}_{\text{Measurement}}}$$

Symbol	Description
$F\left(S_{\mathrm{Z}}(\lambda_{\mathrm{Z}})\right)$	SD DUT
λ_{Z}	wavelength scale for the measurement of SD DUT
$F\left(S_{\mathrm{C}}(\lambda_{\mathrm{C}})\right)$	SD Calibration
λ_{C}	wavelength scale for the measurement of SD Calibration
$s(\lambda_{\mathrm{s}})$	the (rel.) spectral responsivity of the detector
λ_{s}	wavelength scale for the spectral responsivity measurement
$s_{\mathrm{T}}(\lambda)$	the spectral responsivity of the target function (e.g. $V(\lambda)$)
λ	nominal wavelength scale

Using the characteristic value:

The general $\mathrm{V}(\lambda)$ mismatch index, f_1', uses a general calculation not directly related to the spectral mismatch correction factor. However, as demonstrated in (Krüger *et al.*, 2022), one can use a statistical method to analyse the relationship between the index and the spectral mismatch correction factor.

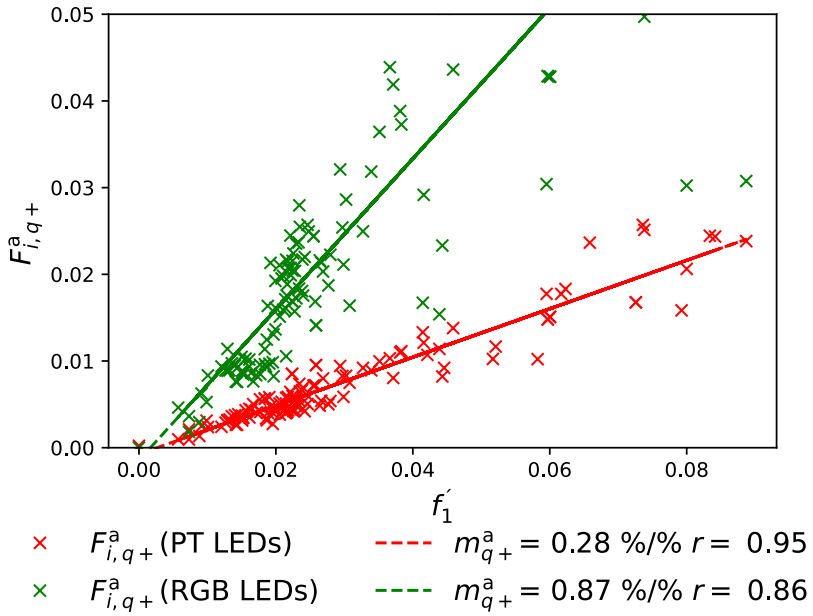


Figure 4: Relation between the absolute deviation due to spectral mismatch and the f_1' value for phosphore type white LEDs and RGB-based white LEDs. (Krüger *et al.*, 2022)

The quantity $F^{\mathrm{a}}_{i,q+}$ represents the 95 % quantile (index q+) for the absolute (upper index a) deviation of the spectral mismatch correction factor minus one ($|F-1|$) for the photometer i based on a large test set of spectral distributions. The slope of the points is given by the m-values in Figure 4, whereas the r-value represents the square root of the coefficient of determination for the regression.

Example: Using this information in a measurement uncertainty budget, the spectral mismatch correction factor F can be modelled as $F_{\mathrm{PT}} \sim \mathcal{U}(1, 0.28 \cdot f_1')$ for measurements of phosphor-type white LEDs and $F_{\mathrm{RGB}} \sim \mathcal{U}(1, 0.87 \cdot f_1')$ for measurements of RGB-type white LEDs.

3.6 Linearity index, f_3

With the notation of this document, the linearity index is defined based on the relative deviation between actual value and the given value at a certain input setting:

$$f_3(Y) = \left| \frac{Y}{Y_{\max}} \frac{X_{\max}}{X} - 1 \right| \tag{21}$$

This means for the output value Y (with the full range value $Y_{\max}$ in the observed measurement range) and the corresponding input value X we calculate the value $f_3(Y)$. In this case the full range value $Y_{\max}$ is corresponding to $X_{\max}$ input value.

The linearity index f_3 is defined as:

$$f_3 = \max_{Y=0.1Y_{\max} \ldots Y_{\max}} \left[f_3(Y) \right] \tag{22}$$

Meaning that we look for the maximum $f_3(Y)$ value in a specific measurement range from 10% full range value to the full range value.

The characteristic function $f_3(Y)$ agrees with the definition of non-linearity of (CIE 237:2020, 2020):

$$N_{\mathrm{L}} = \frac{s(Z) - s(Z_{\mathrm{R}})}{s(Z_{\mathrm{R}})} \tag{23}$$

With the sensitivity $s(Z)$ and the reference sensitivity $s(Z_{\mathrm{R}})$. However, no characteristic values based on this function are defined there.

For the following, we will have a look at some properties of the $f_3(Y)$ definition above.

Properties and remarks:

- $f_3(Y_{\max}) = 0$ (This is more or less an adjustment to define a working point.)
- $f_3(Y \to 0)$ is not defined or significantly depending on the MU of the input values X. Therefore, the calculation is limited to the range $Y = 0.1Y_{\max}$ to $Y_{\max}$.

If one attributes the correction of the non-linearity of a system to multiplication with a correction factor dependent on the output level (Ferrero, Campos and Pons, 2006) then one must use

$$c_{\mathrm{NL}}(Y) = \frac{X}{Y} = \frac{L_{\mathrm{R}}}{L} \tag{24}$$

multiply so that with $X_{\max} = Y_{\max}$, $f_3(Y)$ can be written as:

$$f_3(Y) = \frac{1}{c_{\mathrm{NL}}(Y)} - 1 \tag{25}$$

Whereas L_0 is the luminance of a luminance standard or other known reference (generally X) and L the luminance measurement result for the corresponding setting L_{R} (generally Y).

Thus, with the determination of $c_{\mathrm{NL}}(Y)$ one has determined both the correction function including their uncertainty $u\left(c_{\mathrm{NL}}(Y)\right)$ and the characteristic value, whereby the measurement uncertainty and the influence of the non-linearity correction on the measurement result can also be described.

All this information is not usable in a MU budget. However, with some additional assumptions, one can estimate the influence of the non-linearity roughly.

The manufacturer and, with a few exceptions, the user can also use the information from a linearity measurement, which is needed to determine the non-linearity correction and its test, to estimate a "residual error". However, it makes sense to implement the correction not as a factor but as an offset. This offset correction must be carried out before applying further correction factors.

$$Y = c_Y\left(Y' - Y_0'\right) \tag{26}$$

4 Parameter Estimation

4.1 General Models

4.1.1 Linear Regression

Linear regression will be used often to estimate model parameters, e.g.

- relative temperature coefficients with value pairs (temperature, value)
- dark signal generation rates (integration time, signal @ dark condition)
- evaluation of high dynamic range information (integration time, signal @ light condition)

Calculating the slope α and intercept β from the measurement value pairs, the measurement uncertainty of the slope $u(\alpha)$ has to be estimated too.

Let's start with a linear model:

$$y = \alpha \cdot x + \beta \tag{27}$$

From the device characterization, we get value pairs and measurement uncertainties $(x_i, y_i), \left(u(x_i), u(y_i)\right)$. Using the standard ordinary least squares approach (OLS), we reach for the slope α:

$$\alpha = \frac{S_{xy}}{S_{xx}} \tag{28}$$

Using the standard notation of summation in this case:

$$S_{xy} = \sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) \tag{29}$$

$$S_{xx} = \sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(x_i - \bar{x}\right) \tag{30}$$

But what about $\mathrm{u}(\alpha)$? Here we have three different possibilities:

- Using the empirical standard deviation, we get from the OLS regression model.
- Calculating $\mathrm{u}(\alpha)$ using a MC-Simulation
- Using the approach of (Matus, 2005)

4.1.1.1 Empirical standard deviation

In the first approach, the empirical standard deviation, $s(\alpha)$, (every regression algorithm will return) will be used. This value describes how the model fits (independent from $u(x)$ and $u(y)$!).

$$s(\alpha) = \frac{s_y}{\sqrt{S_{xx}}} \tag{31}$$

$$s_y^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left(y_i - \beta - \alpha x_i\right)^2 = \frac{S_{xx}S_{yy} - S_{xy}^2}{(n-2)S_{xx}^2} \tag{32}$$

4.1.1.2 Monte Carlo Simulation

In the second approach, one can use the MCS. Generating random numbers $\left(\mathcal{N}(x_i, u(x_i)), \mathcal{N}(y_i, u(y_i))\right)$ and calculate the slope α in every trial. The standard deviation of α can be calculated after the simulation. This straightforward but time-consuming approach should be avoided in a complex model.

4.1.1.3 Matus

In the third case, the approach of (Matus, 2005) is used, calculating the slope's standard deviation and the measurement uncertainty information from the input data. This results in the following formulas:

$$\begin{aligned} u^2(\alpha) &= \sum_{i=1}^{n}\left(\frac{\delta\alpha}{\delta x_i}\right)^2 u^2(x_i) + \sum_{i=1}^{n}\left(\frac{\delta\alpha}{\delta y_i}\right)^2 u^2(y_i) \\ &= \sum_{i=1}^{n} c_{xi}^2 u^2(x_i) + \sum_{i=1}^{n} c_{yi}^2 u^2(y_i) \end{aligned} \tag{33}$$

$$\begin{aligned} c_{xi} &= \frac{\left(y_i - \bar{y}\right) - 2\alpha\left(x_i - \bar{x}\right)}{S_{xx}} \\ c_{yi} &= \frac{\left(x_i - \bar{x}\right)}{S_{xx}} \end{aligned} \tag{34}$$

This results in two interesting, exceptional cases:

Table 1:$u(\alpha)$ for different settings

	$u(x_i) \equiv u(x)$ $u(y_i) \equiv 0$	$u(x_i) \equiv 0$ $u(y_i) \equiv u(y)$
$u(\alpha) =$	$\frac{\sqrt{S_{yy}}}{S_{xx}} u(x)$	$\frac{1}{\sqrt{S_{xx}}} u(y)$

How to compare $u(\alpha)$ and $s(\alpha)$ (see also *GUM* H 3.2):

$u(\alpha)$:

- Is identical to the results of a MCS (standard deviation of α)
- But more efficient in the calculation (available in front of a MCS)
- Using $u(\alpha)$ equations, one can make design decisions for the experiments (usable also for General Least Square Models GLS)

$s(\alpha)$:

- Empirical standard deviation, describing how the model fits (independent from $u(x)$ and $u(y)$)

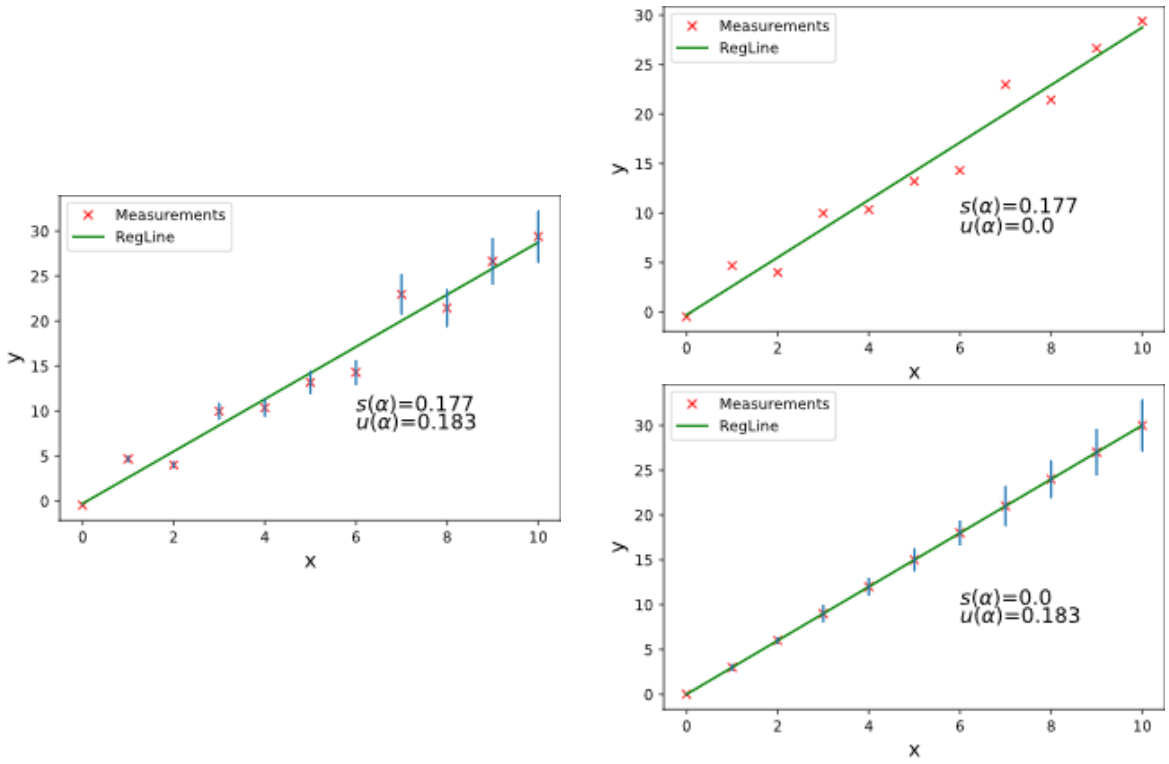


Figure 5: $u(\alpha)$ and $s(\alpha)$ for different settings

I.e. for a simple, practical approach, especially if enough points are available for the regression, one will be able to use $s(\alpha)$. Here, however, one must be aware that one does not use the measurement uncertainty of the points and must trust that due to the large number of measurement points, the realization of the random variables is implicitly included in the regression.

Above all, one must be able to rely on the fact that the mathematical model used represents the underlying physics (see, for example, the use of a polynomial function to generate a wavelength scale for a spectroradiometer from individual measurements of spectral lines).

On the other hand, Matus' approach (Matus, 2005) can be realized quickly so that one can get to the slope and its MU without MCS.

4.2 Examples

For all experiments to investigate the characteristic of ILMDs, great care should be taken to separate the dependency you want to determine from other possible influences. This is not always easy, as the following examples demonstrate.

Problems arise in particular with (some examples for luminance measurements):

- Changing the luminance without changing the spectral distribution of the light source.
- Changing the size of luminous surfaces without changing the luminance.
- Determining angular dependencies (i.e. response non-uniformity) without considering the effect of any angular (or spatial) dependence of the light source.

Furthermore, the following procedure is helpful for experiments (with examples of the temperature sensitivity of a luminance meter):

1. Draw up an essential process for the measurement.
   - What quantity is to be measured? [*luminance*]
   - Which influencing variable [*temperature of the luminance meter*] is to be changed, in which range [*15°C … 35°C*] is this to happen, and how will the change take place (values [*temperature in 5K steps*], times [*settling time about two hours*[4]], control variables [*device temperature, flow and return temperature of the climate control, additional temperature sensor*])?
   - Which critical influencing variables must be kept constant? [*luminance of the reference source*]
2. Carry out a few measurements, test the planned evaluation, and validate the original assumptions (waiting times, etc.). In any case, you should also check the reproducibility when approaching specific measuring points.
3. Generate an automatic process for the measurement and evaluation to get a higher data density.

4.3 Light source stability

The stability of the light sources is determined in so-called burn-in tests. The measurement of a luminance standard is prepared with a stable LMD/ILMD (sufficiently run-in, high repeatability is needed). Shortly after switching on the luminance standard, the measurement starts, whereby as many measurements as possible (small time interval) should be made.

From the data, one can then make specifications for the necessary burn-in time and determine the remaining residual noise for modelling.

4.3.1 Burn-In Conditions for luminance standards

The burn-in process of a luminance standard (SD CIE standard illuminant A) is illustrated in Figure 6. From the data, a necessary run-in time of 3-5 min can be derived. The "residual noise" is <2e-5, whereby at this order of magnitude, it is no longer possible to distinguish between the residual noise of the DUT and that of the measuring device.

[4] When determining temperature dependencies, long setteling times are often required so that the temperature conditions in the device stabilise. In this case, it makes sense to record measured values at short intervals (e.g. one minute) in order to observe the transient behaviour. For the evaluation of the actual measurement, the mean value of the last measurements (before setting the next temperature) can be used.

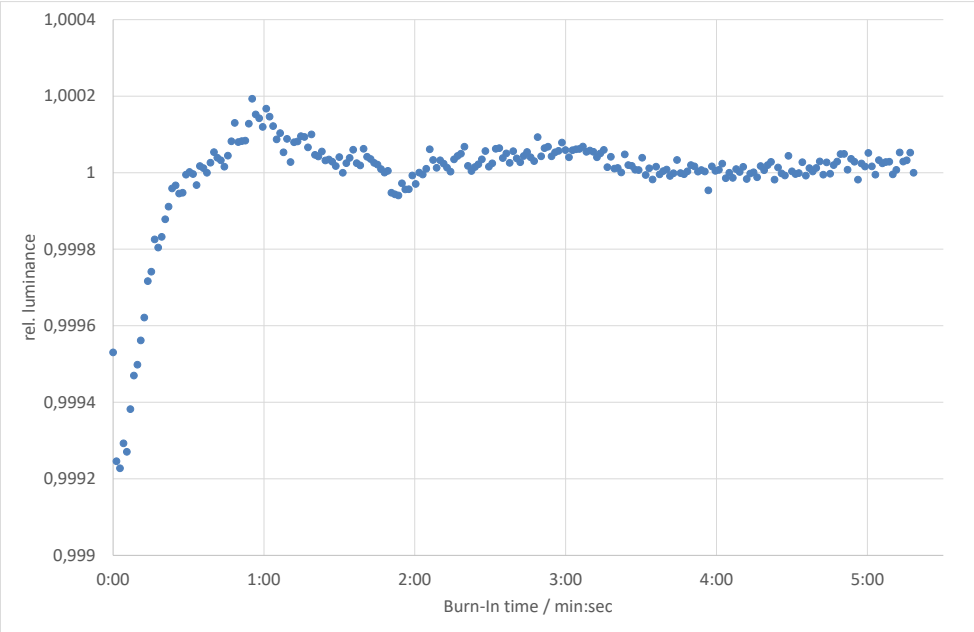


Figure 6: Burn-in results for a luminance standard

4.3.2 Burn-In Observation for DUT's

The burn-in process of a DUT (SD phosphore type LED) is illustrated in Figure 7. From the data, a necessary run-in time of only 1 min can be derived. The "residual noise" is 2.5e-5, whereby at this order of magnitude it is no longer possible to distinguish between the residual noise of the DUT and that of the measuring device.

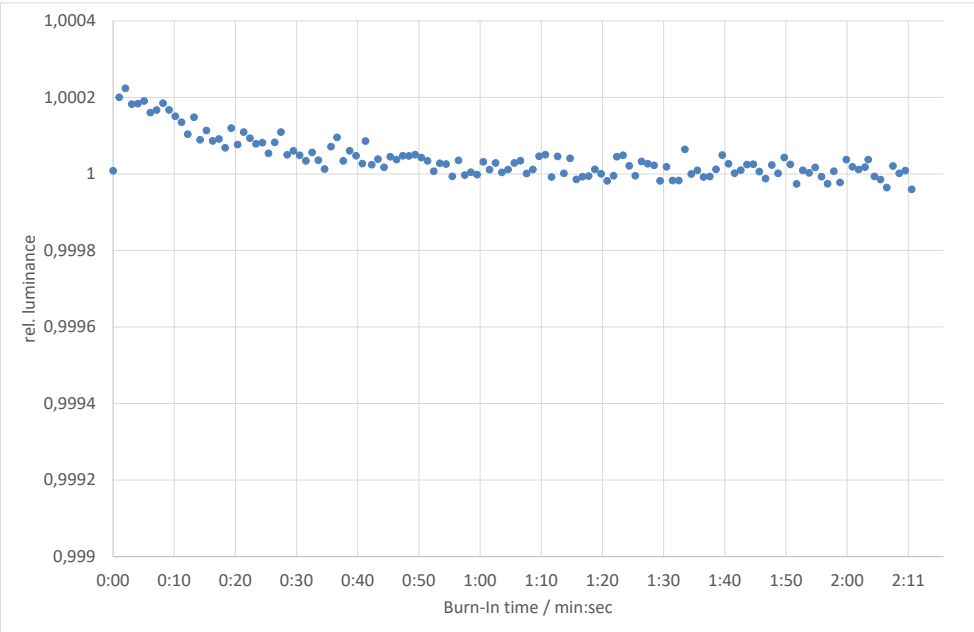


Figure 7: Burn-in results for a DUT

4.4 Temperature Coefficient

For the estimation of the temperature coefficient of a reading x' with respect to a temperature T (ambient temperature T_a or device temperature T_D) usually, a measurement in a temperature chamber is recommended for exact measurements. For rough estimations, measurements during self-heating are also possible.

The measurement design for a linear regression model (see 4.1.1) should be made over the complete temperature operating range for the device. Furthermore, the choice of temperature steps and settling times should ensure that the device under investigation is already in thermal equilibrium during the measurement. This can be guaranteed by so-called pyramid measurements, in which a specific temperature is realized both in the direction of rising and falling temperature.

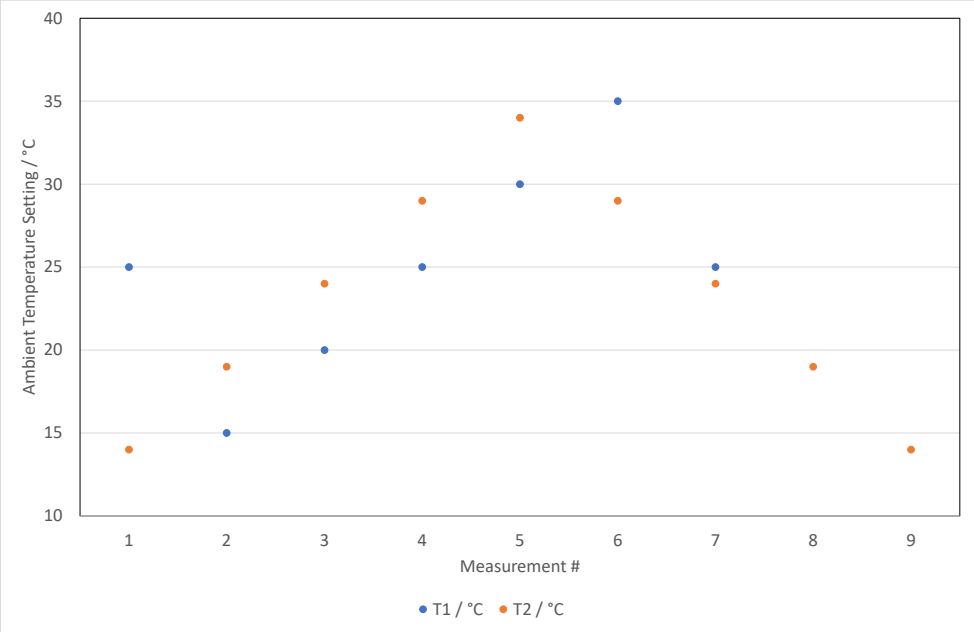


Figure 8: Temperature settings for a pyramid design (T2) and modified pyramid design (T1)

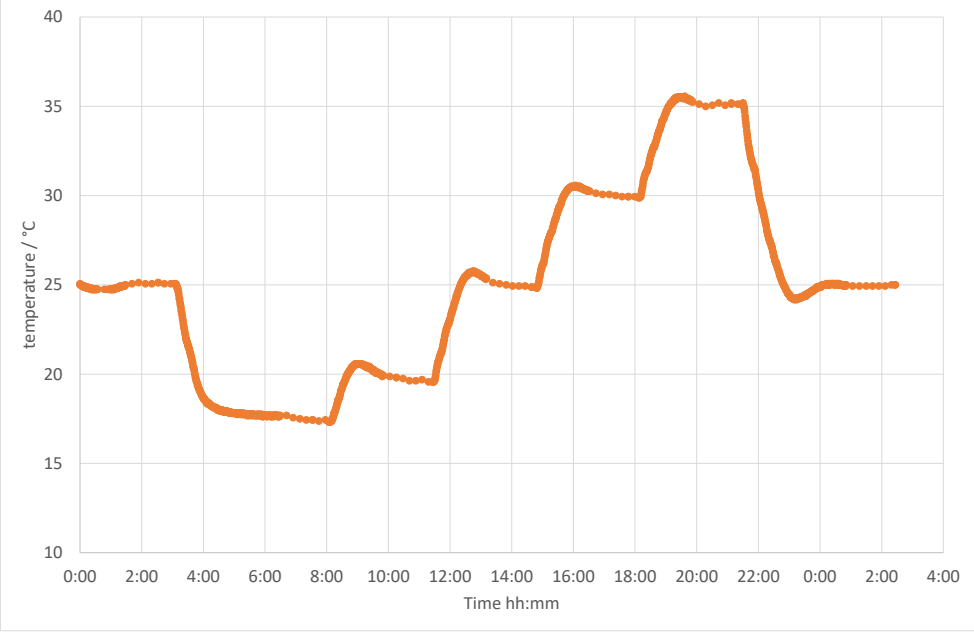


Figure 9: Example temperature profile for the T1-Design from Figure 8

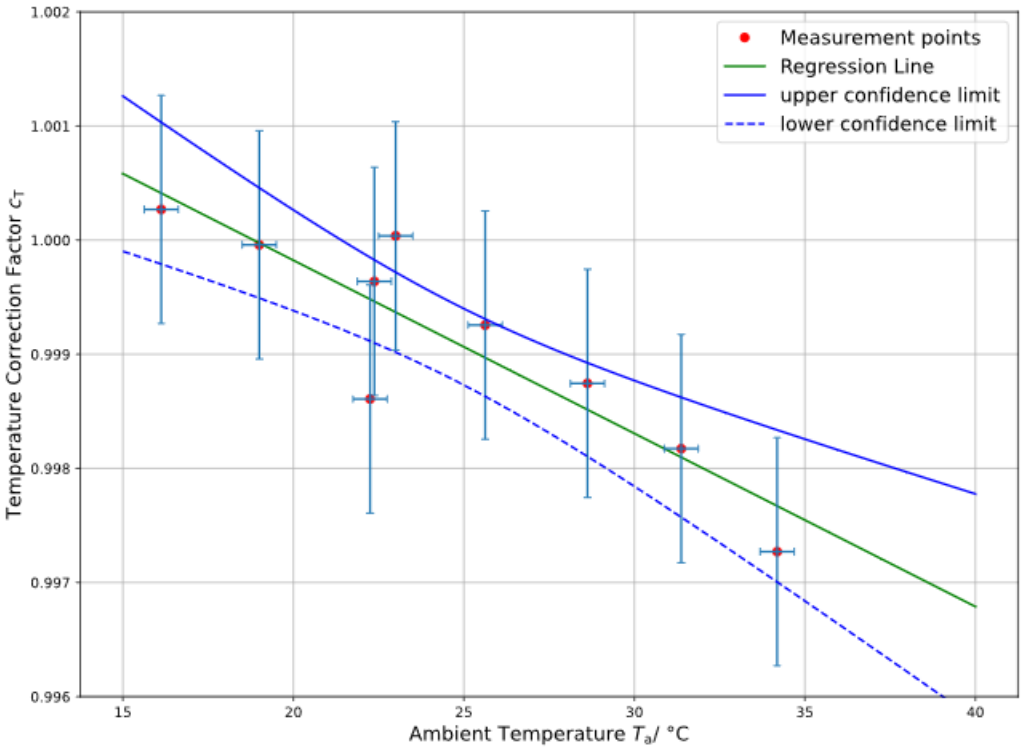


Figure 10: Regression line and confidence interval for a relative temperature coefficient evaluation

Table 2: Measurement data and results for the sample temperature coefficient

	T / °C	Y / A.U.
u(.)	0,5	0,001
α	**- 0,00015254**	
u(α) Matus	**6,095E-05**	
u(α) MCS	**6.070E-05**	
s(α)	**2,8703E-05**	
	22,25	0,998607574
	16,125	1,000268825
	19	0,999958121
	22,375	0,999638246
	25,625	0,999255214
	28,625	0,998744568
	31,375	0,998172623
	34,1875	0,997269868
	23	1,000037044

The Y values in Table 2 represent relative values only. One can normalize the reading to the reference temperature reading, the mean value or other calculations. The only precondition is that the relative values are close to 1 at the end.

For the MU of Y only the repeatability was taken into account. All fully correlated contributions do not affect the measurement uncertainty of the slope.

4.5 Ageing of a luminance standard

One can proceed similarly to describe the ageing of a luminance standard, i.e. the change in luminance between calibrations as a function of time or operating hours.

For this purpose, a table with the date of calibration, current operating hours counter, luminance and standard measurement uncertainty of the luminance is used. In the following, the luminance is normalized to the last known value from the calibration certificates.

The uncertainty in calibrations contains, at least in the case of NMI's, large correlated portions that result from the traceability of the unit. It is, therefore, not sensible in this case to use the MU of the calibrations in the regression analysis, as these would have to be modelled correlated in this case, which leads to no contribution to MU in the slope.

It may therefore make sense to use the empirical standard deviation $s(\alpha)$ from the regression as uncertainty for the slope.

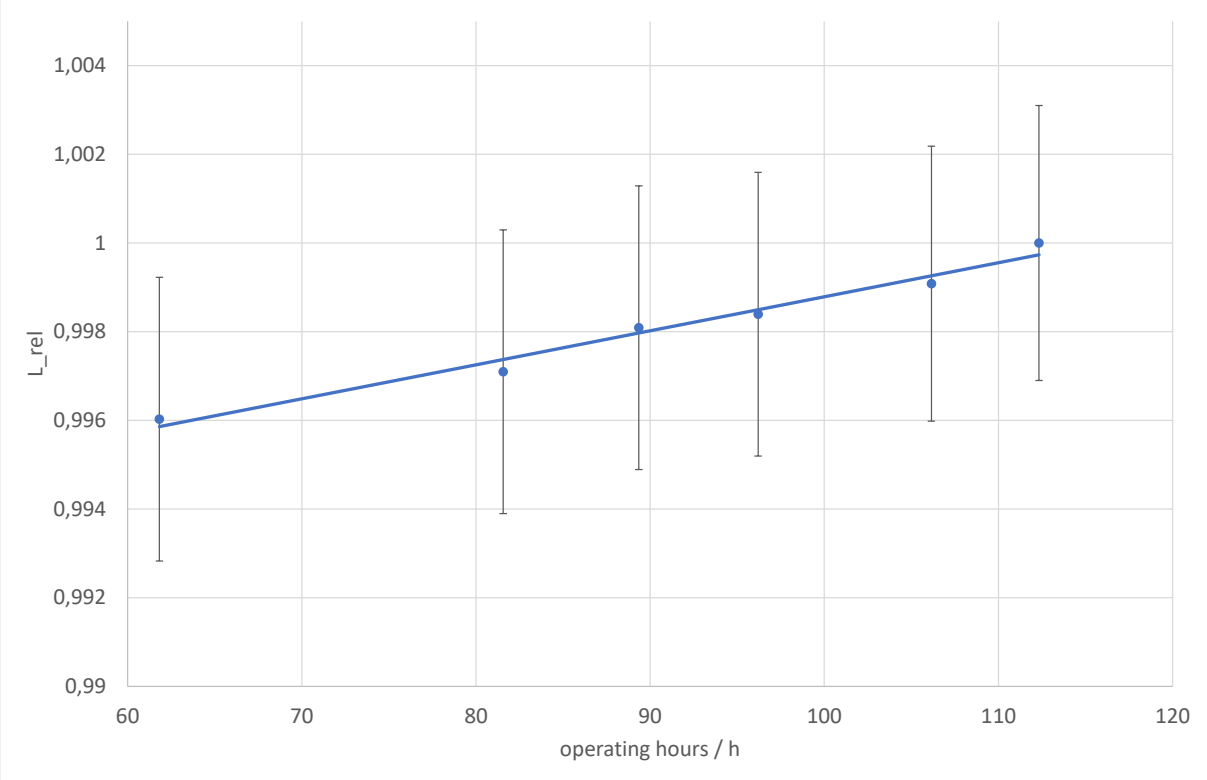


Figure 11: Ageing of a luminance standard over 50 operation hours with standard uncertainties from the calibration sheets

Table 3: Collecting data from calibration certificates of a luminance standard

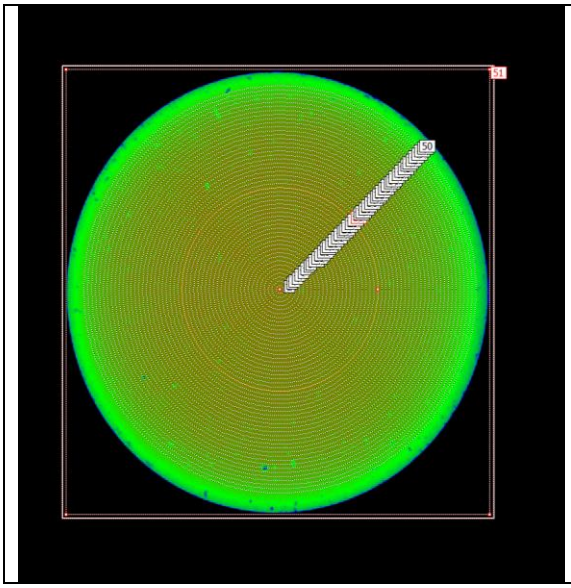
$\alpha_{p,\mathrm{C}}$	**0,0077%**	1/h	
$\mathrm{s}(\alpha_{p,\mathrm{C}})$	**0,00006%**	1/h	
r^2	0,977		
t / days	t_{op} / h	L_{rel}	$u(L_{\mathrm{rel}})$
3500	112,33	1,0000	0,0031
2788	106,15	0,9991	0,0031
2106	96,20	0,9984	0,0032
1469	89,35	0,9981	0,0032
728	81,56	0,9971	0,0032
0	61,82	0,9960	0,0032

4.6 Non-uniformity of a light source

In addition to the run-in behaviour, the radiation behaviour of the luminance standards also plays a role for the measurement, which must be described.

The following influences must be examined:

1. How does the average luminance depend on the size of the evaluation region?
2. How does the average luminance change if the evaluation region is not positioned precisely in the centre?
3. How does the luminance change if the measurement is not exactly perpendicular to the surface of the luminance standard?



In the first step, the luminous surface of the luminance standard is captured with an ILMD, and the average luminance of circular regions of different sizes is evaluated (see picture). Then the relative mean luminance is plotted as a function of area and the area surrounding the calibration conditions (1/4 of the total area) is examined.
If you want to calculate the homogeneity properties of the ILMD, you can also combine different images that show the luminance standard on different areas of the ILMD (multiple displacement of the ILMD relative to the luminance standard while maintaining the perpendicular position). As a rule, however, this is not necessary.

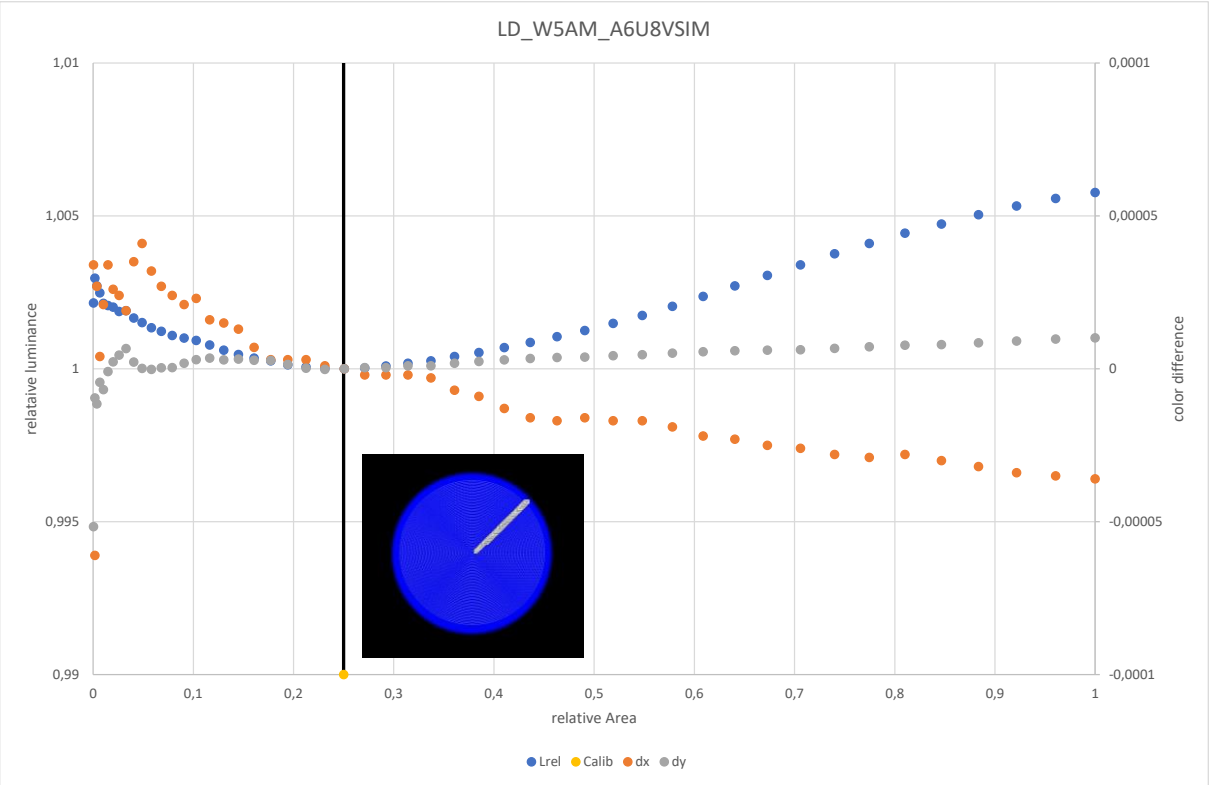


Figure 12: Relative luminance change for different evaluation region sizes (L^3 luminance standard)

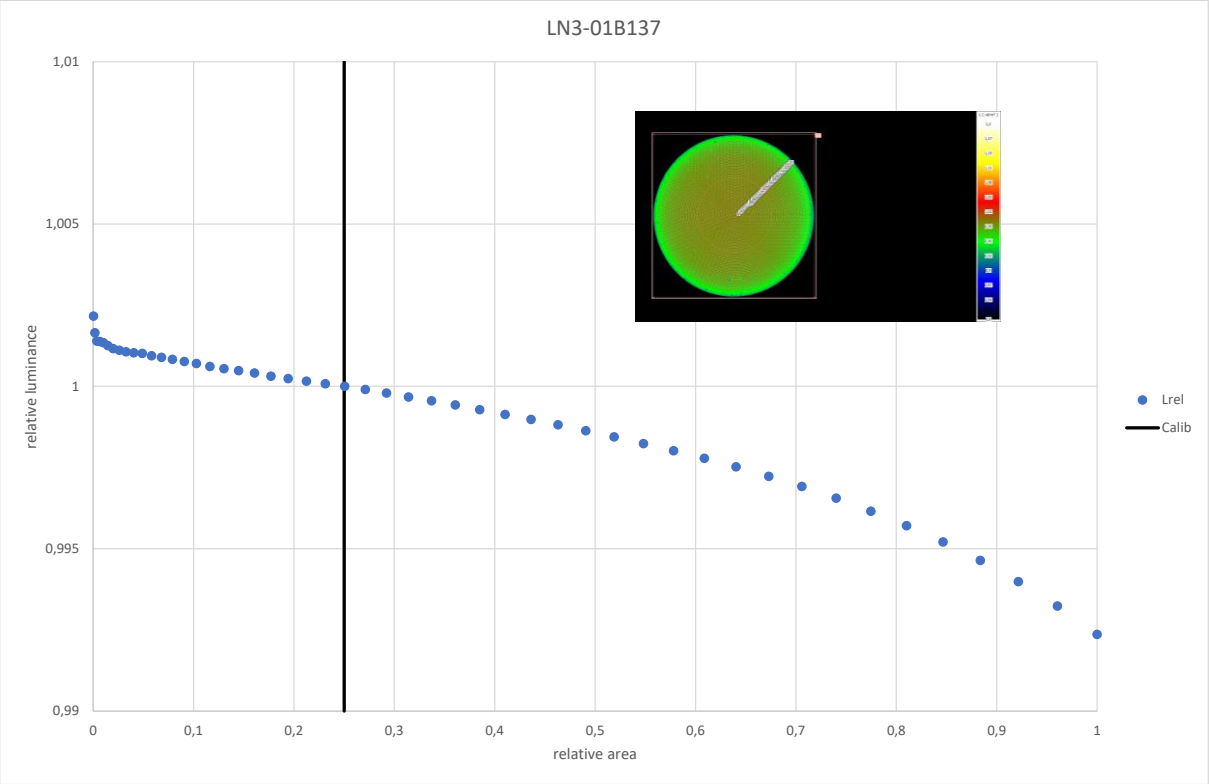


Figure 13; Relative luminance change for different evaluation region sizes (LN3 luminance standard)

4.7 Non-Linearity Correction

Let's assume we make several measurements of a luminance standard with different integration times of the luminance measuring device:

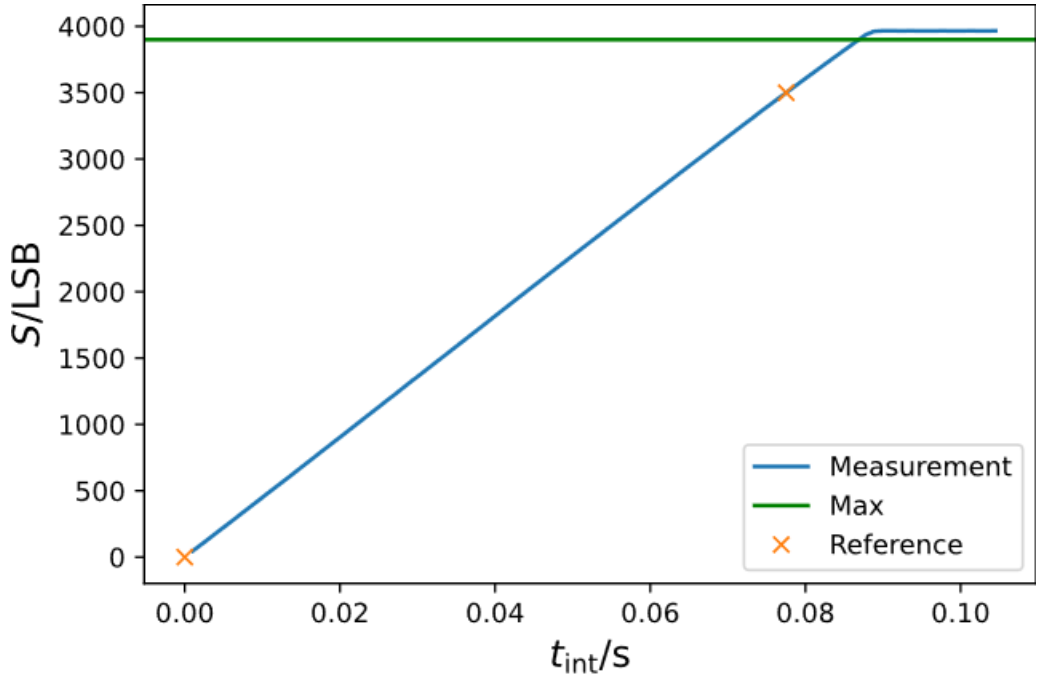


Figure 14: Measurement of a luminance standard with different integration times

We define a maximum load and two reference values. The line through the reference points represents the ideal behaviour of the device. Any deviation from this line is called non-linearity.

Remark: The manufacturer will do this (as demonstrated in Figure 12) with the Analogue-Digital-Converter readings (called counts and stated in digital numbers (DN) or least significant bit (LSB)) and not with the final measurement results. In this case, the manufacturer can avoid a range change and probably use the non-linearity information for different measurement ranges. If this is impossible, the manufacturer can switch between different non-linearity corrections for different measurement ranges (amplifier gain settings).

The corrected value S of the internal measurement can be described based on the reading S' with $S = f_{\mathrm{NL}}(S')$. The correction function f_{NL} can be described using a polynomial or a look-up table. The use of the polynomial is described here.

$$f_{\mathrm{NL}}(x) = \sum_{i=0}^{N_{\mathrm{p}}} \alpha_i x^i \qquad (35)$$

With α_i the polynomial coefficients of the polynomial with grade N_{p} and x is the variable of the domain, used for evaluation.

Remark: The dark signal correction should be applied before, e.g. $S = f_{\mathrm{NL}}(S' - S'_0)$

The question for the MU evaluation is now how to implement this correction. We have two possibilities here:

- Use a MCS for the regression and estimate the polynomial coefficients, including their correlation.
- We can use the range calibration approach (Kessel, Kacker and Sommer, 2010)

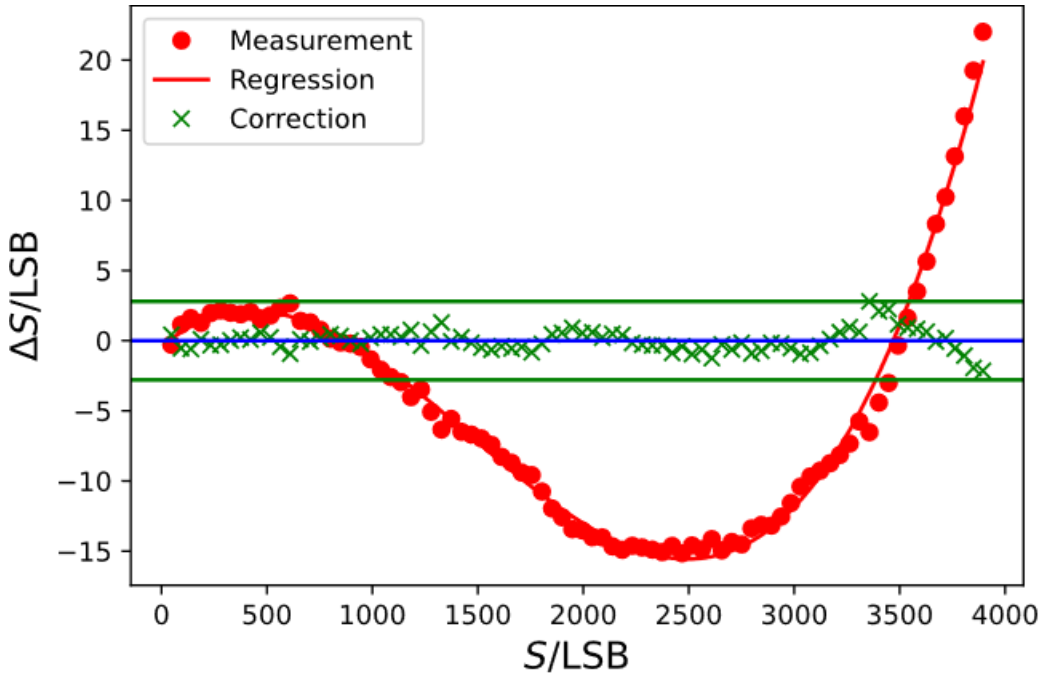


Figure 15: Description of the non-linearity and the result of a non-linearity correction

Table 4: Coefficients of the full regression polynomial for Figure 13 (with domain scaling [0,4000]→[-1,1])

Coef.	x^0	x^1	x^2	x^3
Mean	2004,20	1986,10	-1,55	16,65
StdDev	0,47	0,64	0,40	0,81
Rel.StdDev	0,02%	0,03%	-25,93%	4,88%

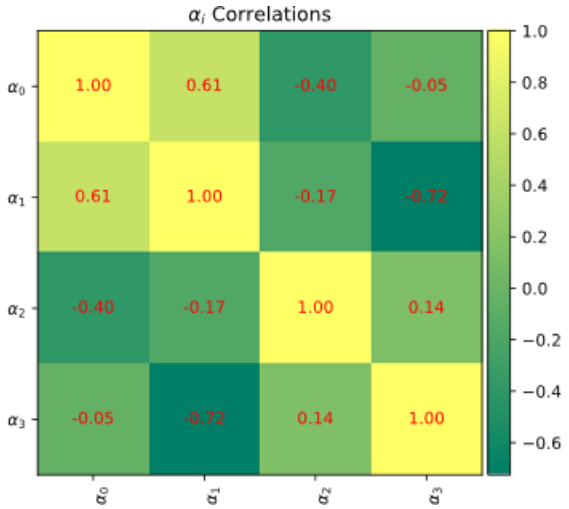


Figure 16: Coefficients of correlation for the polynomial coefficients

5 Monte Carlo Simulation

According to *GUMS1* the Monte Carlo Simulation (MCS) is one way to calculate the measurement uncertainty of a measurand.

This is neither the place to explain the mathematical foundations of MCS nor to go into its subtleties. The following is only a plausible basic introduction, which should be sufficient to carry out and evaluate the first MCS on your own.

The reader will find a more detailed introduction on the engineering level with practical examples (including source code) in (Amelin, 2015; Ciaburro, 2020; Dunn and Shultis, 2022; Stevens, 2022).

5.1 Basics

Assume we have a series of measurements with N readings taken one after the other. Now we can determine the mean and the standard deviation of these readings.

$$\hat{\mu} = \bar{x} = \sum_{i=1}^{N} x_i \; \hat{\sigma} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2} \qquad (36)$$

With the MCS as it is used for the determination of the combined uncertainty of a measurement, we do precisely the opposite. We know the value (mean) and the uncertainty (standard deviation) of the measured input variable (e.g. from the readings and calibration sheets) and generate a series of individual values (assuming a probability density function (PDF) - often the PDF of a normal distribution) representing the possible spread of values for this simulated quantity which might be seen in real measurements. We calculate the output quantity or variables via the measurement model with these individual values (usually from different input variables). Now we can again determine the mean value and standard deviation for the output variable and then have an estimate for our output variables and their measurement uncertainty.

Table 5: Analogy between multiple measurements (left) and the generation of random numbers in an MCS (right)

Index	Measurement
1	9,986
2	10,001
3	9,968
4	9,976
5	9,885
6	10,141
7	10,133
8	9,738
9	9,858
10	10,047
$\hat{\mu}$	9,973
$\hat{\sigma}$	0,117

μ	9,973
σ	0,117
Index	Simulation
1	9,789
2	10,11
3	10,136
4	9,965
5	9,646
6	10,158
7	9,959
8	10,102
9	9,919
10	9,927
$\hat{\mu}$	9,971
$\hat{\sigma}$	0,156

The numerical examples in Table 5 roughly reproduce the procedure (with far too few numbers). First, you have ten measured values, from which you determine the mean and standard deviation (left half of the table). In the MCS (right half of the table), you then take these data and generate ten random numbers with the parameter's mean value and standard deviation using a random generator. If you determine the mean value and standard deviation for these generated random numbers, then you get approximately the values that you previously obtained in the evaluation of the measurement series. The more random numbers you generate, the better the agreement.

5.2 Code Example

The implementation of the MCS can be found in the GitHub repository of this project under the folder https://github.com/empir19nrm02/empir19nrm02/tree/main/empir19nrm02/MC

Examples for the usage can be found in the Jupyter Notebook https://github.com/empir19nrm02/empir19nrm02/blob/main/empir19nrm02/Jupyter/MCSim_PM.ipynb

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