A self-study Course in Gas Modulation Refractometry – An optical technique for assessment of gas refractivity, gas density, and pressure

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2021

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What is refractometry and refractivity

Refractometry is a technique for assessment of

- index of refraction (n) or, more often,
- refractivity (n-1)

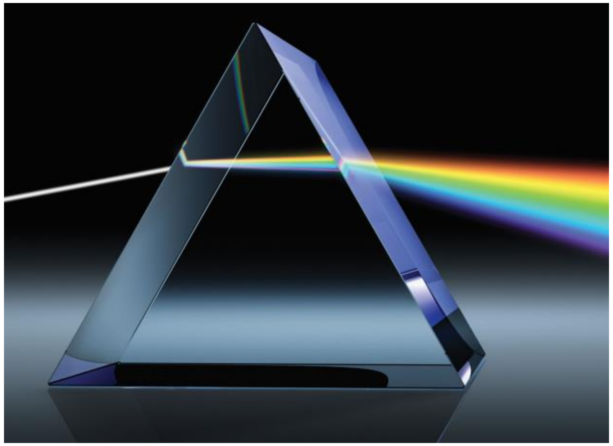
where n is the index of refraction of the material/substance.

The index of refraction constitutes a dimensionless number that describes how fast light travels through the material/substance. It is defined as the ratio of the speed o flight in vacuum, c, and the speed (e.g. the phase velocity) of light in the medium, v, i.e. as c/v.

THE INDEX OF REFRACTION OF A SOLID MATERIAL

The index of refraction of a glass material, n, and its wavelength dependence (dispersion), $n(\lambda)$ can be determined by assessment of the deflection angle when light is refracted at an interface.

Typical values of index of refraction of glass: $n = 1.5 – 1.9$.



THE INDEX OF REFRACTION OF A GAS

The indices of refraction of gases are much smaller, close to unity.

At 0 C, at 1 atm, and at 589 nm

For air: $n = 1.000293$

For CO_2: $n = 1.00045$

For He: $n = 1.000036$

How to assess indices so close to unity?

Measure the refractivity (n-1), since this differs more among various gases.

For air: $n-1 = 2.93 \times 10^{-4}$

For CO_2: $n-1 = 4.5 \times 10^{-4}$

For He: $n-1 = 3.6 \times 10^{-5}$

This can be done by refractometry.

WHAT IS THE PURPOSE OF REFRACTOMETRY

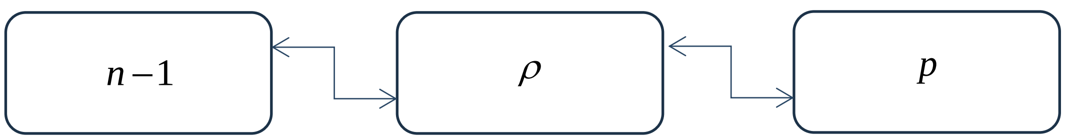
Regarding gases, refractometry can not only be used to assess refractivity; it can also be used to assess gas molar density and pressure.

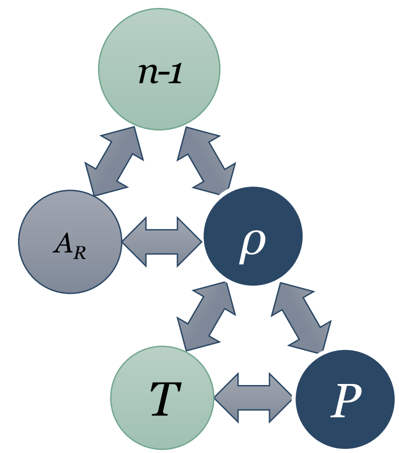
How can refractivity be related to the pressure of a gas?

For an ideal gas, the refractivity of a gas (n-1) is proportional to the molar density of molecules (ρ) in the gas.

The pressure of such a gas (P) is proportional to the molar density of molecules (ρ) through the ideal gas law.

In its simplest form…





$$\rho = \frac{2}{3A_R} \cdot (n-1) \qquad P = RT\rho$$

A_R is the molar polarizability [units: m^3/mol]

R is the molar gas constant [units: J/(K mol)],

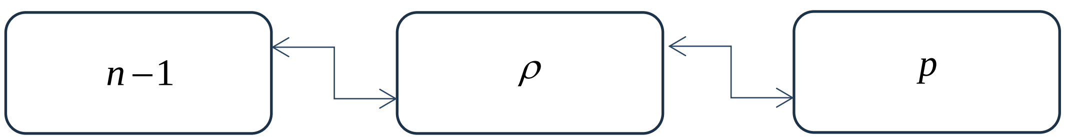
which is given by the product of Avogadro's number, N_A, and Boltzmann's constant, k_B.

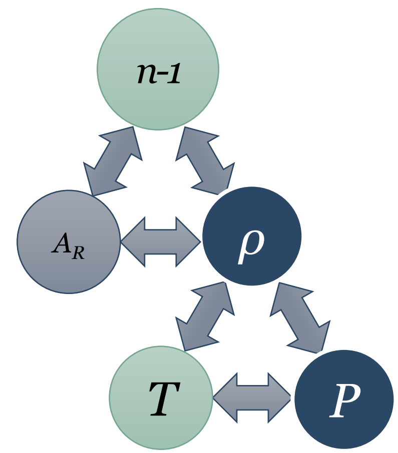
How can refractivity be related to the pressure of a gas?

For a real gas, the refractivity of a gas (n-1) is a function of the molar density of molecules (ρ) in the gas.

The pressure of such a gas (P) is a function of the molar density of molecules (ρ), related through an equation of state.

In this case…





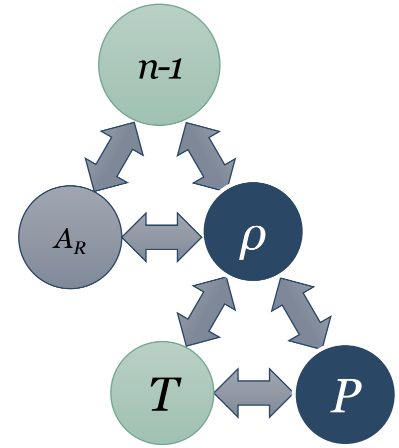
$$\rho = \frac{2}{3A_R} \cdot (n-1)\left[1 + \tilde{B}_\rho \cdot (n-1) + \ldots\right] \qquad P = RT\rho[1 + B_p(T)\rho + \ldots]$$

A_R is the molar polarizability [units: m^3/mol]

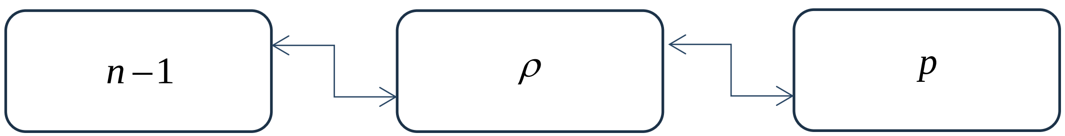
R is the molar gas constant [units: J/(K mol)]

The B_r and B_p are so called virial coefficients. These terms are of minor importance for sub-atmospheric pressures.

TYPICAL CONDITIONS FOR A STANDARD GAS



For an ideal gas, for which we have



$$\rho = \frac{2}{3A_R} \cdot (n-1) \qquad P = RT\rho$$

For a typical value for the molar polarizability, A_R , (for N_2), which is 4.4 × 10^{-6} m^3/mol,

and using the value of the molar gas constant (R), which is 8.314 mol^{-1} K^{-1} mol^{-1},

this implies that, at room temperature, and under atmospheric pressure conditions

(i.e. for P = 1 atm = 10^5 Pa), we get

ρ = 40 mol/m^3

n-1 = 2.7 × 10^{-4}.

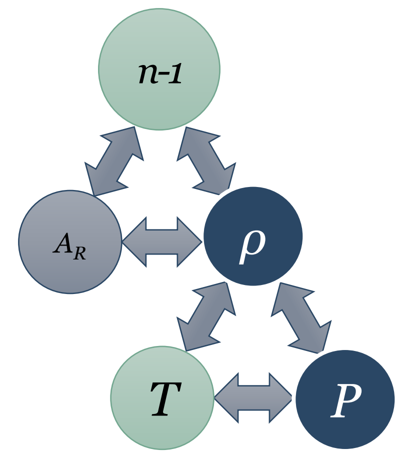
This implies that it should be possible to assess pressure by the use of refractometry.

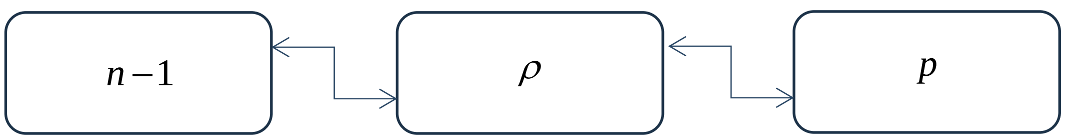
Can refractometry be used to create the unit of Pascal (as a primary standard)?

Since the pressure of a gas (P) is a function of the molar density of molecules (ρ), which, in turn, is a function of the refractivity of a gas (n-1),

if we can measure n-1 and the temperature accurately, and if we know all molecular and physical constants, we can create the unit of Pascal in our laboratory.

This is nothing but a primary standard.





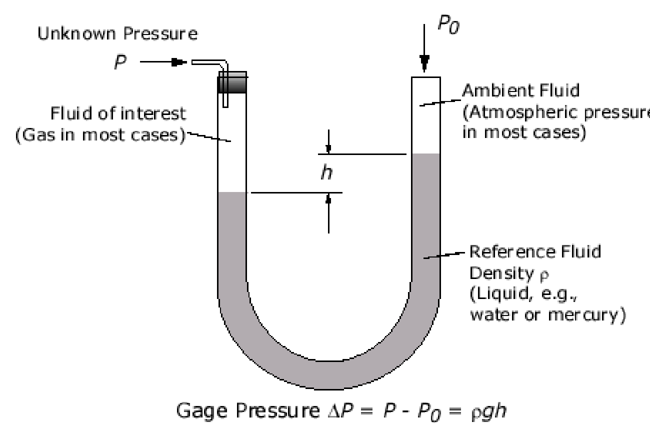
$$\rho = \frac{2}{3A_R} \cdot (n-1)\left[1 + \tilde{B}_\rho \cdot (n-1) + \ldots\right] \qquad P = RT\rho[1 + B_p(T)\rho + \ldots]$$

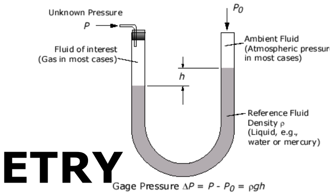
This would then replace all "old-fashioned" mechanical devices that today are use of primary standards.

PRIMARY STANDARD OF TODAY – HOW MUCH IS 1 PASCAL?

The Pascal (symbol: Pa) is the SI derived unit of pressure used to quantify internal pressure, stress, Young's modulus and ultimate tensile strength. The unit, named after Blaise Pascal, is defined as one newton per square meter.

Current realizations of the Pascal rely on piston gauges (also known as pressure balances) and liquid manometers containing toxic mercury, both of which measure force per area.





ONE JUSTIFICATION FOR DEVELOPING REFRACTOMETRY FOR ASSESSMENT OF PRESSURE – A PRIMARY STANDARD

- Their performance however has remained essentially unchanged over the past few decades and they suffer from practical and environmental limitations.
- Therefore, if possible, they should be replaced by more “modern” systems.
- Since the SI system was redefined in May 2019 (eliminating the uncertainty of the Boltzmann constant, and thereby the uncertainty of the molar gas constant), it should be possible to develop photon-based primary standards of pressure that are limited only by the accuracy of the assessments of refractivity, the temperature, and quantum calculations (molecular constants such as molar polarizabilities).
- In the longer term, such primary standards could be miniaturized, providing faster and calibration-free pressure measurements for industry at a fraction of the present cost

How to measure the refractivity of a gas – can one use a Michelson interferometer

Refractivity is most conveniently assessed by interferometry.

The simplest means is to utilize a Michelson interferometer.

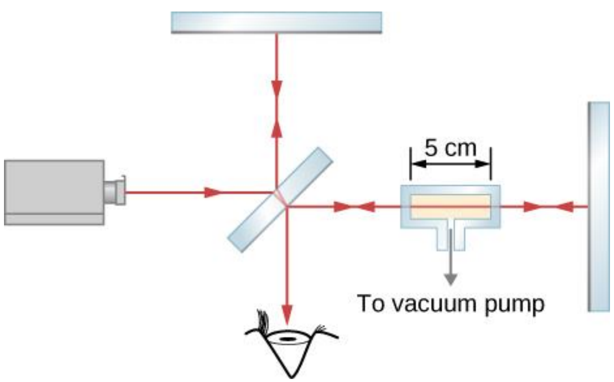
Principle: It creates an interference pattern that is either bright or dark in the center depending on the difference in optical length of the two arms.

In vacuum, it is bright whenever $2L_1 - 2L_2 = N\lambda$

If gas is let into one arm, it becomes dark and then bright again

One such "fringe" has passed whenever $2n_1L_1 - 2L_2 = (N+1)\lambda$

This implies that $2(n_1 - 1)L_1 = \lambda$, or that $n - 1 = \dfrac{\lambda}{2L}$.

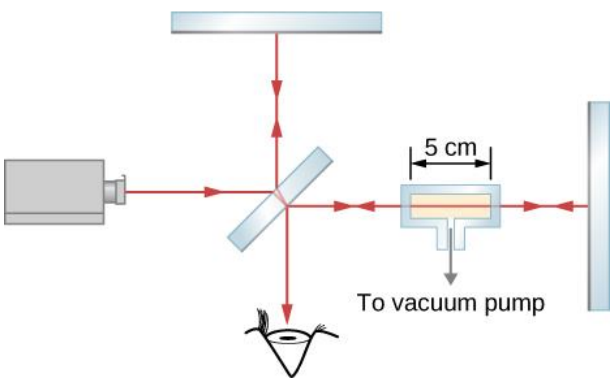


How to measure the refractivity of a gas utilizing a Michelson interferometer

From $n-1=\frac{\lambda}{2L}$ we can estimate when one "fringe" has passed:

For λ = 600 nm and L = 5 cm, this takes place when n-1 = 6×10^{-6}.

If it is possible to detect 1/10 of a fringe, this corresponds to a smallest detectable refractivity, $(n-1)_{min}$, of 6×10^{-7}.



How well can Refractometry be used to assess the presence of gas? Resolution of a Michelson interferometer for assessment of pressure

If P = 1 atm (10^5 Pa) corresponds to (n-1) of 2.7×10^{-4}

A smallest detectable refractivity, $(n\text{-}1)_{min}$, of 6×10^{-7} corresponds to

a pressure of 0.002 atm, or 200 Pa.

Hence, 1 atm of gas can only be assessed with a resolution of 0.2%.

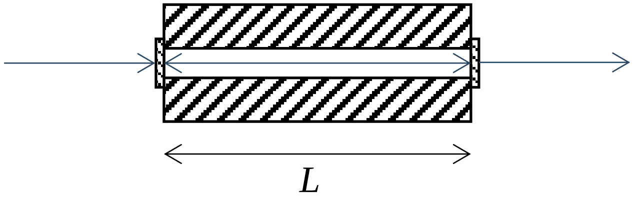
Hence, refractometry by a Michelson interferometer can detect pressure, but not very sensitively.

A good pressure gauge should be able to have a sub-ppm or mPa resolution.

Maybe not worth doing – other pressure gauges can do it much better.

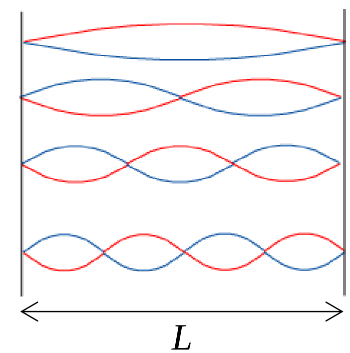
Maybe doable by some other interferometric technique.

IMPROVED MEANS – ASSESS REFRACTIVITY BY THE USE OF A FABRY PEROT (FP) CAVITY



A Fabry Perot cavity consist of two highly reflective mirrors separated by a given distance, possibly with gas inside.

Only light of given wavelengths/frequencies can exist in such a cavity.



Only standing waves, i.e. those for which $q\lambda_q = 2nL$ or $\boxed{\nu_q = q\frac{c}{2nL}}$.

Transmission is given by the Airy-function,

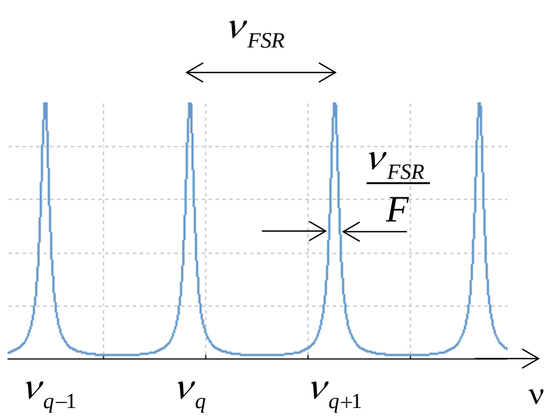
$$T = \frac{1}{1+\left(\frac{2F}{\pi}\right)^2 \sin^2\left(\frac{\pi\nu}{\nu_{FSR}}\right)}$$

where F is the finesse, given by $\frac{\pi\sqrt{R}}{1-R}$,

where R is the reflectivity of the mirrors and

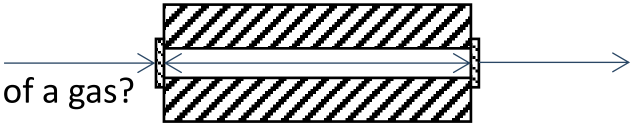
ν_{FSR} is the free-spectral range (FSR), given by $\frac{c}{2nL}$.

The width of a transmission mode is given by $\frac{\nu_{FSR}}{F}$.



Refractivity by the use of a Fabry Perot (FP) Cavity

How can a Fabry-Perot cavity be used to measure the refractivity of a gas?



Look at the ***shift*** of a cavity mode, $\Delta\nu_N$, as gas is let in or out of the cavity. This is given by

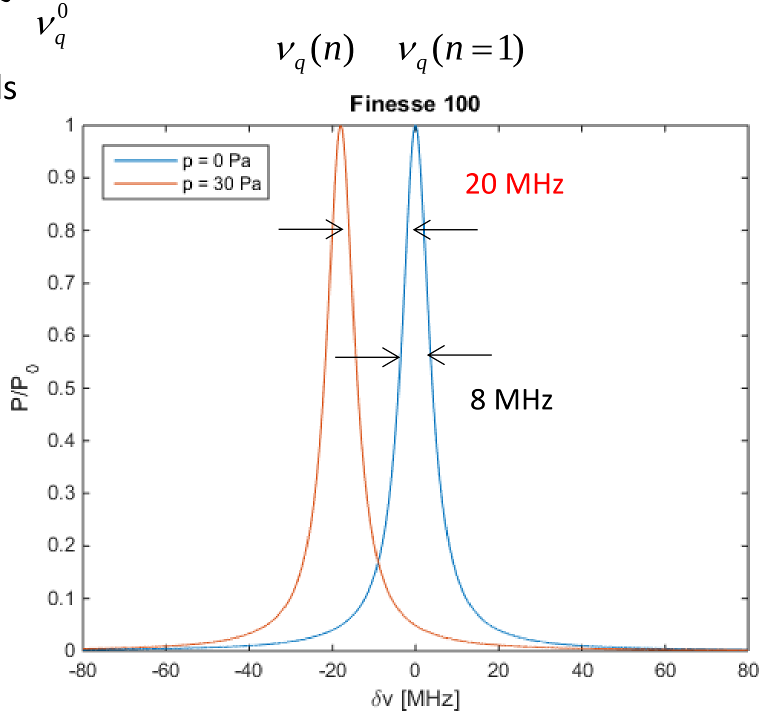
$$\Delta\nu_q = \nu_q(n=1) - \nu_q(n) = q\frac{c}{2L}\left(1-\frac{1}{n}\right) = \nu_q^0\frac{n-1}{n}$$

Hence, refractivity can be assessed as $n-1 = n\frac{\Delta\nu_q}{\nu_q^0} \approx \frac{\Delta\nu_q}{\nu_q^0}$

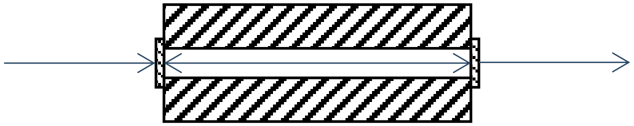
Example 1: A change the pressure of **30 Pa** corresponds to a change in refractivity $n-1$ of 10^{-7} and a mode shift, $\Delta\nu_q$, of 20 MHz (for 1.5 µm light).

Figure: Cavity with a FSR of 800 MHz, a moderate finesse of 100 (R ~ 97 %), gives a mode width of 8 MHz.

This can clearly be detected.



Refractivity by the use of a Fabry Perot (FP) Cavity



What is required to measure smaller amounts of gases?

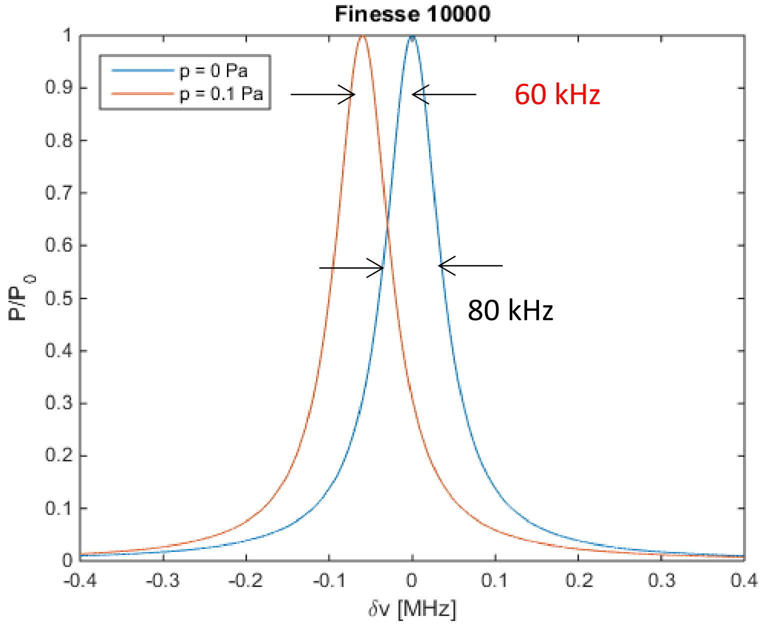
Use a cavity with a higher finesse, e.g. 10 000 (R ~ 99.97 %).

This leads to more narrow transmission modes (80 kHz)

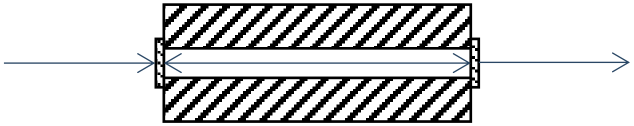
Example 2: A change of the pressure in the cavity of **0.1** Pa implies that $n-1$ changes 3×10^{-10} whereby the mode shift, $\Delta \nu_q$ becomes 60 kHz.

What does this look like?

See figure.



Locking of the Laser to a Cavity Mode



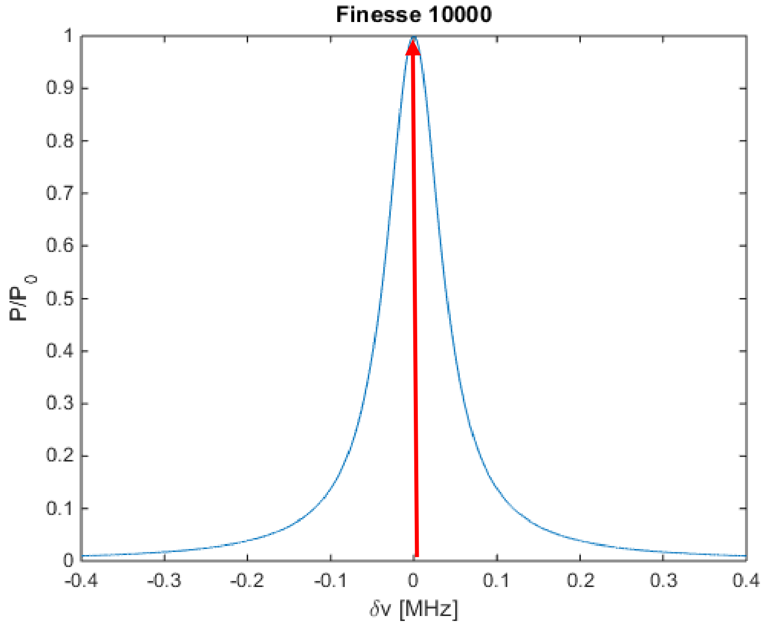
Can we really detect such shifts of a cavity mode?

How can we measure such shifts?

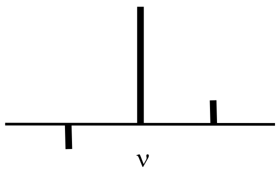
Solution: **lock** the frequency of a narrow linewidth tunable laser to a cavity mode.

If the cavity mode shifts, the frequency of the laser will follow the frequency of the cavity mode.

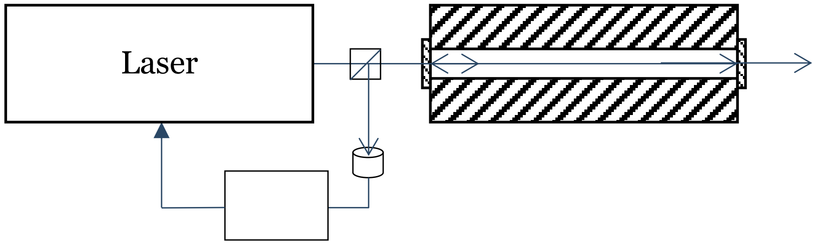
The **frequency of the laser** carries information about the cavity mode and thereby the refractivity



How to lock the Laser to a Cavity Mode

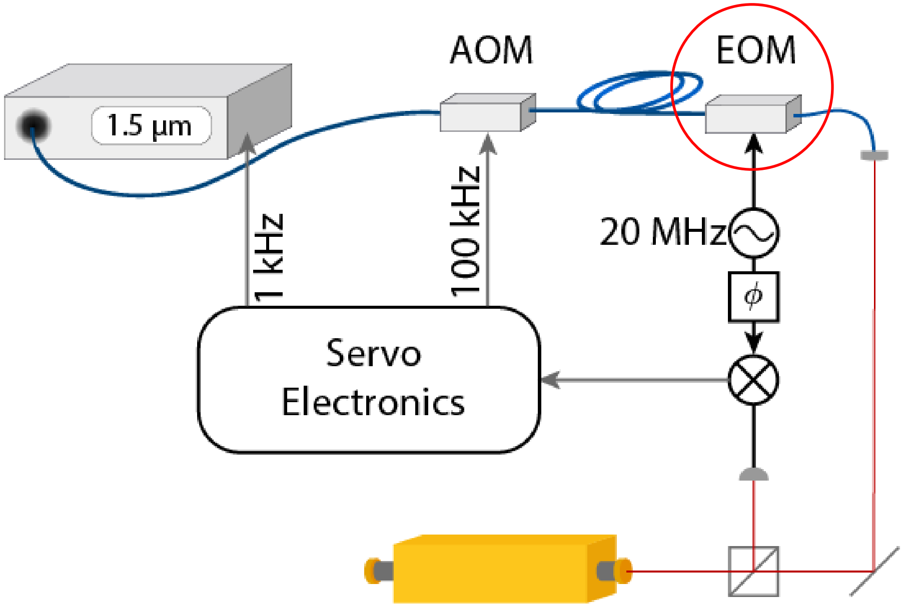


Locking is performed by the Pound-Drever-Hall (PDH) technique.



Servo electronics for locking by use of the Pound Drever Hall (PDH) technique

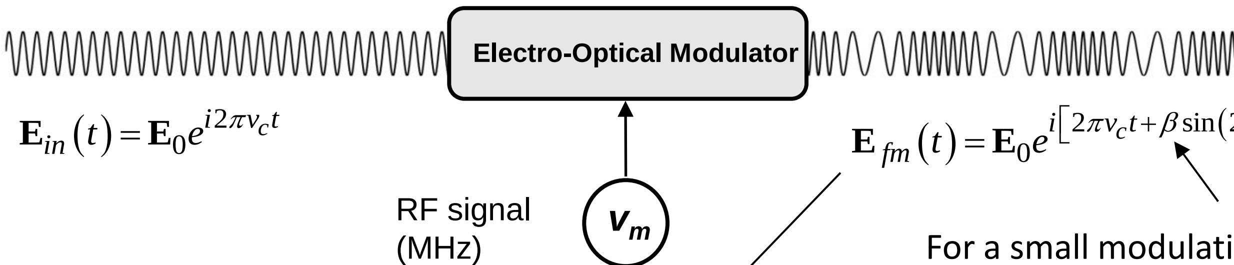
In this technique, the laser light is modulated by a fast electro-optic modulator (EOM) to produce "sidebands".



How can an EOM Produce Sidebands of Light?

An RF signal with a frequency of ν_m can modulate the index of refraction of the EOM.

This implies that the phase of the light will be modulated.



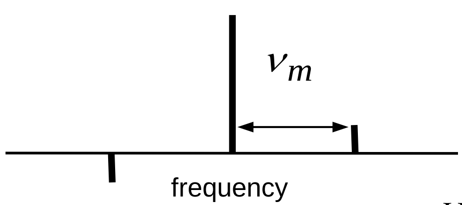
$$\mathbf{E}_{in}(t) = \mathbf{E}_0 e^{i 2\pi \nu_c t}$$

$$\mathbf{E}_{fm}(t) = \mathbf{E}_0 e^{i\left[2\pi \nu_c t + \beta \sin(2\pi \nu_m t)\right]}$$

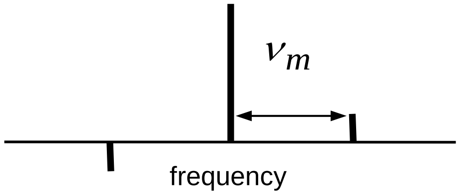
For a small modulation index

For small modulation indices, the light seems to consist of three (in general many) modes.

$$\mathbf{E}_{fm}(t) = \mathbf{E}_0 \left[-\frac{\beta}{2} e^{i 2\pi (\nu_c - \nu_m) t} + e^{i 2\pi \nu_c t} + \frac{\beta}{2} e^{i 2\pi (\nu_c + \nu_m) t} \right]$$



Locking of the Laser to a Cavity Mode

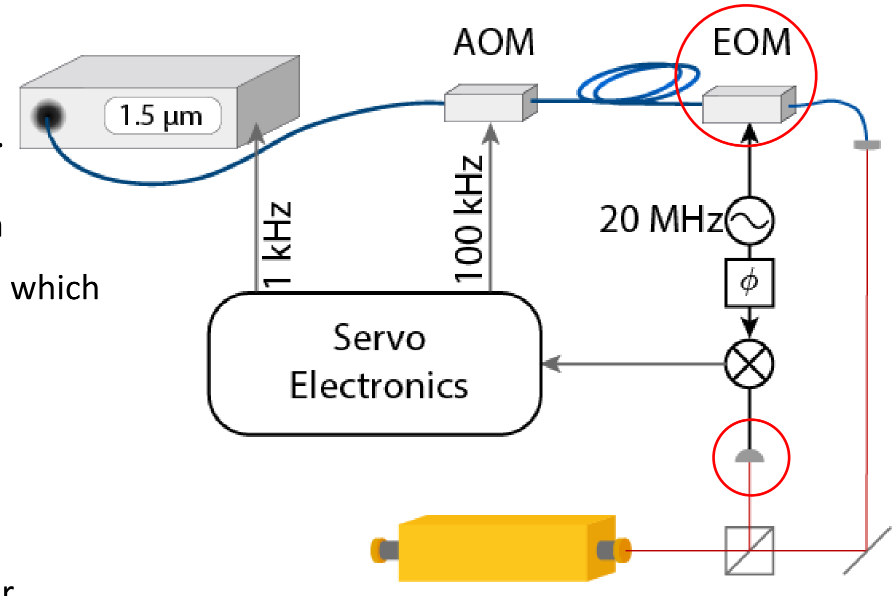


Locking performed by the Pound-Drever-Hall technique.

The laser light is modulated by a fast electro-optic modulator (EOM) that splits the laser light into several frequencies, i.e. to produce “sidebands”.

Under locked conditions, the sidebands are fully reflected from the cavity while the “carrier” is partly reflected.

A detector detecting the reflected light will thus detect three modes, with mode separations of ν_m.

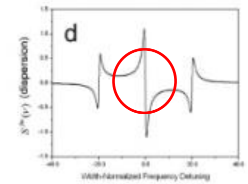


Close to locking, the sidebands are reflected from the cavity and serve as references with respect to which the phase of the “carrier” light is measured

The “carrier” will interact with each sideband and create two beat signals at ν_m.

These beat signals can be detected by the detector.

Locking of the Laser to a Cavity Mode



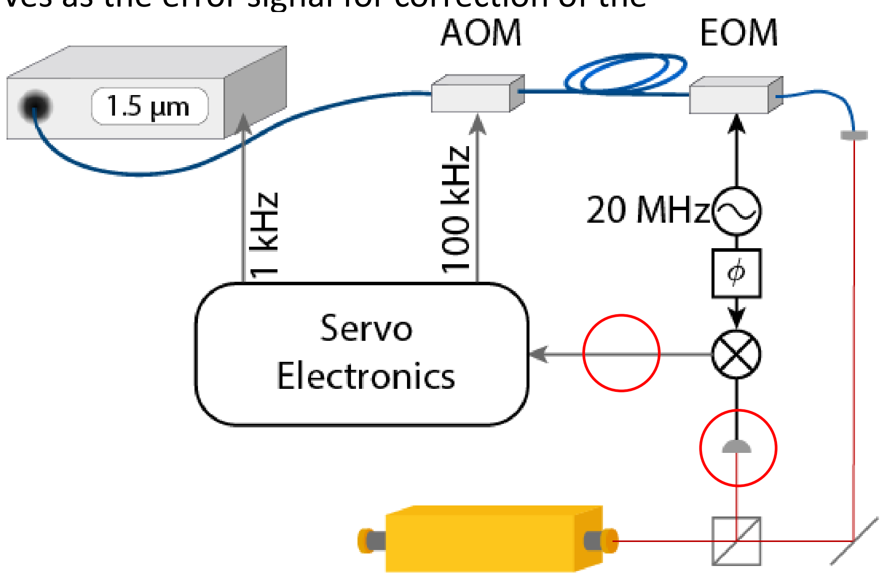
If the laser frequency is exactly on the cavity mode, the two beat signals will cancel.

If it is not exactly on resonance, the cavity will shift the phase of the carrier and the two beat signals will no longer cancel.

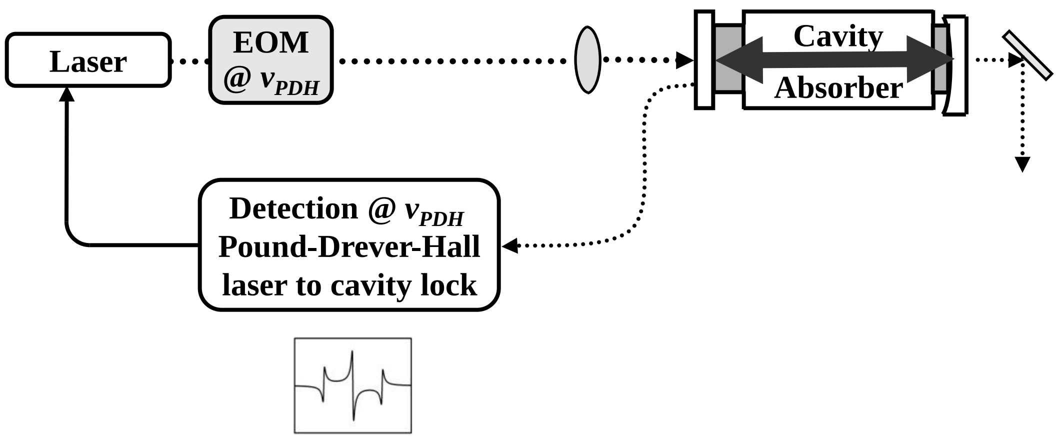
After demodulation at the modulation frequency, this produces a sharp response for laser frequencies around the cavity frequency, which serves as the error signal for correction of the frequency of the laser light. See figure.

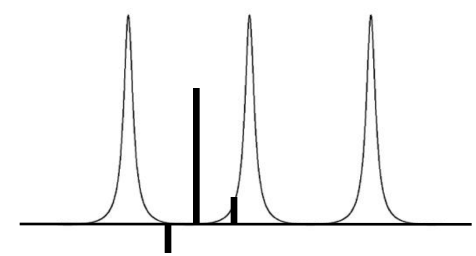
Servo electronics will steer the frequency of the laser to that of the cavity mode.

Hence, the laser is "locked" to the cavity mode.



Locking of the Laser Light to a Cavity Mode





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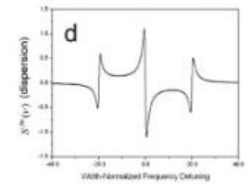
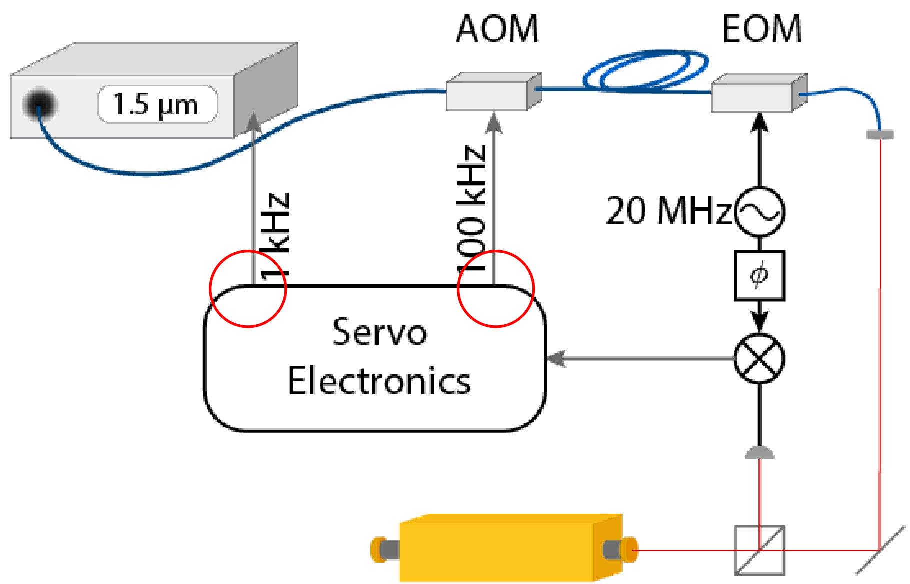


The “slow” parts of the feedback go to the laser.

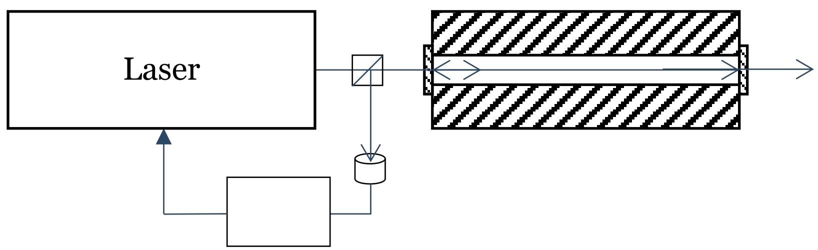
To increase the bandwidth for the feedback to the laser, the fastest part of the error signal is fed to an acousto optic modulator (AOM) that can shift the frequency of the laser light faster, often up to 100 kHz rates.

This locks the laser to the cavity mode “stronger”, to within some hundreds of Hz.



The Shift of the Cavity Mode is Transferred to the Laser Light

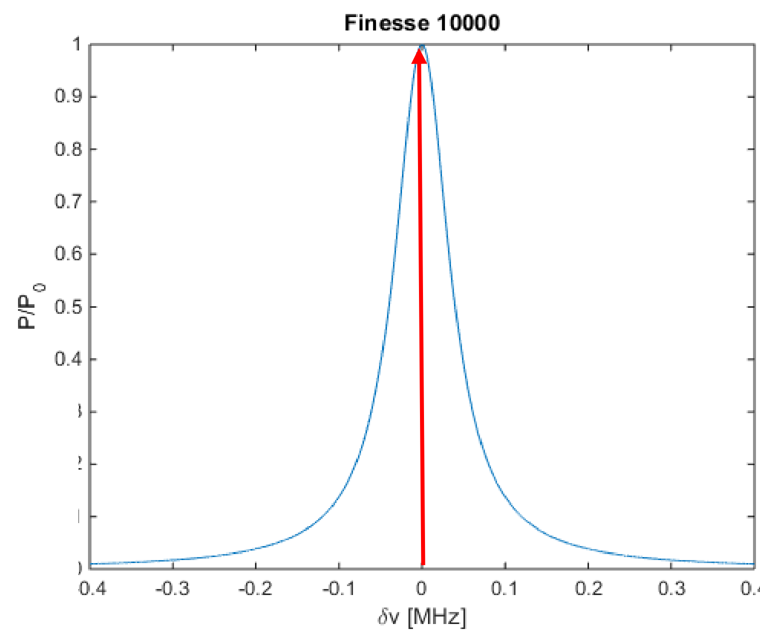
When the frequency of the laser is locked to a cavity mode...



Servo electronics for locking by use of the Pound Drever Hall (PDH) technique

If the cavity mode shifts, the frequency of the laser will follow the frequency of the cavity mode.

The **frequency of the laser** carries information about the cavity mode and thereby the refractivity.



How to Measure a Shift of the Frequency of Laser Light

How can we measure a shift of the laser light of 60 kHz? The frequency of the light is ~ 2×10^{14} Hz.

It corresponds to a change in frequency from $2.0000000000 \times 10^{14}$ to $2.00000000060 \times 10^{14}$ Hz.

It not possible if we look at the wavelength of the light

This changes from 1.50000000000 μm to 1.49999999955 μm.

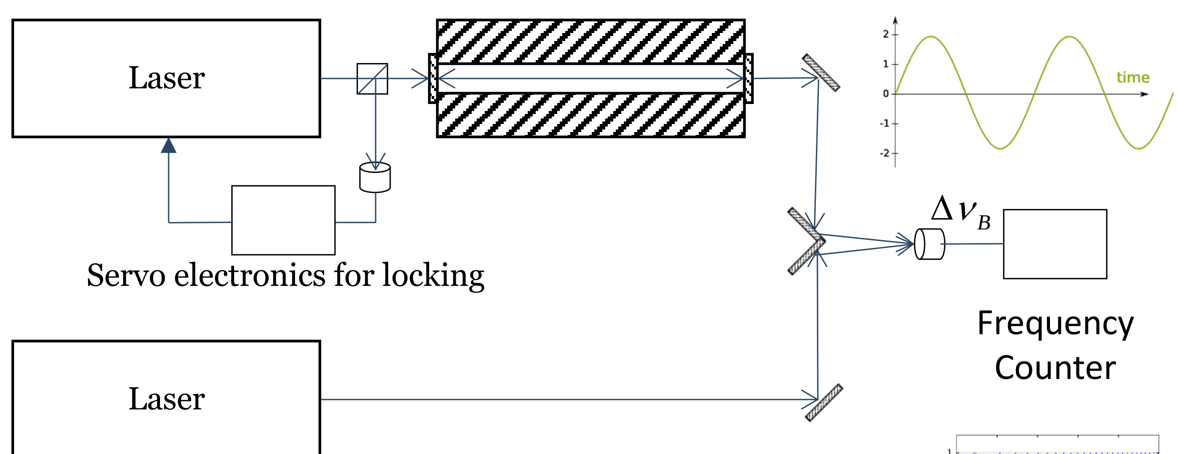
No instrument has this resolution.

How to Measure a Shift of the Frequency of Laser Light

Solution: Beat the light that is locked to the cavity with a that of another laser beam, with almost the same frequency, on a photo diode.

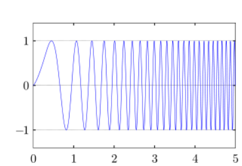
This will produce a detector signal that is modulated at the beat frequency that can be measured.

The frequency of this beat signal, ν_B , will then change an amount, $\Delta\nu_B$, that is equal to the shift of the mode, $\Delta\nu_q$.



This is the basis of FP-based refractometry.

For a changing pressure of the gas



A practical Problem: Drifts

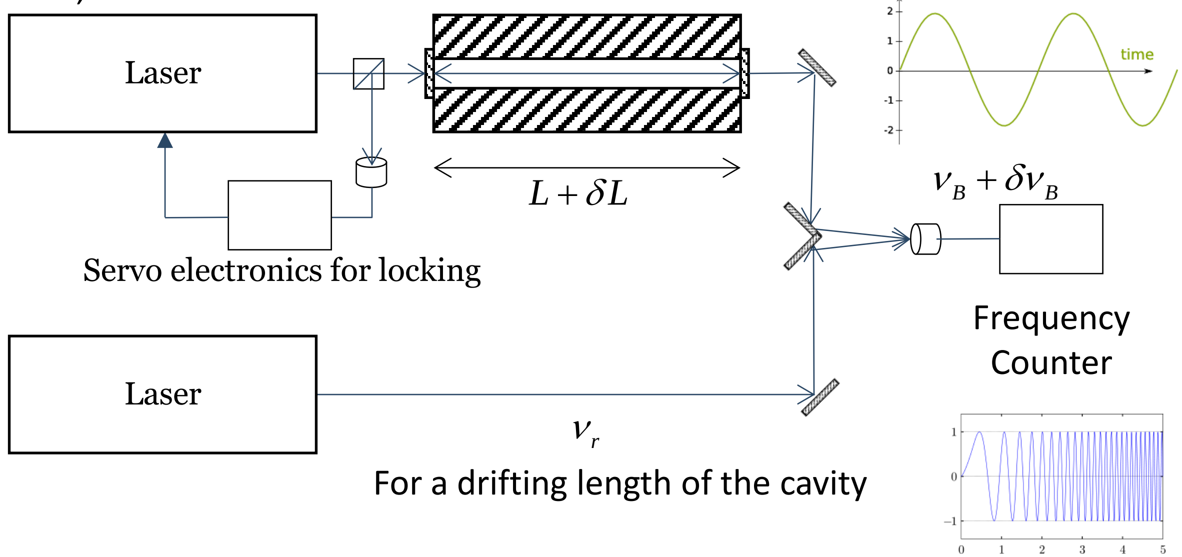
Practical problem (1):

- The length of the cavity will drift.

This implies that the frequency of its modes and the detected beat frequency will drift.

A drift (i.e. a change) in length of a 30 cm long cavity during a measurement of 1 pm (a millionth of a hair strand) corresponds to shift in the frequency that, in turn, corresponds to (an error in) the assessment of refractivity of 3×10^{-12} and a pressure of (for N_2) 1 mPa, respectively.

This implies that FPC-based refractometers are extremely sensitive to tiny fluctuations, disturbances, and drifts.



A practical Problem: Drifts

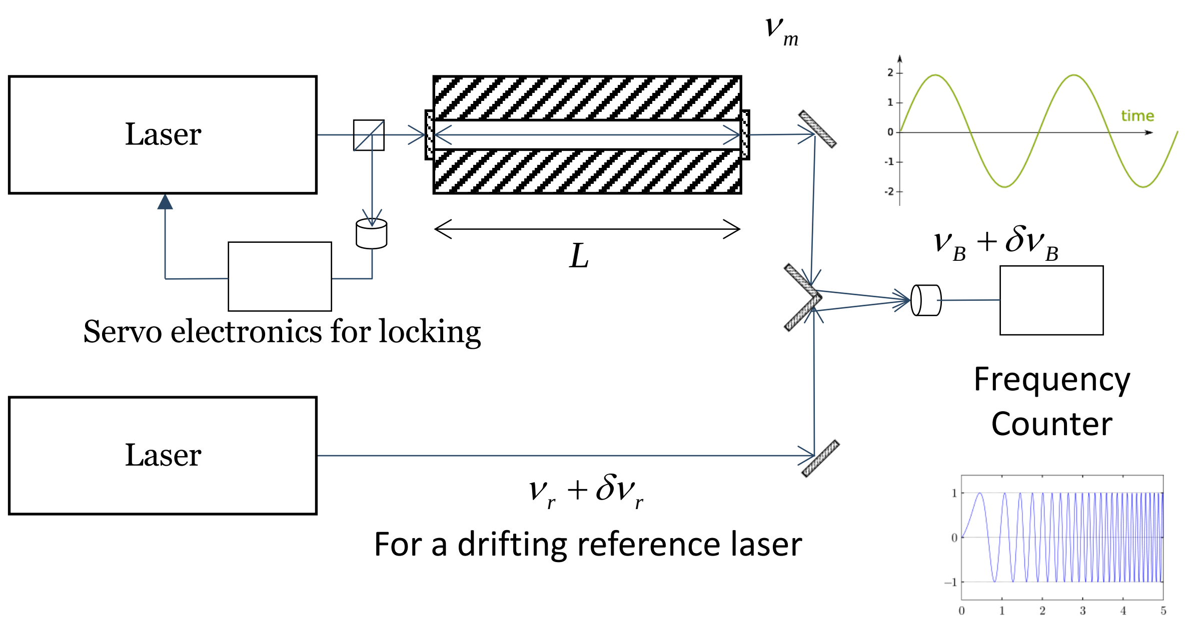
Practical problem (2):

- The frequency of the reference laser can drift.

This implies that the detected beat frequency will drift.

A normal (non-stabilized) laser can easily drift more than 1000 MHz. This makes it difficult/impossible to detect beat frequencies in the kHz range.

The drifts of the laser must be reduced at least 10^6 times.



A PRACTICAL PROBLEM: DRIFTS

Practical problems:

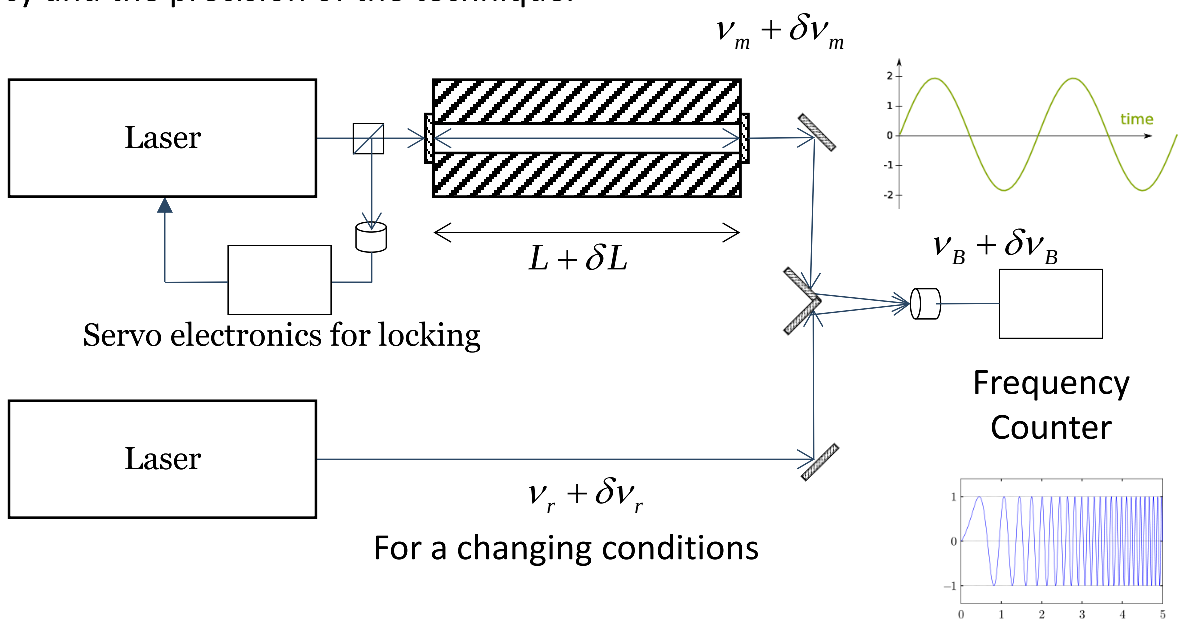
- The length of the cavity will drift.

This implies that the frequency of its modes will drift.

- The frequency of the reference laser will drift.

Both these imply that the measured beat signal will drift.

- This limit both the accuracy and the precision of the technique.



The First Part of the Solution: Use A Dual Cavity

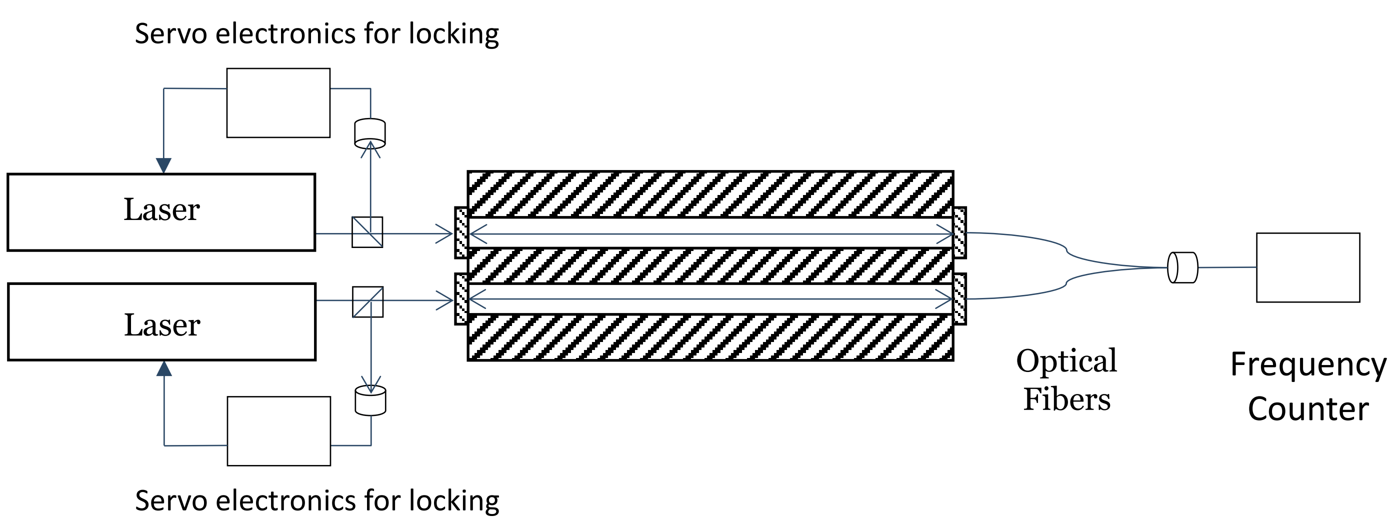
First remedy to this:

- Use two cavities, one measurement cavity (with gas) and one reference (vacuum).
- One laser locked to each.

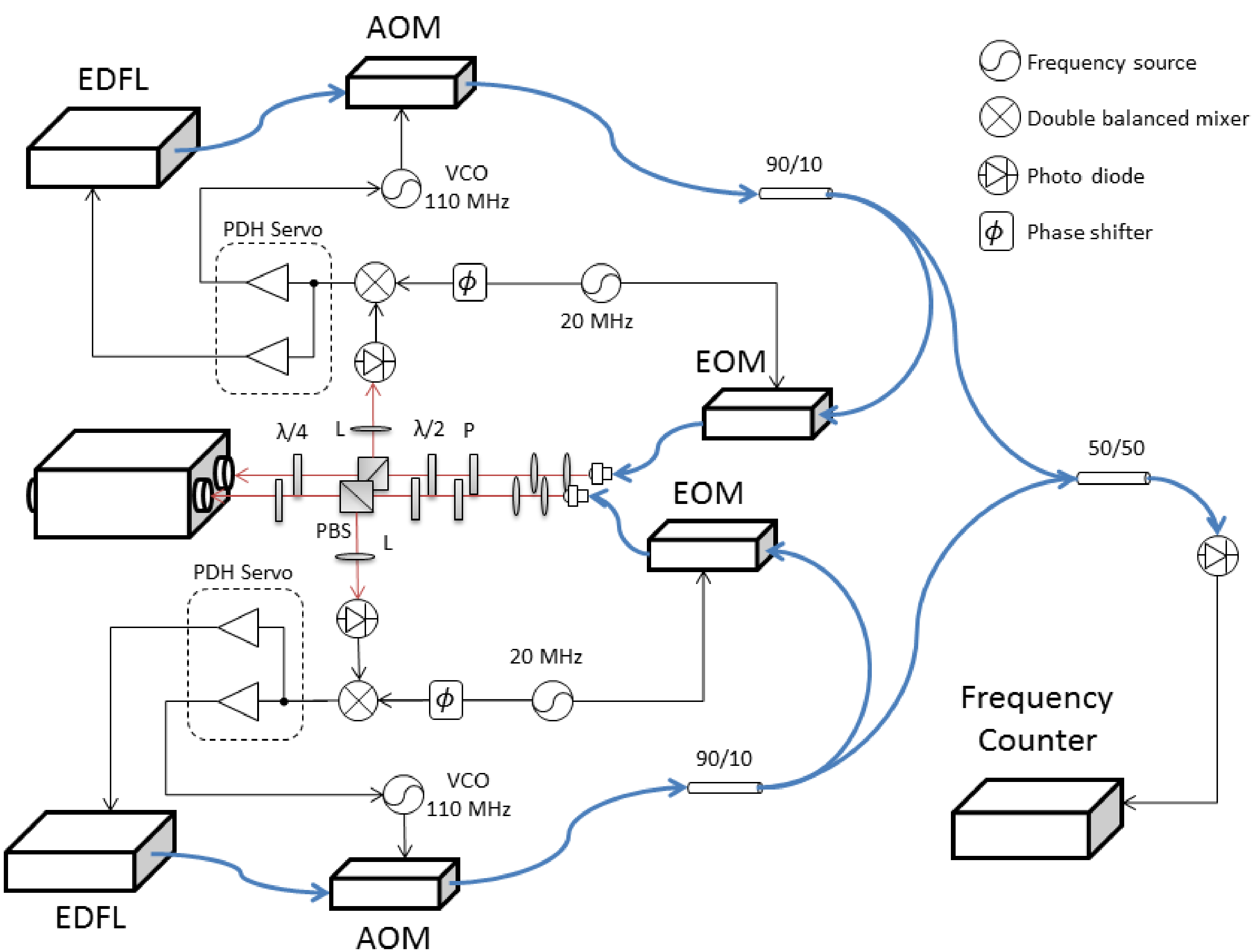
This implies that

1. there is no uncontrolled drift of the reference laser, and
2. drifts of the length of the two cavities will presumably cancel.

Dual Fabry-Perot Cavity (DFPC) Refractometry



Typical/Early Realization of a Dual Fabry-Perot Cavity Spectrometer



EDFL: Er-doped Fiber Laser, $\lambda = 1.55$ nm

Systems still affected by drifts

Practical problem (3):

Typical DFPC-based refractometers are still limited by drifts.

Most experimentalists build their system in huge temperature-stabilized (vacuum) chambers.

This makes them difficult to move.

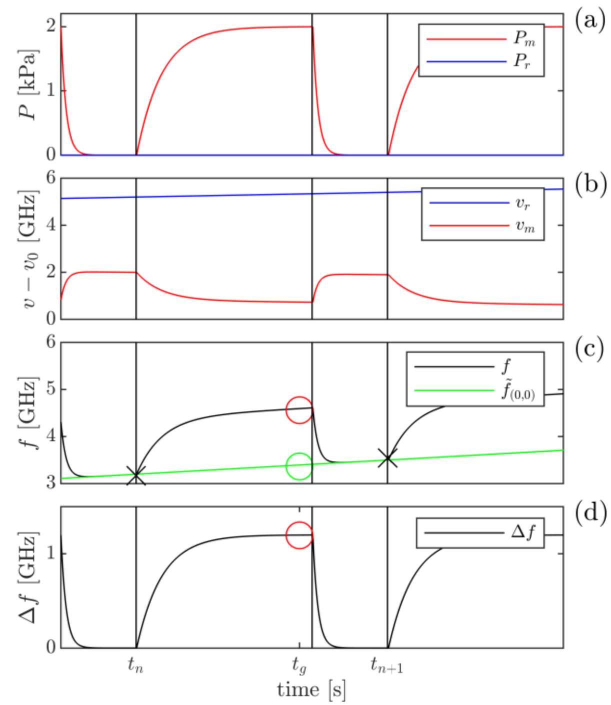
The Umeå University research group has developed a methodology that reduces the influence of drifts and disturbances.

This allows users to build and use systems without such extreme precautions.

The Second Part of the Solution: Use Gas modulation (GAMOR)

The remedy to this: Gas Modulation Refractometry (GAMOR)

- Modulate the amount of gas in the measurement cavity. One cavity (the measurement cavity) with alternating gas and vacuum and the other (the reference cavity) with constant pressure.
- Panel (a): Red curve: the pressure in the measurement cavity, P_m. Blue curve: the pressure in the reference cavity, P_r.
- Panel (b): Cavity mode frequencies: Red curve: the frequency of the mode addressed in the measurement cavity, ν_m. Blue curve: the frequency of the mode addressed in the reference cavity, ν_r.
- Panel (c): Black curve: beat frequency, f, defined as $\nu_r - \nu_m$. Green curve: interpolated behavior of the empty cavity beat frequency, $f_{(o,o)}$.
- Panel (d): Black curve: Drift corrected shift in beat frequency, Δf, defined as $f - f_{(o,o)}$.



$$\nu_q = q\frac{c}{2nL}$$

The Second Part of the Solution: Use Gas modulation (GAMOR)

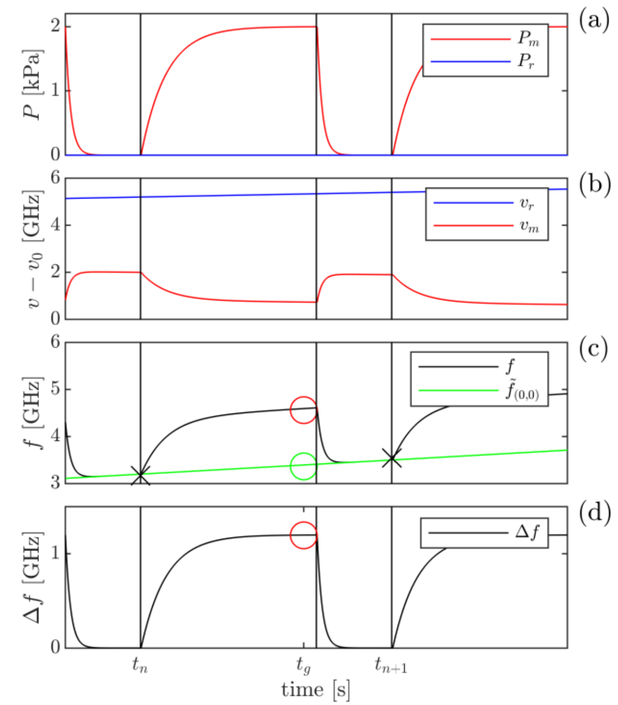
- By this, it is possible to constantly refer the beat frequency measured with the measurement cavity being filled with gas to the beat frequency measured with measurement cavity being empty.

And by this

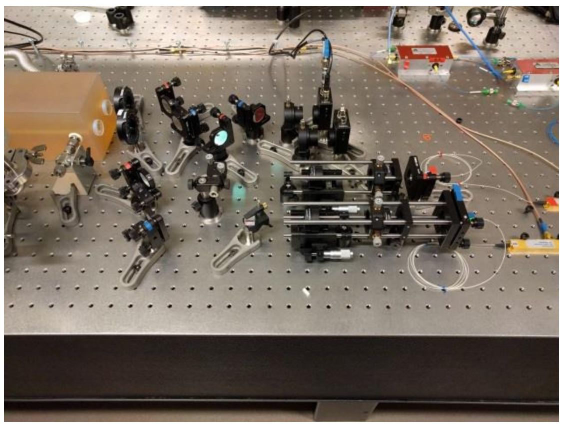
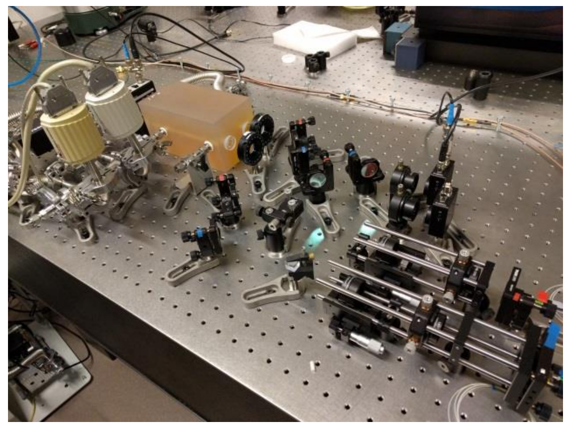
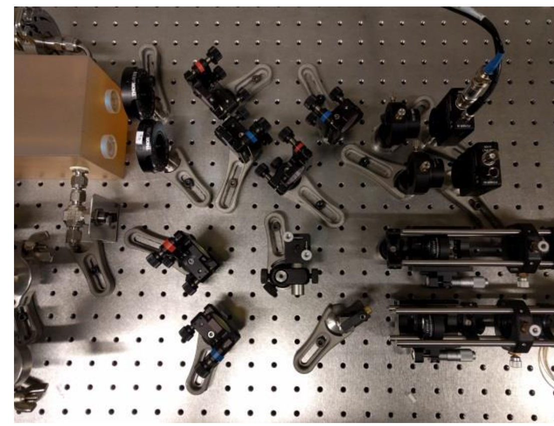
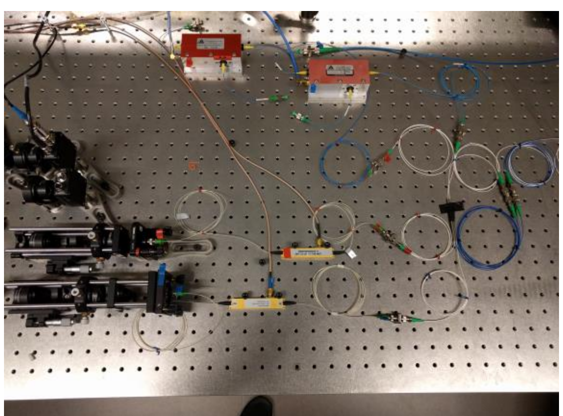
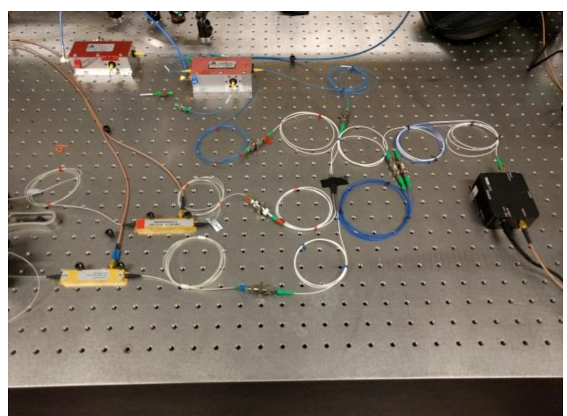
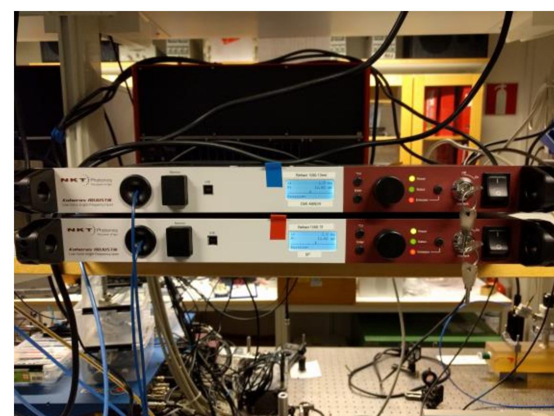
- Reduced the influence of drifts within the system on the assessment of changes of beat frequency considerably.

This is the basis of the technique utilized by us:

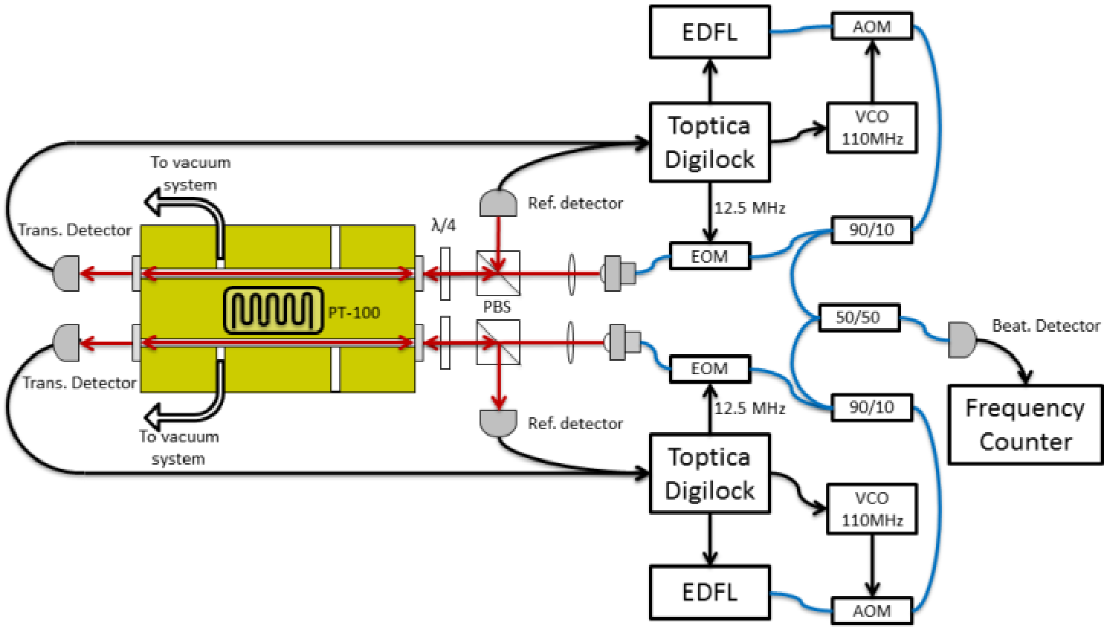
- Referred to as Gas Modulation Refractometry (GAMOR)



Early Realization of a Dual Fabry-Perot Cavity based GAMOR Spectrometer

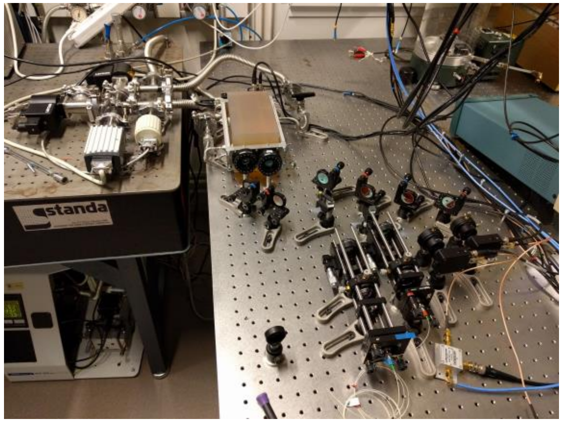







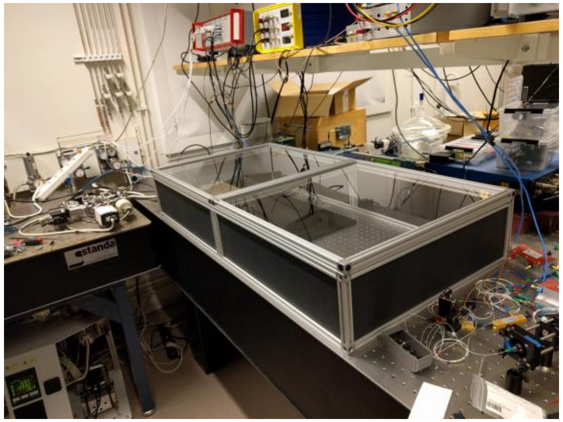
More Recent Realization of a Dual Fabry-Perot Cavity Spectrometer

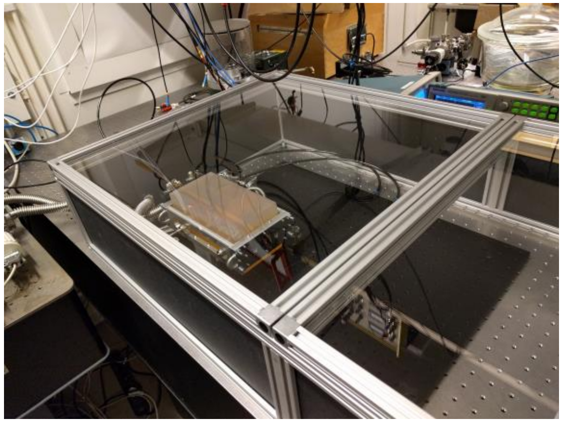


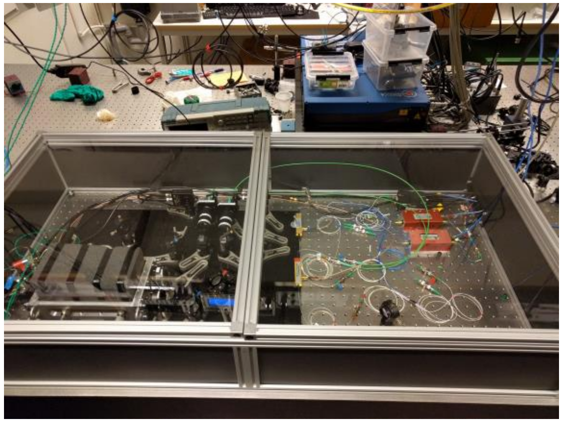
Schematic illustration of the Dual Fabry-Perot Refractometer.
Blue lines: optical fibers; Red lines: free-space light paths; **Black** lines: electrical wires;
EDFL: Er-doped Fiber Laser, λ = 1.55 μm, scannable over 6 GHz;
AOM: Acousto-optic modulator;
EOM: Electro-optic modulator;
PBS: polarizing beam splitter cube;
$\lambda/4$: quarter-wave plate;
90/10 and 50/50: beam splitter / merger;
Trans. / Ref. / Beat Detector: Detectors for the transmitted light, the reflected light and the beat signal.
Pt-100: thermo coupler.
Mirrors are pressed against the cavity; no gluing, no optical contacting. Allows for cleaning.

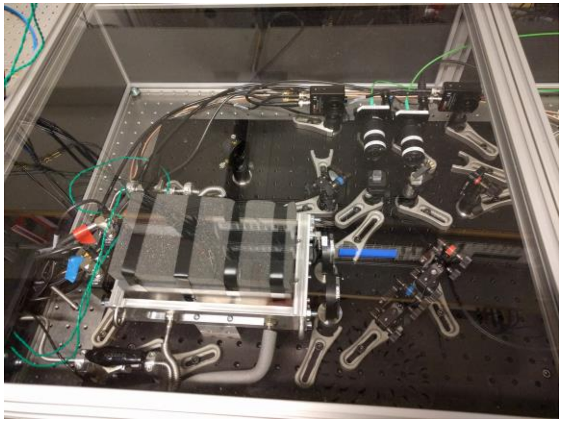
Next Generation of a Dual Fabry-Perot Cavity based GAMOR Spectrometer

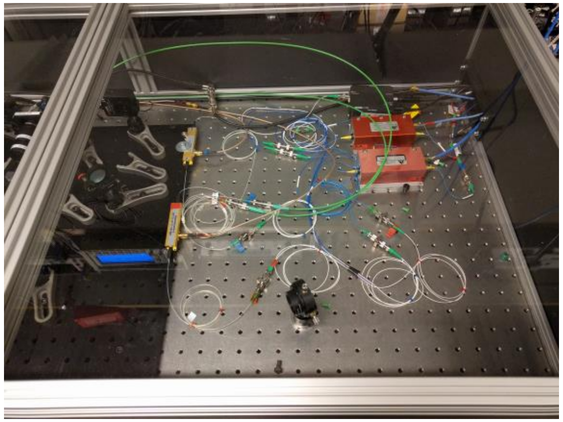












GAS MODULATION REFRACTOMETRY (GAMOR)

The research concerned with development of the GAMOR methodology is presented in a series of papers.

- I. Silander, T. Hausmaninger, M. Zelan, and O. Axner, "*Gas modulation refractometry for high-precision assessment of pressure under non-temperature-stabilized conditions*“, J. Vac. Sci. Technol. A 36, 03E105 (2018), https://doi.org/10.1116/1.5022244 .
- I. Silander, T. Hausmaninger, C. Forssén, M. Zelan, and O. Axner, “*Gas equilibration gas modulation refractometry for assessment of pressure with sub-ppm precision*”, J. Vac. Sci. Technol. B **37**, 042901 (2019), https://doi.org/10.1116/1.5090860 .
- I. Silander, C. Forssén, J. Zakrisson, M. Zelan, and O. Axner, "*Invar-based refractometer for pressure assessments*“, Opt. Lett. **45**, 2652-2656 (2020), https://doi.org/10.1364/OL.391708 .
- O. Axner, I. Silander, C. Forssén, J. Zakrisson, and M. Zelan, “*Ability of gas modulation to reduce the pickup of fluctuations in refractometry*”, J. Opt. Soc. Am. B 37, 1956 (2020), https://doi.org/10.1364/JOSAB.387902 .
- J. Zakrisson, I. Silander, C. Forssén, M. Zelan, and O. Axner, "*Procedure for robust assessment of cavity deformation in Fabry–Pérot based refractometers*“, J. Vac. Sci. Technol. B **38**, 054202 (2020), https://doi.org/10.1116/6.0000375 .

Gas Modulation Refractometry (GAMOR)

- Isak Silander, Clayton Forssén, Johan Zakrisson, Martin Zelan, Ove Axner. "*An Invar-based Fabry-Perot cavity refractometer with a gallium fixed-point cell for assessment of pressure*". ACTA IMEKO, 9, 5, pp. 293-298 (2020) https://doi.org/10.21014/acta_imeko.v9i5.987 .
- Martin Zelan, Isak Silander, Clayton Forssén, Johan Zakrisson, Ove Axner. "*Recent advances in Fabry-Perot-based refractometry utilizing gas modulation for assessment of pressure*", ACTA IMEKO, 9, 5, pp. 299-304 (2020) https://doi.org/10.21014/acta_imeko.v9i5.988 .
- Clayton Forssén, Isak Silander, David Szabo, Gustav Jönsson, Martin Bjerling, Thomas Hausmaninger, Ove Axner, Martin Zelan. "*A transportable refractometer for assessment of pressure in the kPa range with ppm level precision*". ACTA IMEKO, 9, 5, pp. 287-292 (2020) https://doi.org/10.21014/acta_imeko.v9i5.986 .

GAS MODULATION REFRACTOMETRY (GAMOR)

Additional papers that deal with gas modulation refractometry are.

- O. Axner, I. Silander, T. Hausmaninger, and M. Zelan,“*Drift-Free Fabry-Perot-Cavity-based Optical Refractometry -- Accurate Expressions for Assessments of Gas Refractivity and Density*”, e-print arXiv:1704.01187v2 (2017), https://arxiv.org/abs/1704.01187v2 .
- I. Silander, T. Hausmaninger, M. Zelan, and O. Axner, “Fast Switching Dual Fabry-Perot Cavity Optical Refractometry - Methodologies for Accurate Assessment of Gas Density”, e-print arXiv:1704.01186v2 (2017), https://arxiv.org/abs/1704.01186v2 .
- M. Zelan, I. Silander, T. Hausmaninger, and O. Axner, ”*Fast Switching Dual Fabry-Perot-Cavity-based Optical Refractometry for Assessment of Gas Refractivity and Density - Estimates of Its Precision, Accuracy, and Temperature Dependence*”, e-print arXiv:1704.01185v2 (2017), https://arxiv.org/abs/1704.01185v2 .

GAS MODULATION REFRACTOMETRY (GAMOR)

A short course in the GAMOR methodology can be based on three of these papers, *viz.*

- Paper 1: "*Gas modulation refractometry for high-precision assessment of pressure under non-temperature-stabilized conditions*“, I. Silander, T. Hausmaninger, M. Zelan, and O. Axner, J. Vac. Sci. Technol. A 36, 03E105 (2018), https://doi.org/10.1116/1.5022244 .
- Paper 2: "*Invar-based refractometer for pressure assessments*“, I. Silander, C. Forssén, J. Zakrisson, M. Zelan, and O. Axner, Opt. Lett. **45**, 2652-2656 (2020), https://doi.org/10.1364/OL.391708 .
- Paper 3: "*Procedure for robust assessment of cavity deformation in Fabry–Pérot based refractometers*“, J. Zakrisson, I. Silander, C. Forssén, M. Zelan, and O. Axner, J. Vac. Sci. Technol. B **38**, 054202 (2020), https://doi.org/10.1116/6.0000375 .

Read these.

From these, you should be able to get a basic understanding of the methodology.

In particular, you should be to assess

- How well can it be developed to a technique for assessment of refractivity, gas density, and pressure?
- What are its benefits and advantages of using GAMOR?

PAPER 1

Gas modulation refractometry for high-precision assessment of pressure under non-temperature-stabilized conditions

Isak Silander and Thomas Hausmaninger
Department of Physics, Umeå University, SE-901 87 Umeå, Sweden

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Measurement Science and Technology, RISE Research Institutes of Sweden, SE-501 15 Borås, Sweden

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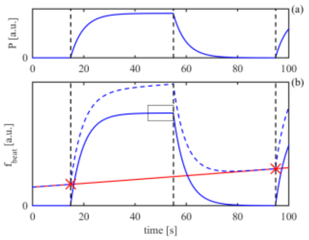
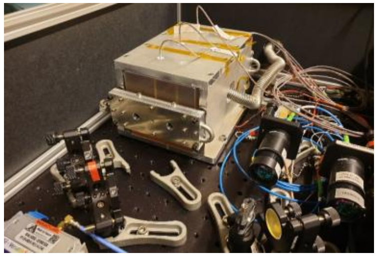
(Received 12 January 2018; accepted 28 March 2018; published 17 April 2018)

The authors report on the realization of a novel methodology for refractometry—GAs modulation refractometry (GAMOR)—that decreases the influence of drifts in Fabry Perot cavity refractometry. The instrumentation is based on a dual Fabry-Perot cavity refractometer in which the beat frequency between the light fields locked to two different cavities, one measurement and one reference cavity, is measured. The GAMOR methodology comprises a process in which the measurement cavity sequentially is filled and evacuated while the reference cavity is constantly evacuated. By performing beat frequency measurements both before and after the finite-pressure measurement, zero point references are periodically created. This opens up for high precision refractometry under nontemperature-stabilized conditions. A first version of an instrumentation based on the GAMOR methodology has been realized and its basic performance has been scrutinized. The refractometer consists of a Zerodur cavity-block and tunable narrow linewidth fiber lasers operating within the C34 communication channel (i.e., around 1.55 μm) at which there are a multitude of fiber coupled off-the-shelf optical, electro-optic, and acousto-optic components. The system is fully computer controlled, which implies it can perform unattended gas assessments over any foreseeable length of time. When applied to a system with no active temperature stabilization, the GAMOR methodology has demonstrated a 3 orders of magnitude improvement of the precision with respect to conventional static detection. When referenced to a dead weight pressure scale the instrumentation has demonstrated assessment of pressures in the kilo-Pascal range (4303 and 7226 Pa) limited by white noise with standard deviations in the $3.2N^{-1/2}$–$3.5N^{-1/2}$ mPa range, where N is the number of measurement cycles (each being 100 s long). For short measurement times (up to around 10^3 s), the system exhibits a (1σ) total relative precision of 0.7 (0.5) ppm for assessment of pressures in the 4 kPa region and 0.5 (0.4) ppm for pressures around 7 kPa, where the numbers in parentheses represent the part of the total noise that has been attributed to the refractometer. As long as the measurement procedure is performed over short time scales, the inherent properties of the GAMOR methodology allow for high precision assessments by the use of instrumentation that is not actively temperature stabilized or systems that are affected by outgassing or leaks. They also open up for a variety of applications within metrology; e.g., transfer of calibration and characterization of pressure gauges, including piston gauges. https://doi.org/10.1116/1.5022244



Paper 1

Gas modulation refractometry for high-precision assessment of pressure under non-temperature-stabilized conditions

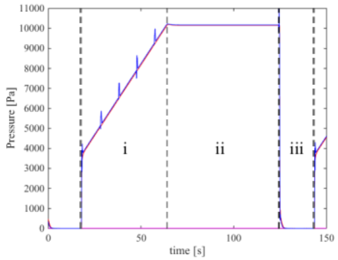


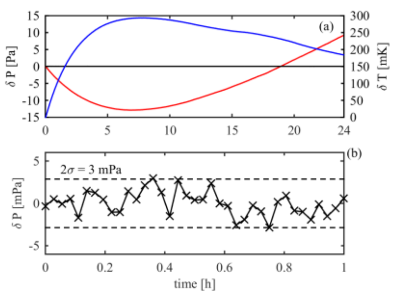
This paper presents the basis of the Gas Modulation Refractometry (GAMOR) technique.

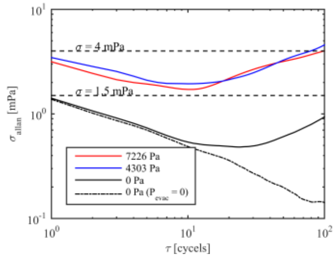
Concentrate mainly on the “Principles and Procedure” and the “Result” sections.

Make sure you understand

- How the GAMOR methodology is carried out; what gas supply procedure is applied to the DFPC; when is data measured; how is the data treated/evaluated?
- Explain what Fig. 5 displays and how this describes the GAMOR methodology.
- What does Fig. 6 show?
- What does Fig. 8 show?
- What advantages do we claim that the GAMOR methodology brings (why do we expect that it brings in some advantages and benefits) and what is the physical justification for its expected advantages?
- Which evidence for its advantages is given?
- It is not trivial for an unexperienced novice to fully understand all arguing and conclusions, but spend some time thinking about what information about the performance of the system Fig. 10 provides and which conclusions one can draw from this.







Paper 2

2652 Vol. 45, No. 9 / 1 May 2020 / *Optics Letters* **Letter**

Optics Letters

Invar-based refractometer for pressure assessments

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[2]*Measurement Science and Technology, RISE Research Institutes of Sweden, SE-501 15 Borås, Sweden*
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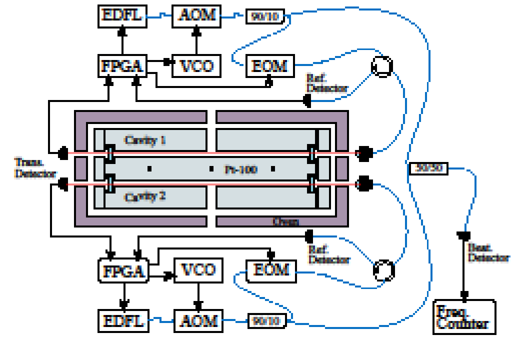
Received 5 March 2020; accepted 6 April 2020; posted 9 April 2020 (Doc. ID 391708); published 30 April 2020

Gas modulation refractometry (GAMOR) is a methodology that can mitigate fluctuations and drifts in refractometry. This can open up for the use of non-conventional cavity spacer materials. In this paper, we report a dual-cavity system based on Invar that shows better precision for assessment of pressure than a similar system based on Zerodur. This refractometer shows for empty cavity measurements, up to 10^4 s, a white noise response (for N_2) of 3 mPa $s^{1/2}$. At 4303 Pa, the system has a minimum Allan deviation of 0.34 mPa (0.08 ppm) and a long-term stability (24 h) of 0.7 mPa. This shows that the GAMOR methodology allows for the use of alternative cavity materials.

https://doi.org/10.1364/OL.391708



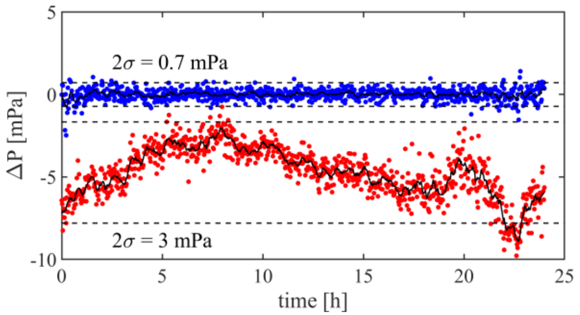
PAPER 2
INVAR-BASED REFRACTOMETER FOR PRESSURE ASSESSMENTS

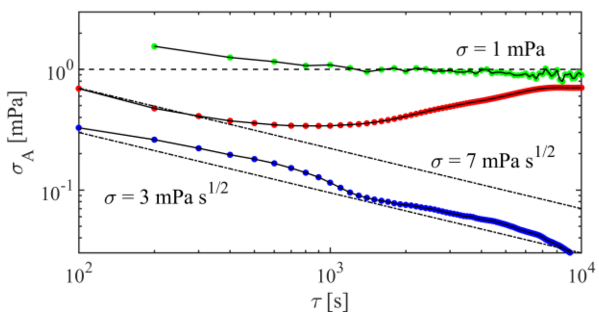
This paper presents the realization of a GAMOR-based refractometry instrumentation based on a cavity made of a non-conventional material - Invar.

A refractometry system based on a Fabry-Perot cavity made out of a metallic material has some advantages and disadvantages with respect to those made out of glass material (e.g. Zerodur)?

Make sure you understand

- Which are the advantages of using a metallic material for the cavity spacer?
- Virtually all FP-based refractometers have utilized various types of glass cavity spacer materials: very few have dared to construct a refractometry system made of a metallic material. What kept most people from doing that?
- Why did the Umeå University research group dare to do so?
- How much better could they make this system (as compared to the Zerodur-based system used in paper 1)?





PAPER 3

Procedure for robust assessment of cavity deformation in Fabry–Pérot based refractometers

Cite as: J. Vac. Sci. Technol. B **38**, 054202 (2020); doi: 10.1116/6.0000375
Submitted: 3 June 2020 · Accepted: 1 September 2020 ·
Published Online: 24 September 2020

View Online Export Citation CrossMark

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ABSTRACT

A novel procedure for a robust assessment of cavity deformation in Fabry–Pérot (FP) refractometers is presented. It is based on scrutinizing the difference between two pressures: one assessed by the uncharacterized refractometer and the other provided by an external pressure reference system, at a series of set pressures for two gases with dissimilar refractivity (here, He and N_2). By fitting linear functions to these responses and extracting their slopes, it is possible to construct two physical entities of importance: one representing the cavity deformation and the other comprising a combination of the systematic errors of a multitude of physical entities, viz., those of the assessed temperature, the assessed or estimated penetration depth of the mirror, the molar polarizabilities, and the set pressure. This provides a robust assessment of cavity deformation with small amounts of uncertainties. A thorough mathematical description of the procedure is presented that serves as a basis for the evaluation of the basic properties and features of the procedure. The analysis indicates that the cavity deformation assessments are independent of systematic errors in both the reference pressure and the assessment of gas temperature and when the gas modulation refractometry methodology is used that they are insensitive to gas leakages and outgassing into the system. It also shows that when a high-precision (sub-ppm) refractometer is characterized according to the procedure, when high purity gases are used, the uncertainty in the deformation contributes to the uncertainty in the assessment of pressure of N_2 with solely a fraction (13%) of the uncertainty of its molar polarizability, presently to a level of a few ppm. This implies, in practice, that cavity deformation is no longer a limiting factor in FP-based refractometer assessments of pressure of N_2.

Paper 3
Procedure for robust assessment of cavity deformation in Fabry–Pérot based refractometers

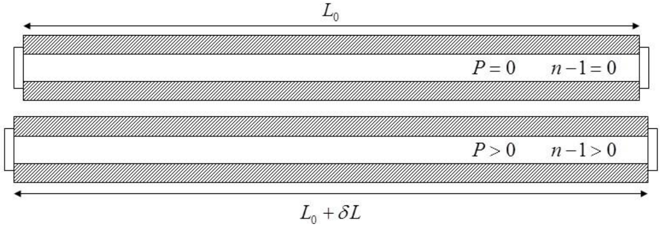
This paper presents a novel procedure for assessment of cavity deformation in Fabry–Pérot based refractometers.

In the presence of cavity deformation:

$$\nu_m^{(0)} = q_0 \frac{c}{2L_0} \quad \text{without gas}$$

$$\nu_m^{(g)} = q_0 \frac{c}{2n(L_0 + \delta L)} \quad \text{with gas}$$

The physical length of the cavity increases with pressure, given by $\delta L = \kappa P$



Therefore. the refractivity is given by : $(n-1)_i = \frac{\overline{\Delta f} + \overline{\Delta q}}{1 - \overline{\Delta f}} - \frac{\delta L}{L_0} = \frac{\overline{\Delta f} + \overline{\Delta q}}{1 - \overline{\Delta f} + \varepsilon}$ where $\varepsilon = (\delta L / L_0)/(n-1)$

Hence, refractivity can only be correctly assessed if we know the cavity deformation.

Paper 3
Procedure for robust assessment of cavity deformation in Fabry–Pérot based refractometers

It has been shown to be extraordinarily difficult to assess cavity deformation with the accuracy needed for future pressure assessments.

One way to deal with the problem of having an unknown cavity deformation (here referred to as conventional means) is to first measure the refractivity of the cavity when a given (but not necessarily known) pressure of a gas with a known molar polarizability is used.

This gives a value of the cavity deformation at that particular pressure.

If then the same pressure of another gas (whose molar polarizability is likewise known) is applied to the refractometry, the pressure can be assessed.

Make sure you understand

- Why this (in theory) can provide an assessment that is not affected by cavity deformation.
- What practical problems are associated by this conventional methodology?
- The paper provides a methodology that claims that it can circumvent these problems.
- Which are the main “pillars” of the novel methodology (what is it that we do that no one else has done before that can mitigate the problems)?

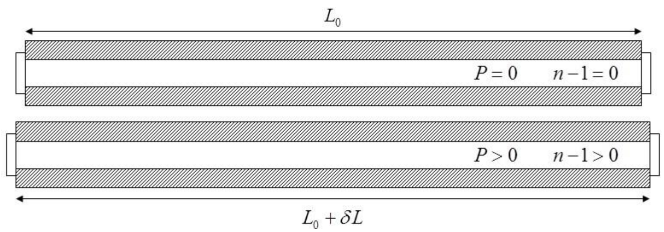
Paper 3
Procedure for robust assessment of cavity deformation in Fabry–Pérot based refractometers

This paper presents a novel procedure for assessment of cavity deformation (ε) in Fabry–Pérot based refractometers.

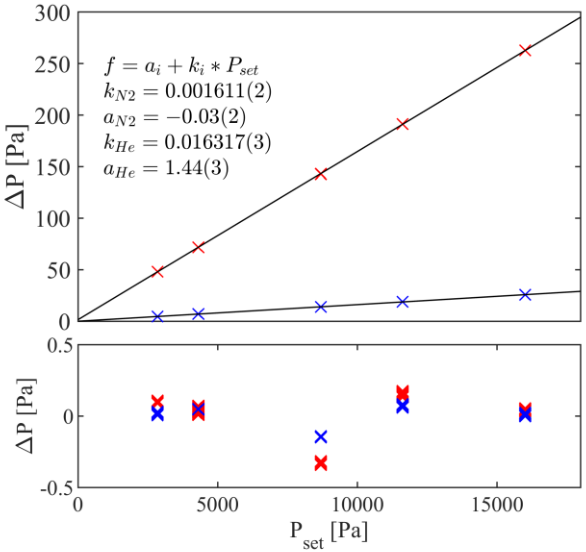
The procedure provides a robust characterization with small amount of uncertainties that is

- independent of systematic errors in the reference pressure, and gas temperature.
- and insensitive to gas leakages and outgassing processes.

It is predicted that when a high-precision refractometer is characterized according to the procedure, the uncertainty in the deformation will contribute to the uncertainty in the assessment of pressure with solely a fraction (13 %) of the uncertainty of the molar polarizability of N_2, presently is about 1 ppm.



$$(n-1)_i = \frac{\overline{\Delta f} + \overline{\Delta q}}{1-\overline{\Delta f}} - \frac{\delta L}{L_0} = \frac{\overline{\Delta f} + \overline{\Delta q}}{1-\overline{\Delta f} + \varepsilon}$$



Final Assessments

When you have finalized this self-study course, you should be able to answer the following specific questions:

1. What are the basic principles of the GAMOR methodology – How is it carried out?
2. In the papers, it is claimed and demonstrated that the novel technique provides improvements in comparison with conventional refractometry techniques for assessment of refractivity, density, and pressure.
    a. Why is the novel technique expected to be better than the conventional ?
    b. What is the logical or physical justification for this?
3. There are both advantages and disadvantages associated by using a cavity spacer of a metallic material for refractometry.
    a. Why did the Umeå University research group dare (and succeed) in using an Invar cavity (instead of classical glass materials)?
4. Which are the main "pillars" of the novel methodology for assessment of cavity deformation in FP-based refractometers presented in paper 3 (what is it that we do that no one else has done before that can mitigate the problems)?

if you have adequate answers to these questions, you have gain a first level of knowledge of the GAMOR methodology.