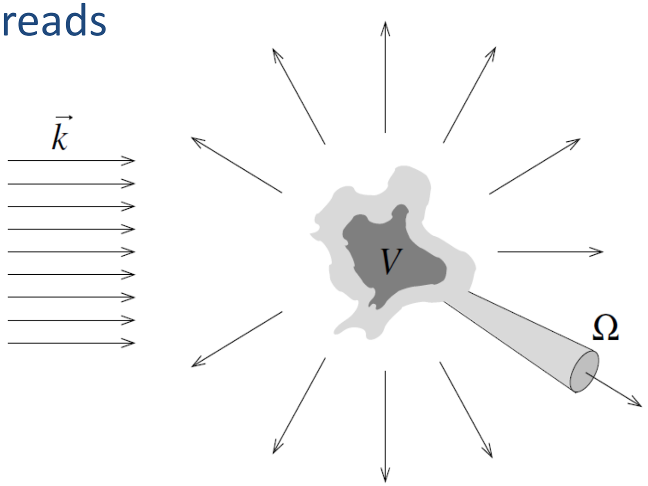
Voraussichtlicher Plan der Kurse

No.	Datum	
1	21.10.19	Einführung
2	24.10.19	Räumliche Symmetrien in Physik und Chemie, Drehimpulsoperator
3	28.10.19	Drehimpulsoperator, formale Definition, Eigenwerte und Eigenfunktionen
4	28.10.19	Kopplung der Drehimpulse, Clebsch-Gordan-Koeffizienten
5	04.11.19	Clebsch-Gordan-Koeffizienten: Eigenschaften
6	11.11.19	Drehungen im euklidischen Raum , Drehoperator
7	14.11.19	Eulersche Winkel, Wigner D-Matrizen
8	18.11.19	Irreduzible Tensoren, Wigner-Eckart-Theorem
9	21.11.19	Algebra der Tensoroperatoren, Auswertung von Matrixelementen
10	25.11.19	Grundlagen der Streutheorie, Lippmann-Schwinger-Gleichung
11	28.11.19	Greensche Funktion, Wirkungsquerschnitt
12	02.12.19	Streuamplitude, Wirkungsquerschnitt
13	05.12.19	Bornsche Näherung, Feynman-Diagramme
14	09.12.19	Niedrigenergie-Streuung, S-Wellen Streuung, Kalte Atome
15	12.12.19	Zeitabhängige Streutheorie, Ausbreitung von Wellenpaketen
16	16.12.19	S-Operator und S-Matrix

Lippmann-Schwinger equation

We have derived the Lippmann-Schwinger equation which reads for the correct asymptotics $r \rightarrow \infty$ as:

$$\psi_k{}^{(+)}(\boldsymbol{r}) = e^{i\boldsymbol{kr}} + \int G_0{}^{(+)}(E,\boldsymbol{r},\boldsymbol{r}')V(\boldsymbol{r}')\psi_k(\boldsymbol{r}')d\boldsymbol{r}'$$



Where the free-particle Green's function is given by:

$$G_0{}^{(+)}(E,\boldsymbol{r},\boldsymbol{r}') = -\frac{\mu}{2\pi\hbar^2}\frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}\prime|}}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

We have determined the asymptotic behaviour of the wavefunction:

$$\psi_{\boldsymbol{k}}{}^{(+)} = e^{i\boldsymbol{kr}} + f(\boldsymbol{k}',\boldsymbol{k})\frac{e^{ikr}}{r}$$

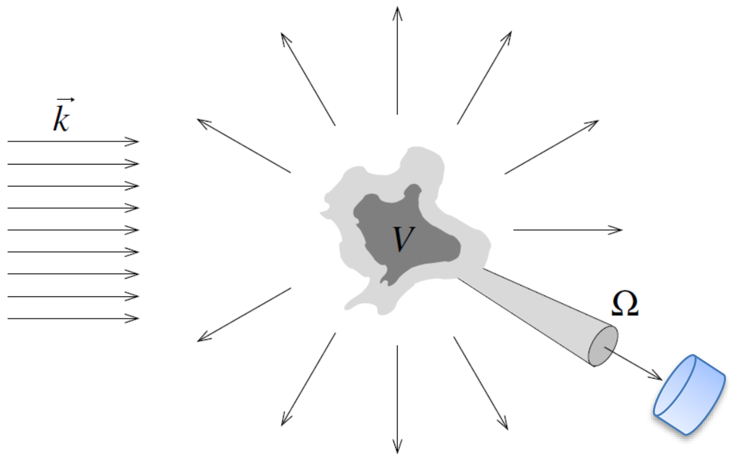
Where the scattering amplitude $f(k',k) = -\frac{\mu}{2\pi\hbar^2}\int e^{-i\boldsymbol{k}'\boldsymbol{r}'}V(\boldsymbol{r}')\psi_{\boldsymbol{k}}{}^{(+)}(\boldsymbol{r}')d\boldsymbol{r}'$ is directly related to the cross section:

How the scattering amplitude is related to observable physical properties?

Differential cross section

Definition: Cross section is the number of particles scattered (per unit of time) into the element of solid angle, divided by the incident flux:

$$d\sigma = \frac{dn}{j_0} \qquad dn = j_{sc}\, r^2\, d\Omega$$



Please, note: cross section has dimension *length*^{2}!

By employing solution of the Schrödinger equation:

$$\psi_{\boldsymbol{k}}^{(+)} = e^{i\boldsymbol{kr}} + f(\boldsymbol{k}', \boldsymbol{k})\frac{e^{ikr}}{r}$$

incident wave — scattered outgoing wave

We found the cross section: $$\frac{d\sigma}{d\Omega} = |f(k', k)|^2$$

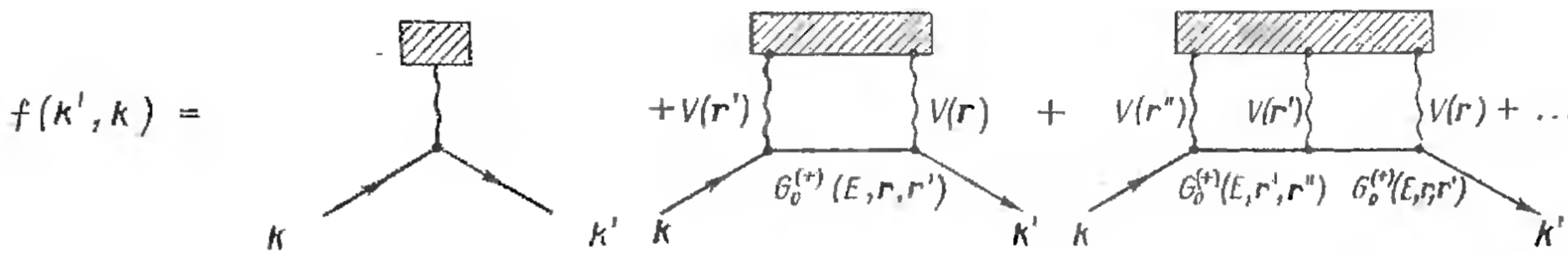
But! The problem is still not solved! We still don't know how to calculate scattering amplitude!

Born series

We have found the scattering amplitude by performing series expansion:

$$f(k',k) = \underbrace{-\frac{\mu}{2\pi\hbar^2}\int e^{-i\boldsymbol{k}'\boldsymbol{r}}V(\boldsymbol{r})e^{i\boldsymbol{k}\boldsymbol{r}}\,d\boldsymbol{r}}_{f^{(1)}(k',k)} \underbrace{-\frac{\mu}{2\pi\hbar^2}\int e^{-i\boldsymbol{k}'\boldsymbol{r}}V(\boldsymbol{r})G_0^{(+)}(E,\boldsymbol{r},\boldsymbol{r}')V(\boldsymbol{r}')e^{i\boldsymbol{k}\boldsymbol{r}'}d\boldsymbol{r}\,d\boldsymbol{r}'}_{f^{(2)}(k',k)}+\ldots$$

To deal with this expansion one may use simple Feynman diagram rules:



The first term in the expansion is the amplitude in the Born approximation:

$$f(k',k) = -\frac{\mu}{2\pi\hbar^2}\int e^{-i\boldsymbol{k}'\boldsymbol{r}}V(\boldsymbol{r})e^{i\boldsymbol{k}\boldsymbol{r}}\,d\boldsymbol{r} = -\frac{\mu}{2\pi\hbar^2}\int e^{i\boldsymbol{q}\boldsymbol{r}}V(\boldsymbol{r})\,d\boldsymbol{r}$$