Preliminary lecture plan

No	Date	Title of the lecture
1	09.04.2018	Motivation, Maxwell equation, Vector and scalar potentials, gauge invariance
2	16.04.2018	Hamiltonian for a particle in a field
3	23.04.2018	Time-dependent perturbation theory
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5	07.05.2018	Electric dipole approximation, selection rules, spin of photon
6	28.05.2018	Bound-electron dynamics in EM fields, Rabi oscillations, Bloch sphere
7	04.06.2018	Photoionization and radiative recombination
8	11.06.2018	Rayleigh and Thomson scattering of light
9	11.06.2018	Twisted light, vector potential, basic properties
10	18.06.2018	Fundamental atomic processes with twisted light, new selection rules
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Perturbation approach

In order to find solutions of the Hamiltonian of an atom in EM field we need to apply the perturbation theory:

$$i\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 - \frac{Ze^2}{r} - i\hbar\frac{e}{m}\boldsymbol{A}\boldsymbol{\nabla}\right]\Psi(\boldsymbol{r},t)$$

Unperturbed Hamiltonian H_0 (first two terms in the bracket); Perturbation $\lambda H'$ (last term)

We expand the time-dependent wave-function $\Psi(\boldsymbol{r},t)$ in the basis of well-known solutions of the unperturbed Hamiltonian:

$$\Psi(\boldsymbol{r},t) = \sum_k c_k(t)\,\psi_k(r)\,e^{-iE_k t/\hbar}$$

After some algebra we have found the rate for the excitation of an atom under absorption of light:

$$\Gamma_{ab} = \pi\left(\frac{e}{m}\right)^2 A_0^2(\omega_{ab})\left|\langle\psi_b|e^{i\boldsymbol{k}\boldsymbol{r}}\,\boldsymbol{\epsilon}\,\boldsymbol{\nabla}|\psi_a\rangle\right|^2$$

Electric dipole approximation

For the case of interaction of visible light with light atomic systems we can expect that:

$$e^{ikr} = 1 + ikr + \frac{1}{2}(ikr)^2 + \cdots \propto 1$$

This allows us to simplify the transition matrix element:

$$\langle \psi_b | e^{ikr}\, \boldsymbol{\epsilon}\, \boldsymbol{\nabla} | \psi_a \rangle \approx \langle \psi_b | \boldsymbol{\epsilon}\, \boldsymbol{\nabla} | \psi_a \rangle$$

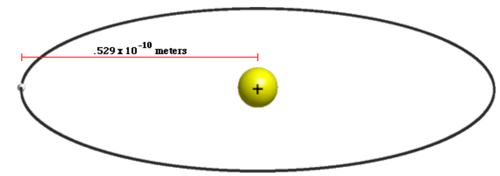
And to write it, after use of the Heisenberg equation of motion, as:

$$\langle \psi_b | \boldsymbol{\epsilon}\, \boldsymbol{\nabla} | \psi_a \rangle = -\frac{\boldsymbol{\epsilon}\, \omega_{ab}}{\hbar} \langle \psi_b | \boldsymbol{r} | \psi_a \rangle$$

We need to calculate this matrix element and to find the selection rules!

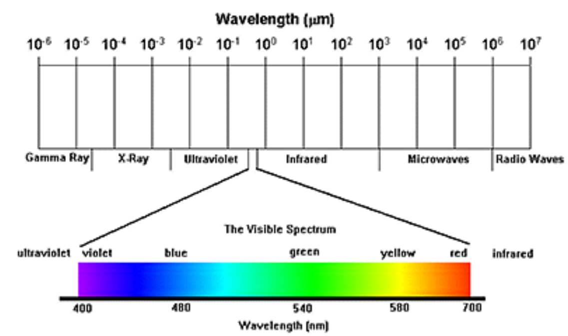
Size of light atom

$r \propto 10^{-8}\ cm$



Wavevector

of visible light $r \propto 10^{-8}\ cm$



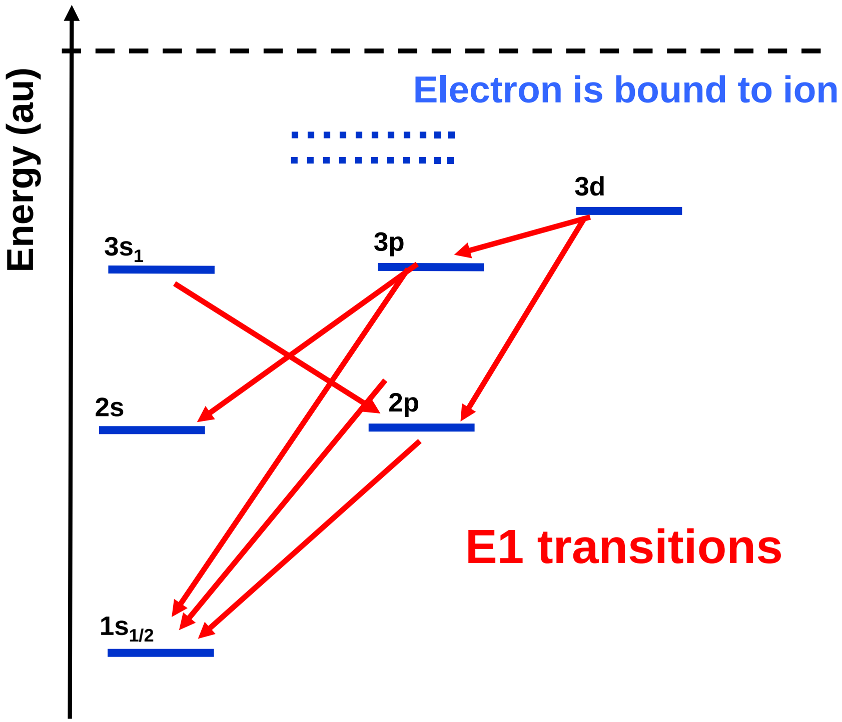
Dipole selection rules

From the analysis of dipole matrix element we found:

$$\langle\psi_b|\boldsymbol{\epsilon}\,\boldsymbol{\nabla}|\psi_a\rangle = -\frac{\epsilon\,\omega_{ab}}{\hbar}\langle\psi_b|\boldsymbol{r}|\psi_a\rangle \propto \sum_q \epsilon_q^*\,\langle l_a m_a\,1q|l_b m_b\rangle\,\langle l_a 0\,10|l_b 0\rangle$$

And from the properties of Clebsch-Gordan coefficients we have determined the selection rules:

$$\Delta l = \pm 1,\;\; m_a + q = m_b$$



What to do if electric dipole transition is forbidden?

Higher-order transitions

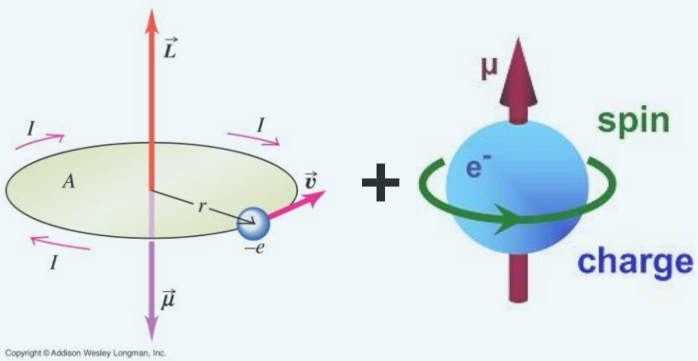
We have to consider the next term in the multipole expansion:

$$e^{ikr} = 1 + ikr + \frac{1}{2}(ikr)^2 + \cdots$$

By analyzing this term we derive the matrix element:

$$M_{ab}^{(M1,E2)} \propto \underbrace{\int \psi_a^+(\mathbf{r}) L_y \psi_b(\mathbf{r}) d\mathbf{r}}_{\text{magnetic dipole (M1) term}} + i\omega_{ab} \underbrace{\int \psi_a^+(\mathbf{r})\, x\, z\, \psi_b(\mathbf{r}) d\mathbf{r}}_{\text{electric quadrupole (E2) term}}$$

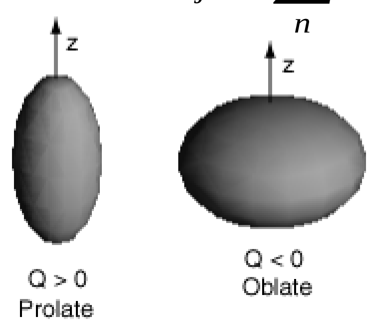
Proportional to magnetic moment of the ion (spin should be added in exact treatment)



$$\hat{\boldsymbol{\mu}} = -\mu_0 (\hat{\mathbf{L}} + g\hat{\mathbf{S}}) / \hbar$$

Proportional to electric quadrupole moment of the ion

$$Q_{ij} = \sum_n q_n \left(3x_i x_j - x^2 \delta_{ij}\right)$$



deviation from spherical shape!

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