

**Modeling in measuring devices calibration
procedures.
(for discussion)**

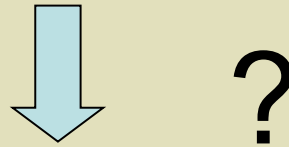
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Introduction

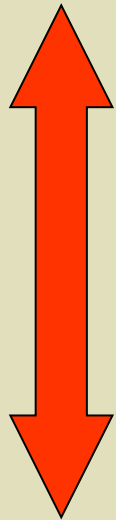
Application of uncertainty evaluation framework to evaluation of data in calibration of measuring instruments



- formulating the measurement model,
- assigning the priory distributions to the input and output quantities,
- calculation of instrumental uncertainty,
- interrelation between calibration and checking compliance with the requirements

The procedure for data evaluation at calibrating MI comprises four steps:

1. constructing measurement models,



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- 2. calculating uncertainties of input quantities,**
- 3. calculating uncertainty of output quantity,**

4. analyzing (interpretation) the results

Classification of measurement models

measurement task	comparison	calibration	testing (control, verification)	others ?
measurement method	<ul style="list-style-type: none"> • direct or indirect measurements, • comparisons (substitution method,...), • 			
field of measurement	physical model + measuring instrument model + measurement conditions			

Calibration [VIM, 2.39]

- **Calibration** - operation that, under specified conditions, in a first step establishes a *relation between the quantity values* with measurement uncertainties *provided by measurement standards* and *corresponding indications (I)* with associated measurement uncertainties and, in a second step, uses this information to establish a *relation for obtaining a measurement result from an indication (II)*.

Expression of calibration

First step

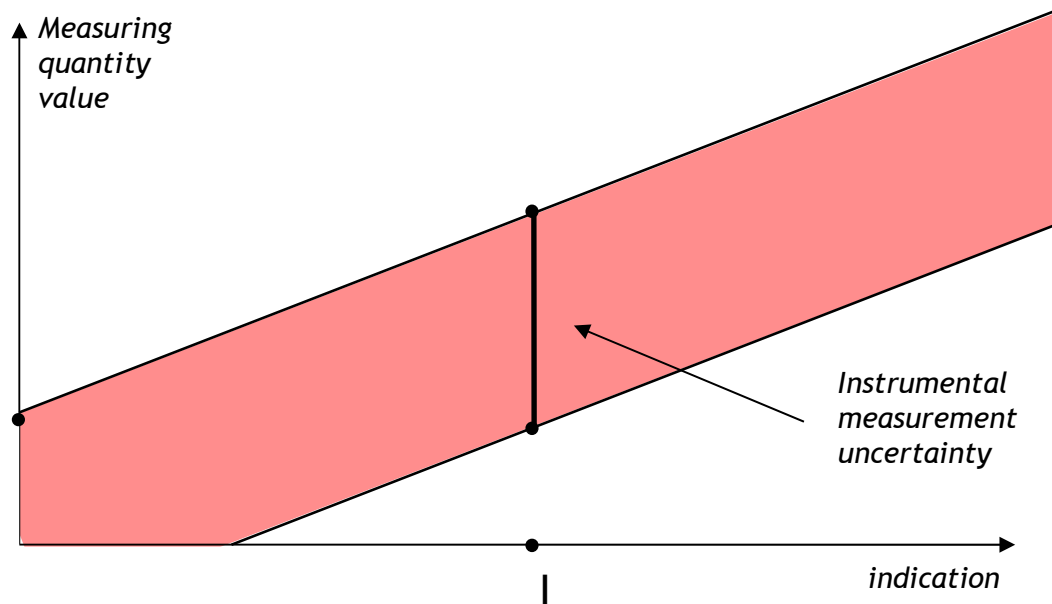
- calibration function
- calibration table

$$I = f(X)$$

Second step

- calibration diagram - graphical expression of the relation between indication and corresponding measurement result

$$X = \tilde{f}^{-1}(I)$$



First step. Modeling

- relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications

$$I = f(X)$$

Error

$$\Delta(X_{ref}) = I_{cal} - X_{ref}$$

Calibration coefficient

$$k = \frac{I_{cal}}{X_{ref}}$$

others

Measurement model

**Characteristic of
calibrating measuring
instrument**

$$\Delta(X_{ref}) = I_{cal} - X_{ref}$$

+

**Measurement procedure
(method):**

**Reference
operating
condition**

**influence quantities
(frequency, temperature,
pressure,...)**

$$\Delta(X_{ref}) = I_{cal} - X_{ref} + G(Z_1, \dots, Z_k)$$

Evaluating measurement uncertainty

prior information measurement results

$$I, X_{ref}, Z_1, \dots, Z_k$$



input quantities' values + associated uncertainties



**Error (instrumental bias)
+ associated uncertainty**

+



prior information about error - maximum permissible measurement error (MPE)



prior distribution of input quantities and MI error

checking consistency of relevant information



posterior distribution of MI error, its best estimate and associated uncertainty

$$\hat{\Delta}(x_{ref}), u(\hat{\Delta}(x_{ref}))$$

The relation for obtaining a measurement result from MI indication: some practical solutions:

- **Correction for systematic bias:**

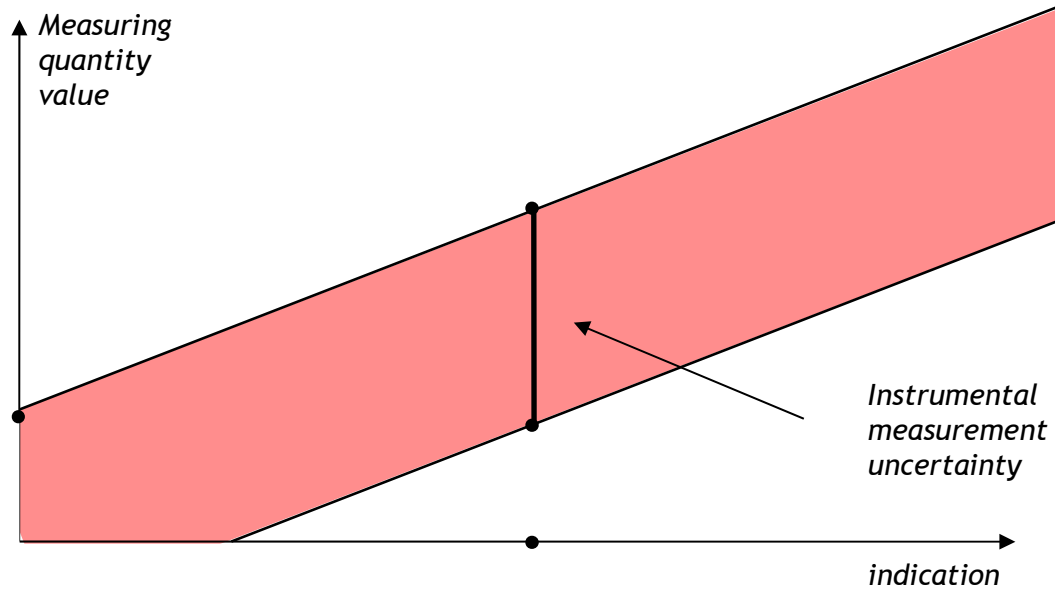
$$I_{cal}^{correct} = I_{cal} - \Delta(X_{ref})$$

$$u_{inst}(x(t)) = u(\Delta(x_{ref}))$$

- **Without correction:**

$$u_{inst} = \sqrt{\Delta^2(X_{ref}) + u^2(\Delta(x_{ref}))}$$

Step II. Modeling

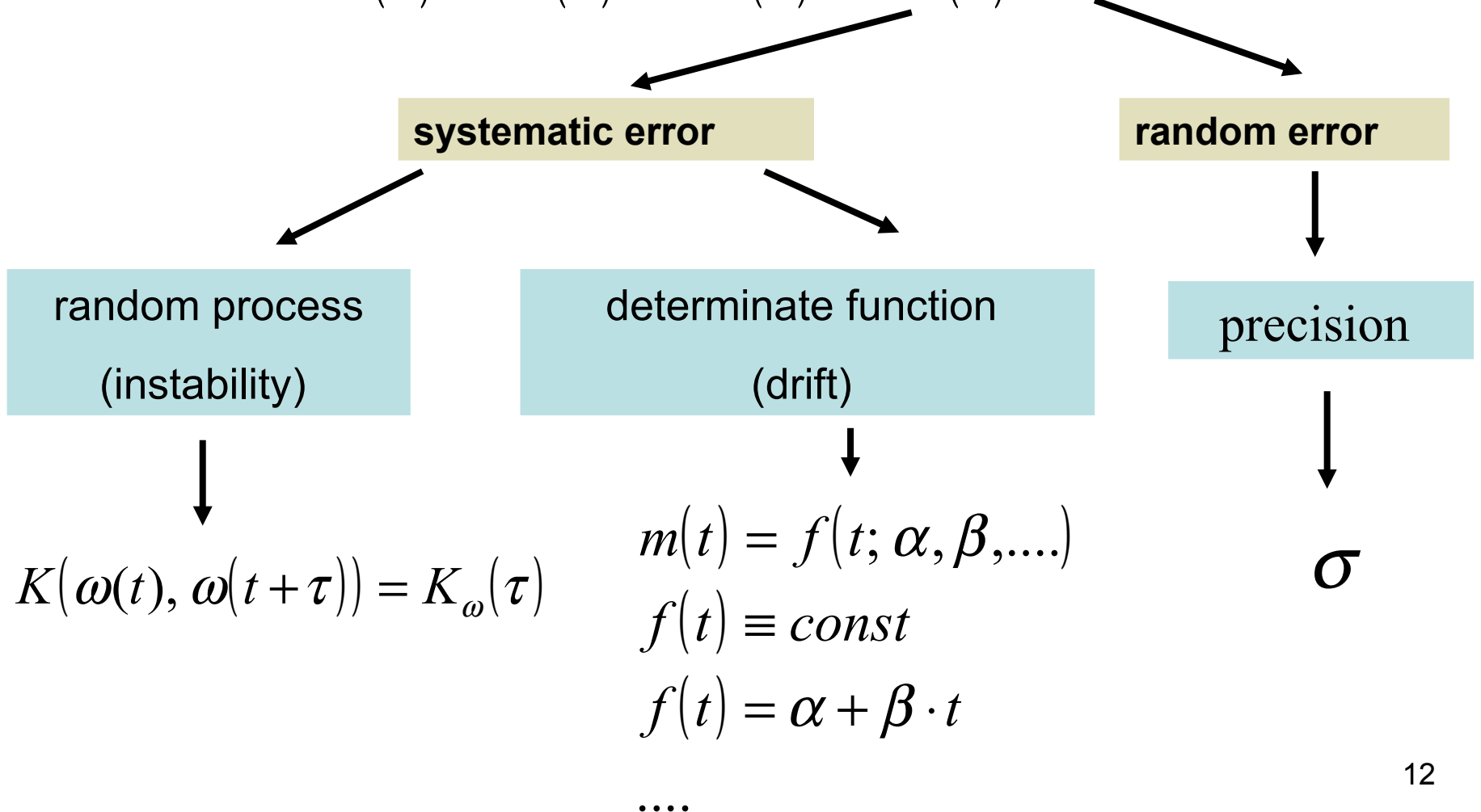


$$u_{instr}(x)$$

$$\hat{\Delta}(t), u(\hat{\Delta}(t)) \quad \xrightarrow{\text{?}} \quad x(t + \tau), u_{inst}(x(t + \tau))$$

Measuring instrument error modeling

$$\Delta(t) = I(t) - X(t) = \omega(t) + \varepsilon$$



prior information

- maximum permissible measurement error (MPE) θ
- precision (SD) σ
- results of previous calibration

I

results of calibration

$$\hat{\Delta}(x_{ref}, t_0), u(\hat{\Delta}(x_{ref}, t_0))$$

II

III

investigation of stability of MI error

Analysis and interpretation of all available information

“evaluation approach”:

$$p(\omega(t+\tau) | \omega(t)) = \frac{p(\omega(t+\tau), \omega(t))}{p(\omega(t))}$$

estimate of MI error and associated uncertainty

$$E(\omega(t+\tau) | \omega(t)), D(\omega(t+\tau) | \omega(t))$$

“checking approach”:

???

$$\Delta(t_0) < \theta$$

$$u_{inst} = \frac{\theta}{\sqrt{3}}$$