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Likelihood and posterior probability mass function for countings

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Abstract

In many high-level measurements (for example, in fields such as time and frequency, optics, ionizing radiations, microbiology and chemistry) as well as in everyday life, counting processes often play an important role. Like any other result of a measurement, also the result of a measurement by counting should be the whole “set of quantity values being attributed to the measurand together with any other available relevant information” [1, definition 2.9]. The result might be expressed as an estimate (the number of item actually counted) and an associated uncertainty (or coverage interval), thus following the frame of the “Guide to the expression of uncertainty in measurement” (GUM) [2], or as a probability density function (for discrete quantities, a probability mass function, pmf, actually), as recommended in the Supplement 1 to the GUM [3].

The issue of uncertainty in countings has received little attention in the metrological literature; specifically, neither of the mentioned documents provide any guidance on the topic. There exists a guide for the uncertainty of counting colonies and other discrete quantities in microbiology [4].

The occurrence of error in counting is real and needs to be addressed. It might occur, for example, that one fails in counting an object because of some reasons, such as human or instrumental errors. In such a case, the measurand, i.e., the number of items intended to be counted, is underestimated. On the other hand, one may count a non-existing object (or count twice an existing object), hence obtaining an overestimate of the measurand.

In a previous paper [5], we proposed a general model for measurements by counting which allows an evaluation of the uncertainty compliant with the general framework of the GUM. The present work considers the same *scenario*, but facing the problem from the viewpoint of Bayesian inference. In particular, we derive a discrete likelihood function of the measurand, given the number

of counted objects, which accounts for the probability of errors, and show how, by means of the Bayes-Laplace’s rule, a posterior pmf can be associated to the measurand, depending on the chosen prior distribution. We discuss the sensitivity of the model to different prior distributions and present the results obtained from the application of the method to theoretical and practical examples.

[1] International Vocabulary of Metrology – Basic and General Concepts and Associated Terms JCGM 200:2012 (JCGM 200:2008 with minor corrections)

[2] Evaluation of measurement data – Guide to the expression of uncertainty in measurement JCGM 100:2008 (GUM 1995 with minor corrections)

[3] Evaluation of measurement data – Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method JCGM 101:2008

[4] MIKES - Centre for Metrology and Accreditation (Helsinki 2003). Uncertainty of quantitative determinations derived by cultivation of microorganisms. Seppo I. Niemelä. Publication J4/2003

[5] W. Bich and F. Pennecchi, Metrologia 2012, 49 (1), 15-19