Sampling based polynomial chaos approaches for uncertainty propagation

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We consider a general problem F(u,y)=0 where u is the solution of the problem and y a set of uncertain parameters whose uncertainty is assumed known in terms of suitable probability distributions. We address the situation in which the parameter-to-solution map u(y) is smooth, however y could be very high (or even infinite) dimensional. In particular, we are interested in cases in which F is a differential operator and y a distributed, space and/or time varying, random field. We aim at effectively propagate the input uncertainty on y onto the solution u of the problem.

In this talk, we consider multivariate polynomial approximations of the parameter-to-solution map u(y). These can be obtained by collocating the equation F(u,y)=0 on sparse grids of Gauss points (zeros of the orthogonal polynomials with respect to the underlying probability density functions), or by discrete least square approximation (regression) starting from random, noise-free evaluations of the map u(y).

We present some recent stability results for the least square approach and we discuss possible strategies to select suitable high dimensional multivariate polynomial spaces, based either on a priori information relying on the smoothness properties of the parameter-to-solution map, or on adaptive, greedy type, algorithms.

Some convergence results and numerical tests will be presented for the case of elliptic partial differential equations with random coefficients.