

# Bayesian Inference of a Linear Mixed Model for Sonic Nozzle Calibration

G. Wübbeler, B. Mickan and C. Elster

Physikalisch-Technische Bundesanstalt, Braunschweig and Berlin, Germany

## Abstract

Critical sonic nozzles are used for the calibration of flow meters and also as highly stable transfer standards [1-3]. We consider the characterization of a sonic nozzle in terms of a linear model between the effective area of the sonic nozzle and the inverse square root of the Reynolds number. The model is identified from calibration data which consist of several measurement series. However, while each series is well described in terms of a linear model, intercept and slope of the linear models obtained for the single series differ significantly. Simple least-squares procedures fail to account for this between series variation and uncertainties obtained by such approaches can turn out significantly underrated.

We apply a linear mixed model [4-6] that addresses the between series fluctuation of intercept and slope by random effects. The particular linear mixed model used for the analysis of the sonic nozzle calibration data is described as well as its identification using Bayesian inference. In this context we also discuss the selection of an appropriate prior. The numerical calculation of the posterior distribution is done by applying Markov Chain Monte Carlo techniques. In addition, we also consider the Laplace approximation method [7].

Results for calibration data will be presented and accompanied by a sensitivity analysis. Particular focus is put on predictions made by the inferred model. We show that resulting prediction intervals cover the observed significant variability of different measurement series in a reasonable way.

## References

- [1] J Wright et al 2007 CIPM key comparison for low-pressure gas flow: CCM.FF-K6 *Metrologia*, 44, 07008
- [2] B Mickan et al. 2006 Comparisons by PTB, NIST, and LNE-LADG in Air and Natural Gas by Means of Critical Venturi Nozzles Agree within 0.05 %, *Proceedings of the 6<sup>th</sup> International Symposium on Fluid Flow Measurement*, Queretaro, Mexico.
- [3] B Mickan and R Kramer 2009 Experiences with sonic nozzles used for different gases and wide range of pressure and temperature conditions, *Proceedings of the 7<sup>th</sup> International Symposium on Fluid Flow Measurement*, Anchorage, Alaska.
- [4] C E McCulloch, S R Searle, J M Neuhaus 2006 *Generalized, Linear, and Mixed Models*, John Wiley & Sons.
- [5] J C Pinhero and M Bates 2000 *Mixed-Effects Models in S and S-PLUS*, Springer Verlag.
- [6] S R Searle, G Casella, C E McCulloch 2009 *Variance Components*, Wiley-Interscience.
- [7] L Tierney and J B Kadane 1986 Accurate approximations for posterior moments and marginal distributions, *J. Amer. Statist. Assoc.*, 81:82–86.