Abstract for MATHMET 2014, International Workshop on Mathematics and Statistics for Metrology, PTB Berlin, March, 24-26, 2014.

Bayesian analysis of a flow meter calibration problem

Gertjan Kok[1], Adriaan van der Veen[1], Peter Harris[2], Ian Smith[2]

Corresponding author's email address: gkok@vsl.nl

[1]VSL, Thijsseweg 11, 2629 JA Delft, the Netherlands
[2]National Physical Laboratory (NPL), Hampton Road, Teddington, Middlesex TW11 0LW, UK

A flow meter is a device for measuring the quantity of fluid (e.g., water) transported through a pipeline, both measured as transported volume in a given time and as transported volume per unit of time (flow rate) [1]. A flow meter has typically (at least) two kinds of outputs: an analogue output for flow rate and a pulse or frequency output for (total) volume. For the latter output a conversion factor is required. This *pulse factor* (often referred to as a "*k*–factor") is determined, meaning that one output pulse is equivalent to a specified volume that has passed the meter (e.g., 10 pulses/L or alternatively 100 mL/pulse). In a flow meter calibration this pulse output is calibrated, i.e., the pulse factor k is the explained variable in the regression model, and flow rate q the regressor variable. For these calibrated pulse factors a regression curve $k(q)$ is determined as a function of the (reference) flow rate. This curve can then be used to correct the flow meter reading at a measured flow rate. A model of the form

$$k_i = \beta_0 + \beta_1 q_i^{r_1} + \ldots + \beta_p q_i^{r_p} + \varepsilon_i , \tag{1a}$$

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2) \tag{1b}$$

is often used to describe flow meter calibration measurements. The $k_i, i = 1, \ldots, n$, denote measurements of the pulse factor for the corresponding flow rates q_i , and $r_1, \ldots, r_p$ denote powers (e.g., -1, 1, 2, 3 with p=4). The powers are known from previous validation work.

In the framework of EMRP project NEW04 "Novel mathematical and statistical approaches to uncertainty evaluation" model (1) is being studied from a Bayesian point of view and the results are being compared with those obtained using a classical approach. In particular prior knowledge has been collected from discussions with an application expert and this knowledge has been interpreted in various ways, so as to investigate the sensitivity of the results to the prior knowledge. An interesting aspect is that the prior knowledge concerns the values k of the curve $k(q)$, whereas in the Bayesian framework prior knowledge distributions are usually attributed to the parameters $\beta_0, \beta_1, \ldots, \beta_p$ and (possibly) σ^2 . A further challenge is statistical calibration [2], i.e., the inverse use of the estimated calibration model to predict the flow rate given observation(s) of the pulse factor.

We present initial results obtained from the study, and present a comparison of the results obtained from the Bayesian and classical approaches to the calibration problem. In a future stage we will relax the conditions of model (1), i.e., we will consider heteroscedastic models and allow for temporal drifts of the regression curves.

1. R. C. Baker, *Flow Measurement Handbook*, Cambridge University Press (2000).
2. P. J. Brown, *Measurement, Regression, and Calibration*, Oxf. stat. sc. ser. (Clarendon Press Oxford, 1993).