Extreme value statistics in metrology – implications of using sample range in acceptance tests

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Abstract

The application of statistical evaluation techniques is ubiquitous during calculation of measurement uncertainty. The corresponding mathematical methods are described in the respective standards and guidelines and are also part of metrological education. However, due to its simple mathematics and ease of use, in general this seems to be restricted to statistics of sample mean and variance. This results in a common misconception regarding the statistical behavior of parameters derived from the extreme values of a sample, like the sample range for instance. For these kinds of parameters the extreme value statistics is valid, which will be introduced against the background of metrological applications. It will be shown, that parameters derived from extreme values, do not follow the rules of sample mean statistics. In particular the central limit theorem, i.e. the convergence of the probability distribution against a normal distribution with increasing sample size, does not hold true. Furthermore the resulting distribution for the parameter will be strongly dependent on the sample size, especially for an initially normal distributed process. For the uniform distribution we will exemplarily present analytical solutions for the statistics of sample range and mid-range. The resulting probability density function (PDF) for the uniform distribution in the interval [0,1] and a sample size of n=10 is shown in Fig. 1.

The implications of the aforementioned effects on metrological applications will be demonstrated and discussed exemplarily for the acceptance tests of coordinate measuring machines (DIN EN ISO 10360) and optical systems based on area scanning (VDI/VDE 2634-2). One of the parameters that has to be evaluated according to these guidelines is the probing error (form), for the measurement of a reference sphere. It is defined as the range of radial deviations from a best-fit sphere. By means of Monte-Carlo simulations the distribution of the probing error according to different guidelines has been calculated under the assumption of an identically normal distributed measurement process. We found that the expected value of these distributions covers nearly two orders of magnitude, making it useless for the comparison of optical and tactile probing techniques. It has to be pointed out however, that this is not to be attributed to the fact, that the comparison would be inherently difficult, but to the choice of the sample range as the characterizing parameter.

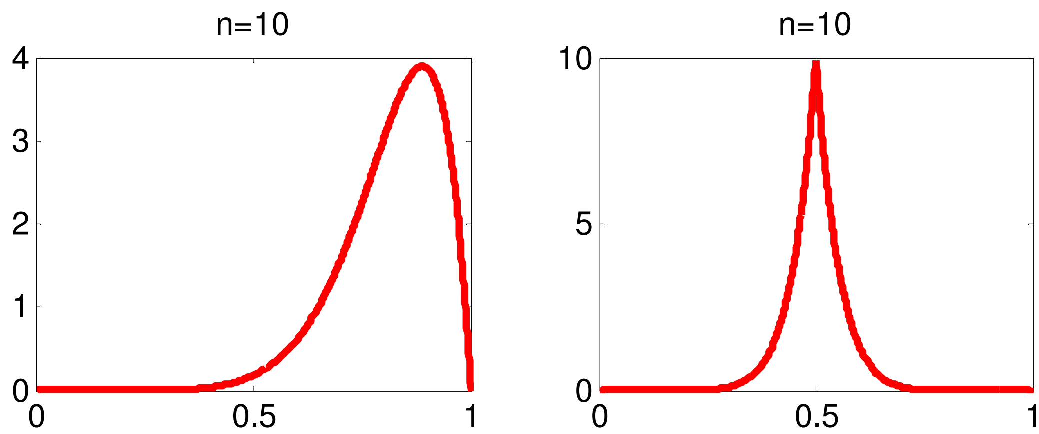

Fig. 1: PDFs of sample range (left) and sample mid-range (right) for a uniform-distribution in the interval [0,1] and a sample size of n=10