**Bayesian framework for proficiency tests using auxiliary information on laboratories**

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Proficiency tests (PT) are interlaboratory comparisons used to evaluate the proficiency of laboratories as external quality controls. Measurement bias is the indicator of the proficiency of a laboratory considered here. The usual way to summarize the results of a PT is to compute a consensus value and its associated uncertainty.

We consider hereby the case when laboratories do not report standard uncertainties or when the uncertainties are not validated independently (*e.g.* by a laboratory accreditation body, see NF ISO 13528 [1]).

We derive a complete Bayesian setting to compute the marginal posterior distribution of the consensus value under auxiliary information on laboratories and prior information on the consensus value. The approach extends the use of weighted mean for the consensus value by a double weighting: the weight of the prior and the weights of the results.

The approach is presented as an alternative method to algorithm A (NF ISO 13528 [1]), when auxiliary information on laboratories is available. Within this framework, it is immediate to tackle multivariate structures of auxiliary information.

This approach is motivated by two typical application frameworks yielding auxiliary information

i. selecting expert laboratories but still willing to combine results from the other participants (can be seen as an extension to the computation of consensus values from expert laboratories with algorithm A, given univariate auxiliary information),
ii. willing to give more weight to laboratories which use a better technique (univariate auxiliary information) or a better measurement process (multivariate auxiliary information).

In these cases the auxiliary information relies on expert knowledge : (i) to select expert laboratories and (ii) to compare techniques or measurement processes to achieve a given measurement.

Managing auxiliary information requires additional previous steps to traditional analysis of PTs

i. to generate expertise on a proficiency test and to select the most relevant information,
ii. to convert expertise into variables : *e.g.* a binary variable (coded 0/1) may encode the fact of being an expert laboratory (1) or not (0), an ordered categorical variable with modalities 1-2-3 may encode the fact that technique 1 is better than technique 2, which is better than technique 3…,
iii. to convert variables into weights of laboratories.

This approach requires a suitable measurement model based on a multiplicative bias

$$
\begin{aligned}
&x_i = \mu + \tau b_i \\
&b_i \sim N\left(0, \sigma_i^2\right) \\
&\sigma_i^2 \left| \mathbf{y} \sim \pi_i \left( \sigma_i^2 \right| \mathbf{y} \right)
\end{aligned}
$$

where $\mu$ is the consensus value, $b_i$ is the laboratory bias assumed centered gaussian and $\tau^2$ is a scaling parameter. In the limit case where $b_i = 0$, $\tau^2$ is set up to the sampling variance of results. The variance $\sigma_i^2$ of the bias is determined by a procedure depending on the nature of the auxiliary information $\mathbf{y}$ and returning a distribution of probability $\pi_i$.

In the univariate case, auxiliary information is given by a binary or ordered categorical observed variable. Then, a simple procedure to assign weights is derived based on the number of categories and the number of laboratories in each category. In the multivariate case, the procedure handles the structure of dependence between auxiliary variables based on the work by Demeyer et al (2010) [2] in which a structural equation model is used to capture the correlation structure of the auxiliary information.

References :

[1] ISO/TC69, Statistical methods for use in proficiency testing by interlaboratory comparisons ISO 13528:2005. International Organization for Standardization (ISO), Geneva, Switzerland, (2005).

[2] S. Demeyer , J.-L. Foulley , N. Fischer , G. Saporta - Bayesian analysis of structural equation models using parameter expansion, in "Statistical learning and data science", January 2012, Chapman & Hall/CRC, pp. 135-145, (isbn: 978-1-4398-6763-1)