Bayesian estimation of measurement trueness and measurement precision.

A Chunovkina, N.Burmistrova

The D.I.Mendeleyev Institute for Metrology , St.Petersburg, Russia

A.G.Chunovkina@vniim.ru

Measurement trueness and measurement precision are the basic properties of any measurement. These properties are quantitatively expressed by a measurement bias, b, and by a variance, σ, of measured quantity values obtained in specified condition of measurement (repeatability condition, intermediate precision condition or reproducibility condition of measurement).

Quite often the bias, an estimate of systematic measurement error, is a parameter of interest during the calibration of measuring instrument. In the process of calibration the measurements are carried out by a measurement standard and measuring system under calibration. The measurements performed by the measurement standard provide an estimate of the measurand value and associated measurement uncertainty $\{x_{ref}, u_{ref}\}$.

This information makes it possible to assign the normal prior pdf $p_{ref}(x)$, to the measurand with the expectations x_{ref} and standard deviation $\sigma_{ref} = u(x_{ref})$. The measured quantity values, $y_1, \dots, y_n$,obtained by the measuring instrument under calibration contain the information about precision of measuring instrument and about systematic bias as well as about measurand itself.

For the non-informative prior distribution $p(b, \sigma)$ and for the assumption of normal distribution of measured quantity values the Bayes' Theorem and marginalization yield the following posterior joint probability density function of σ and *b:*

$$p\left(b, \sigma \middle| y_1, \dots, y_n, x_{ref}, \sigma_{ref}\right) \propto \int \frac{1}{\sigma^{n+1}} exp\left\{-\frac{\sum (y_i - b - x)^2}{2\sigma^2}\right\} p_{ref}(x) dx \propto \quad (1)$$

$$\frac{1}{\sigma^n \sqrt{\sigma_{ref}^2 + \frac{\sigma^2}{n}}} \exp\left\{-\frac{\left(b - (\bar{y} - x_{ref})\right)^2}{2\left(\sigma_{ref}^2 + \frac{\sigma^2}{n}\right)}\right\} exp\left\{-\frac{\sum (y_i - \bar{y})^2}{2\sigma^2}\right\} ,$$

where $\bar{y} = \frac{1}{n} \sum y_i$.

Marginalization of (1) yields posterior distributions $p\left(b \middle| y_1, \dots, y_n, x_{ref}, \sigma_{ref}\right)$, $p(\sigma | y_1, \dots, y_n)$. In general case $p\left(b \middle| y_1, \dots, y_n, x_{ref}, \sigma_{ref}\right)$ can be calculated numerically.

$$p\left(b \middle| y_1, \dots, y_n, x_{ref}, \sigma_{ref}\right) = \int p\left(b, \sigma \middle| y_1, \dots, y_n, x_{ref}, \sigma_{ref}\right) d\sigma \quad (2)$$

The paper considers approximations of $p\left(b \middle| y_1, \dots, y_n, x_{ref}, \sigma_{ref}\right)$ by a normal pdf depending on the parameter $\gamma = \frac{\sigma^2}{n\sigma_{ref}^2}$. The best estimates of the bias and the corresponding coverage intervals, obtained using the approximate pdf are compared with those obtained by (2).