Bayesian treatment of a random effects model for the analysis of key comparisons

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Abstract

Key comparisons are interlaboratory comparison measurements carried out in order to establish the equivalence between national metrology institutes [1]. It has been proposed to apply random effects models for the analysis of key comparison data and to use the resulting estimates of the laboratory effects as the so-called degrees of equivalence between the participating laboratories [2], [3]. In [2], [3] maximum likelihood type estimators of the laboratory effects were taken. Here we consider a Bayesian approach. In contrast to [3] we restrict our treatment to the case of normally distributed laboratory effects. More precisely, the measurements $x_1, \ldots, x_n$ are assumed to be modeled as

$$x_i = \mu + \lambda_i + e_i\,,$$

where μ is the overall mean; the λ_i denote the random effects with $\lambda_i \sim N(0, \sigma_\lambda^2)$ and the e_i the usual measurement errors with $e_i \sim N(0, u_i^2)$. The $u_1, \ldots, u_n$ are the standard uncertainties quoted by the participating laboratories. The unknowns in our problem comprise μ and σ_λ^2 as well as the realized values of the laboratory effects $\lambda_1, \ldots, \lambda_n$.

In our Bayesian inference we use the conditional prior $\lambda_i|\mu, \sigma_\lambda^2 \sim N(\mathbf{0}, \sigma_\lambda^{\mathbf{2}}\mathbf{I})$ for the laboratory effects (which equals their assumed sampling distribution). Prior knowledge about the location parameter μ is modeled by a constant prior, and appropriate priors for the hyperparameter σ_λ are investigated. We prove propriety of the resulting posteriors and provide them in analytical form up to one-dimensional quadrature. We then investigate the properties of resulting credible intervals for the biases. Finally, we present and discuss results obtained for some key comparison data.

References

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