

# Bayesian treatment of a random effects model for the analysis of key comparisons

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## Abstract

Key comparisons are interlaboratory comparison measurements carried out in order to establish the equivalence between national metrology institutes [1]. It has been proposed to apply random effects models for the analysis of key comparison data and to use the resulting estimates of the laboratory effects as the so-called degrees of equivalence between the participating laboratories [2], [3]. In [2], [3] maximum likelihood type estimators of the laboratory effects were taken. Here we consider a Bayesian approach. In contrast to [3] we restrict our treatment to the case of normally distributed laboratory effects. More precisely, the measurements  $x_1, \dots, x_n$  are assumed to be modeled as

$$x_i = \mu + \lambda_i + e_i,$$

where  $\mu$  is the overall mean; the  $\lambda_i$  denote the random effects with  $\lambda_i \sim N(0, \sigma_\lambda^2)$  and the  $e_i$  the usual measurement errors with  $e_i \sim N(0, u_i^2)$ . The  $u_1, \dots, u_n$  are the standard uncertainties quoted by the participating laboratories. The unknowns in our problem comprise  $\mu$  and  $\sigma_\lambda^2$  as well as the realized values of the laboratory effects  $\lambda_1, \dots, \lambda_n$ .

In our Bayesian inference we use the conditional prior  $\lambda_i | \mu, \sigma_\lambda^2 \sim N(0, \sigma_\lambda^2 \mathbf{1})$  for the laboratory effects (which equals their assumed sampling distribution). Prior knowledge about the location parameter  $\mu$  is modeled by a constant prior, and appropriate priors for the hyperparameter  $\sigma_\lambda$  are investigated. We prove propriety of the resulting posteriors and provide them in analytical form up to one-dimensional quadrature. We then investigate the properties of resulting credible intervals for the biases. Finally, we present and discuss results obtained for some key comparison data.

## References

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