A Bayesian framework for the measurement of thermal diffusivity

Authors : A.Allard, N.Fischer, G.Ebrard (LNE), P.Harris, L.Wright, C.Matthews (NPL)

The GUM and its Supplements provide a harmonized and widely used method for the evaluation of uncertainty in metrology. However, the assumptions underlying the GUM procedure is not generally applicable to the case of regression and parametric inverse problems. Indeed, in this case, the measurands are parameters that generally cannot be expressed directly in terms of quantities that are measured or for which information is available. The determination of the thermophysical properties of a material using the laser flash method proposed by Parker et al.[1] constitutes an illustration of an inverse problem. The measurand is the thermal diffusivity of a material for a given temperature. The experimental layout consists in impacting a sample of the material enclosed in a furnace with a laser flash on its front face. This causes a temperature rise on the back face, which is measured using an infra-red detector, and is described by an associated experimental thermogram that represents the temperature as a function of time. The thermal diffusivity cannot be obtained directly but is determined through the identification of this experimental thermogram with a theoretical one, generated from a thermal model. The evaluation of measurement uncertainty associated with such a determination may be performed but is not straightforward.

Within the project EMRP-NEW04 on novel mathematical and statistical approaches to uncertainty evaluation and funded by the European Metrology Research Program (EMRP), a new methodology for the evaluation of the associated measurement uncertainty is proposed. It is based on a Bayesian framework that takes into account both the measurements performed and the available knowledge of the properties of the material. Using a Metropolis-Hastings algorithm, this framework consists of the estimation of two parameters that summarize the thermal exchanges in the experiment, and are used to determine the diffusivity. Prior knowledge of these parameters is formulated as a prior probability distribution. Both informative and uninformative prior distributions are considered in order to address cases where knowledge may be available or not. The use of different prior distributions enables to evaluate the sensitivity of the determination of the diffusivity to the choice of the prior distribution.

Finally, a validation of the proposed framework is performed on the basis of simulated data, and experimental data are considered in order to compare the results with the results obtained with the current state of the art in metrology.

Reference :

[1] W. J. Parker, W. J. Jenkins, C. P. Butler, G. L. Abbott, *Flash method of determining thermal diffusivity, heat capacity and thermal conductivity*, J. Appl. Phys., **32**, 1679-1684 (1961).