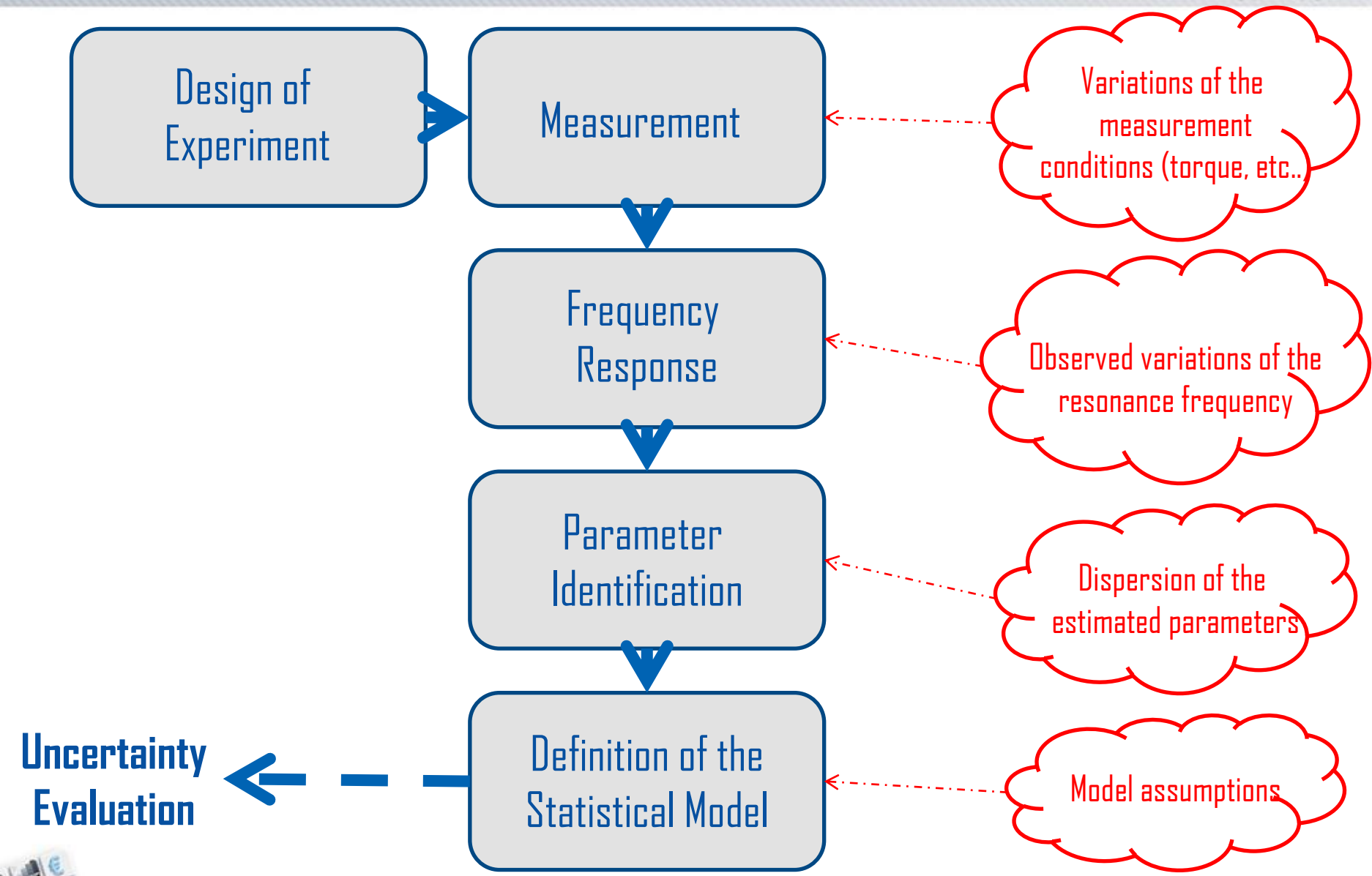
Parameter identification for dynamic calibration of force transducers using chirp excitations and assessment of the associated uncertainty

L. Coquelin, N. Fischer, A-F. Obaton

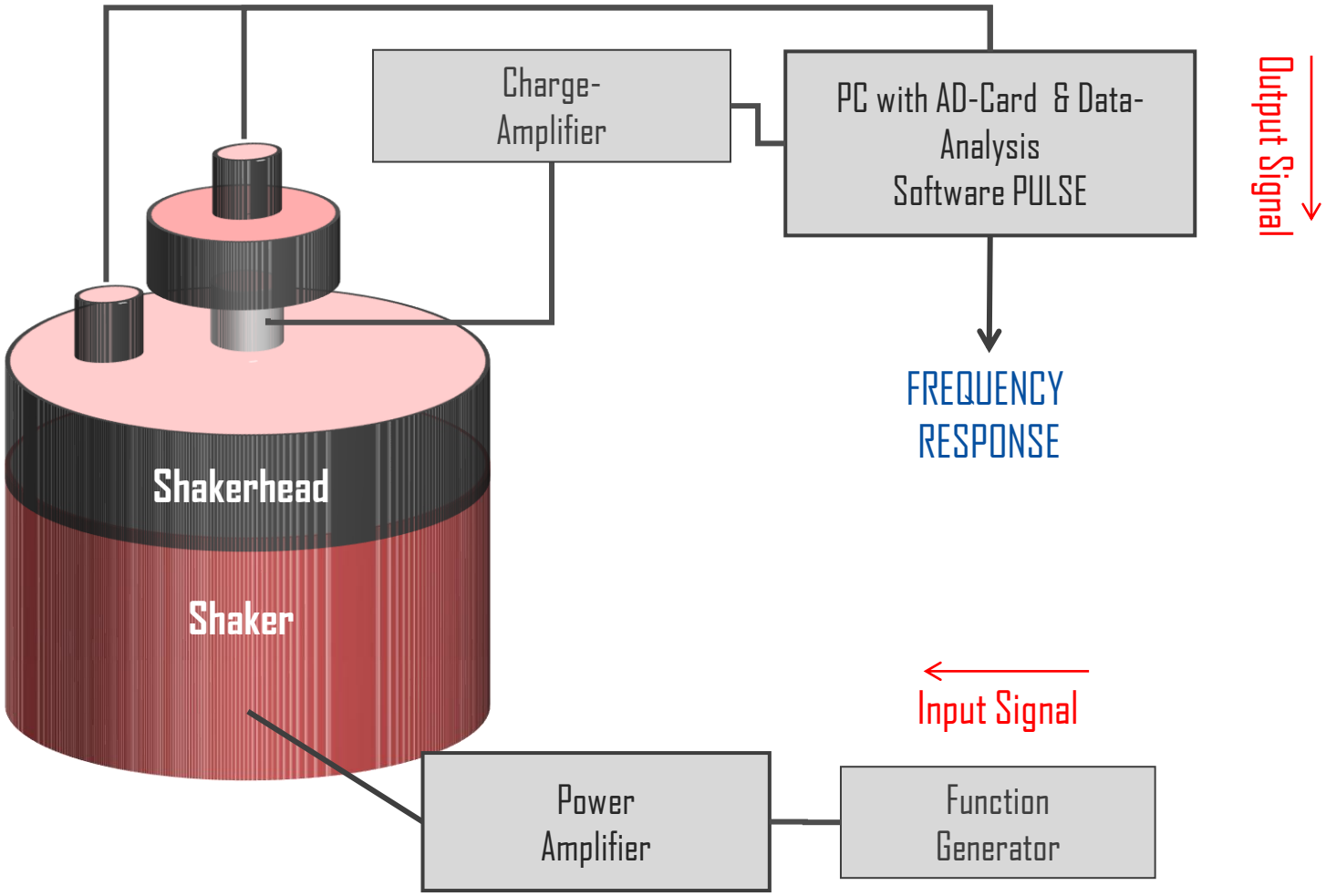
MEASUREMENT & STANDARDS
Keys to COMPETITIVENESS and A SAFER WORLD

National Laboratory of Metrology and Testing

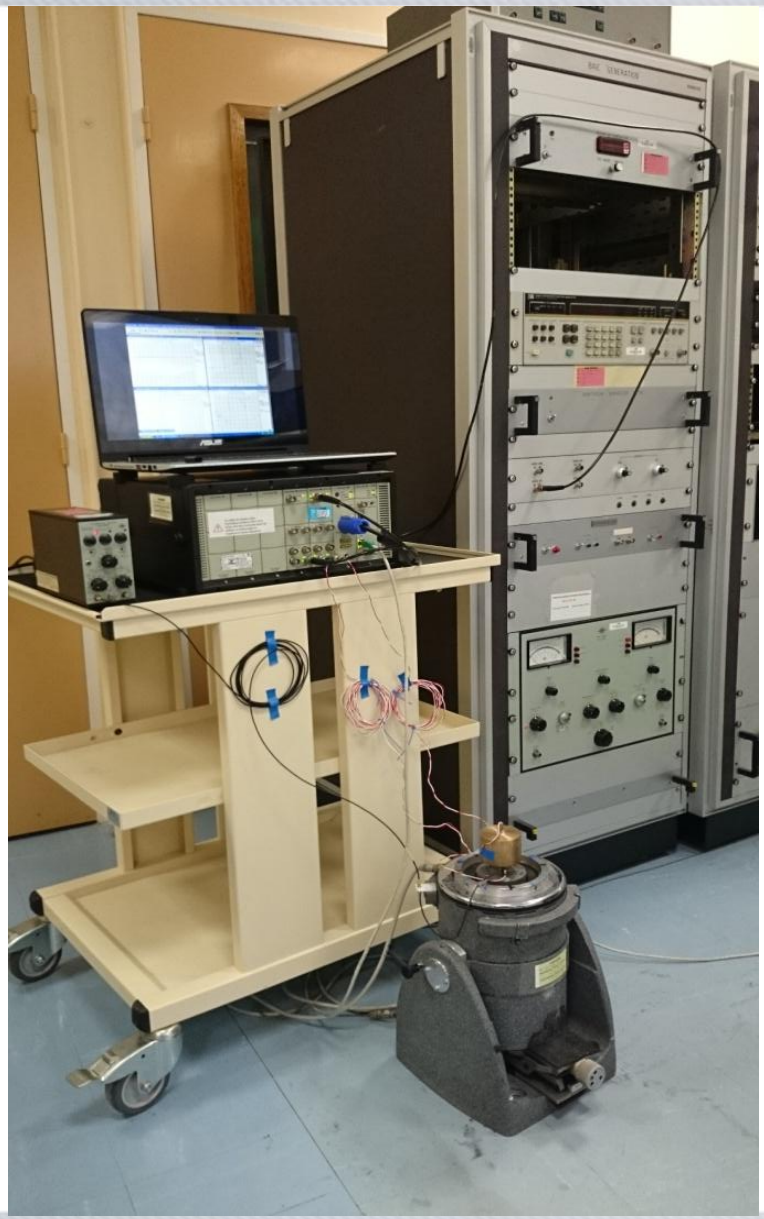
OVERVIEW OF THE PRESENTATION



SCHEMATIC SET UP



EXPERIMENTAL SET UP



SOURCES OF UNCERTAINTY

1. EFFECT OF THE MASS
2. EFFECT OF THE POSITION OF THE BASE ACCELEROMETER
3. EFFECT OF THE MOUNTING

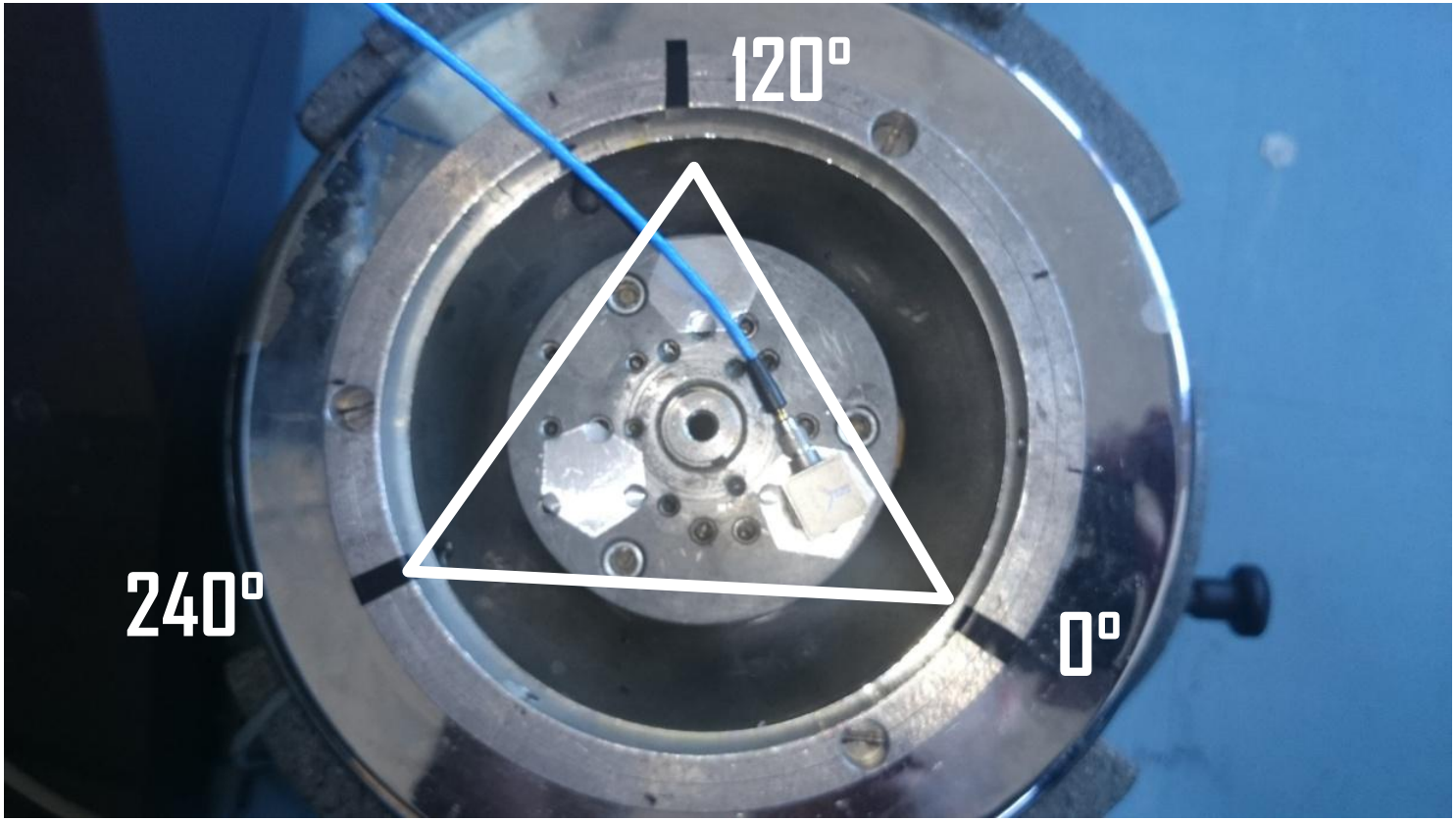
The design of experiment is created to evaluate the sources

2 MASSES

- 400g

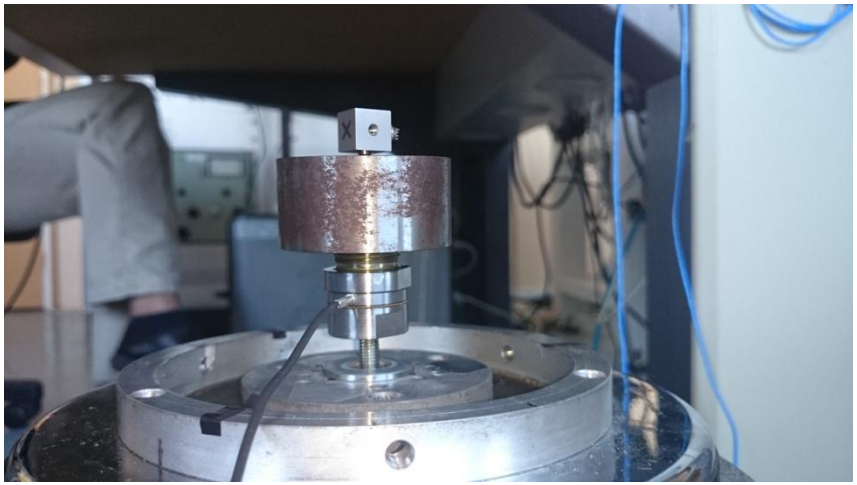
- 1kg

BASE ACCELEROMETER POSITIONS (0°, 120°, 240°)



MOUNTING

- 2 MASSES (400g, 1kg) SCREWED ON THE TOP
- 1 SENSOR KISTLER 9175B
- 1 SCREW WITH 2 DIAMETERS
- 1 WASHER
- 1 SCREW (TO FIX THE SENSOR TO THE SHAKER TABLE)

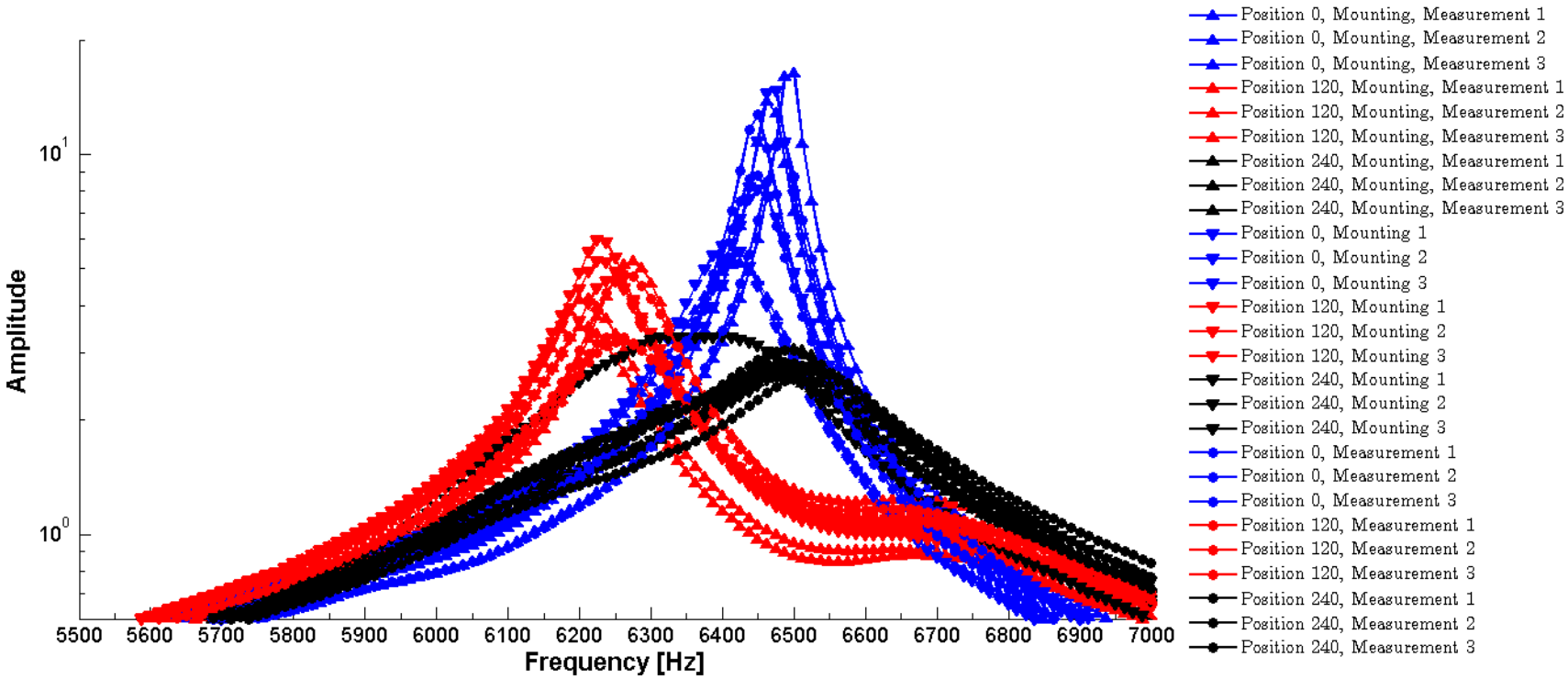


MOUNTED

UNMOUNTED

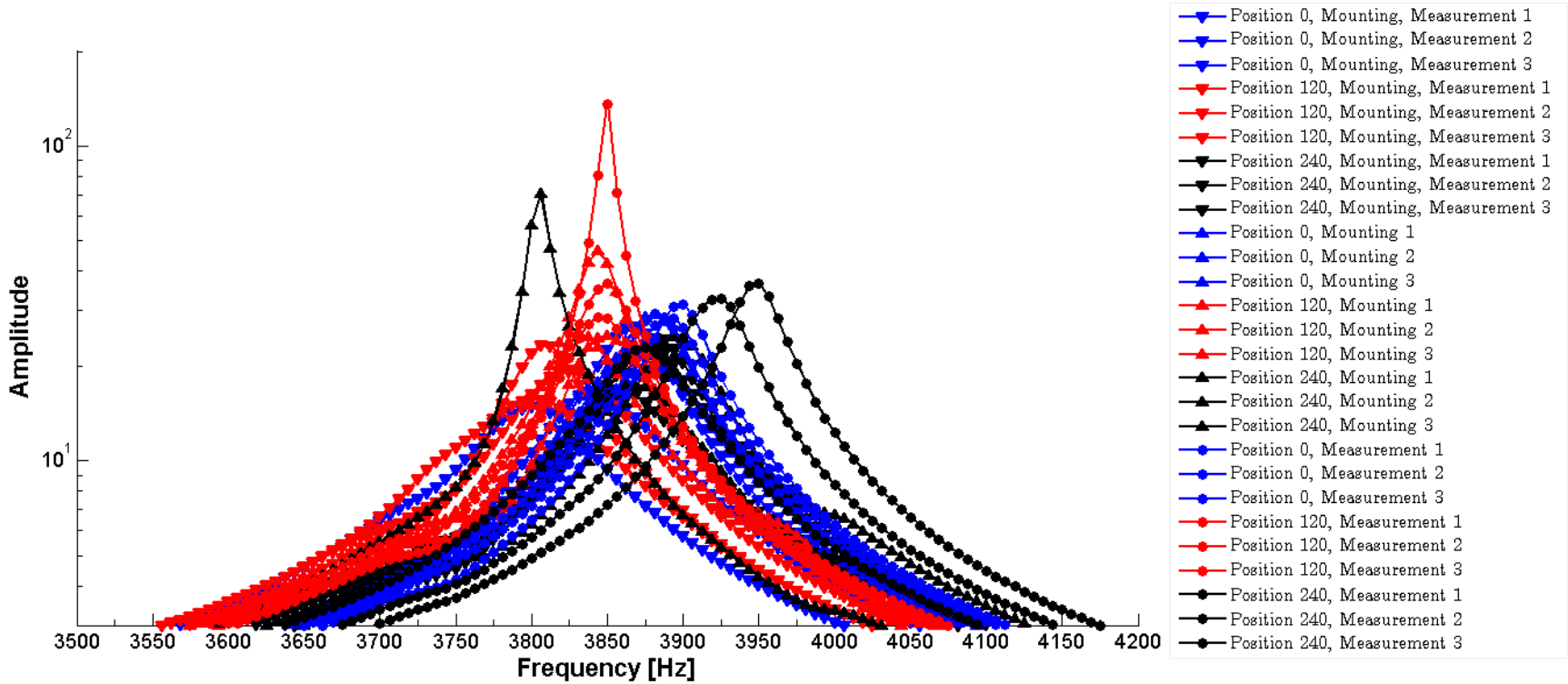
RESULTS FOR THE MASS OF 400g

AMPLITUDE RATIO BETWEEN THE FORCE TRANSDUCER OUTPUT AND THE BASE ACCELEROMETER



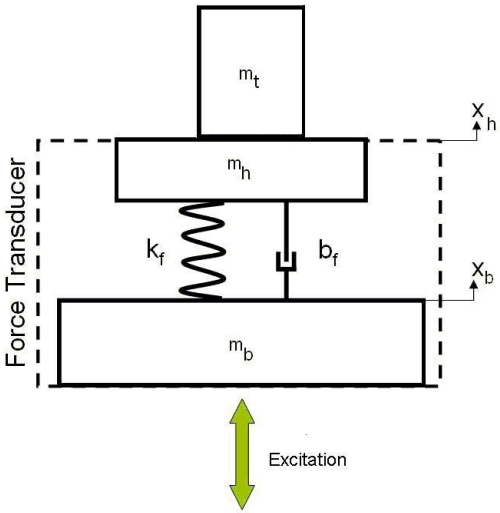
Variations due to the mounting and to the position of the base accelerometer on the shaker table

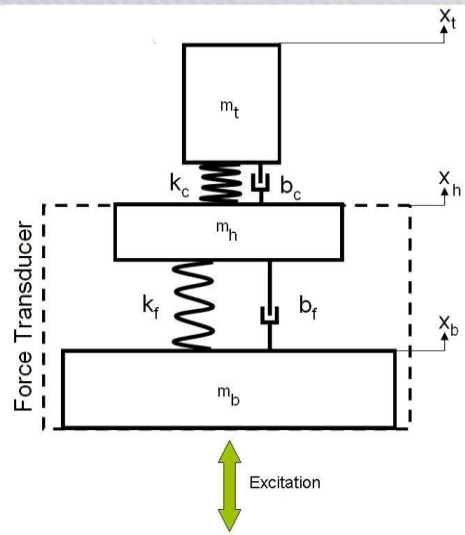
AMPLITUDE RATIO BETWEEN THE FORCE TRANSDUCER OUTPUT AND THE BASE ACCELEROMETER



Variations due to the mounting and to the position of the base accelerometer on the shaker table

SYSTEM MODELING





TRANSFER FUNCTION

Ratio between the force transducer output and the acceleration at the top mass

$$H(s) = -\frac{\rho}{sb_f + k_f}$$

$$H(s) = -\frac{\rho(m_t + m_h)}{sb_f + k_f}\underbrace{\left(1 + \frac{\mu_t s^2}{sb_c + k_c}\right)}_{A(s)}$$

Ratio between the force transducer output and the acceleration at the base mass

$$H(s) = -\frac{\rho}{s^2 + s\frac{b_f}{M} + \frac{k_f}{M}}, M = m_t + m_h$$

$$H(s) = -\frac{\rho A(s)}{s^2 A(s) + \frac{(sb_f + k_f)}{m_h}(A(s) - \frac{\mu_t}{m_h})}$$

3 parameters

5 parameters

PARAMETER ESTIMATION

- Only Kistler 9175b transducer has been measured
  - Coupling between the loaded mass and the transducer is very stiff → Simple Model best-suited

- Weighted Least-Square Method: « *System identification of force transducers for dynamic measurements* », **A. Link and al., IMEKO 2009**

$$\mu = \left(k_f \rho^{-1}/M, b_f \rho^{-1}/M, \rho^{-1}\right)$$

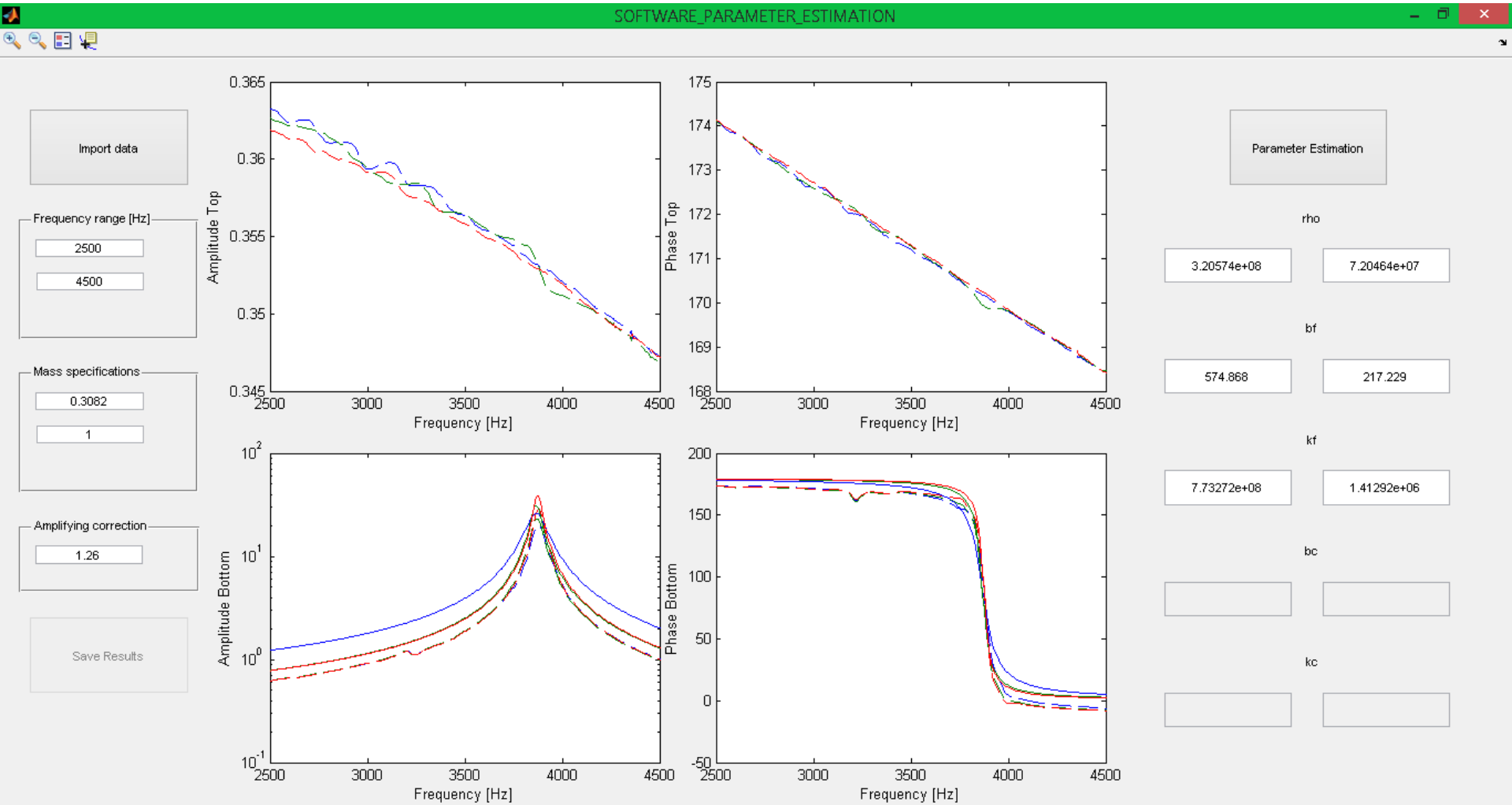
$$\hat{\mu} = \arg\min_{\mu}\{(y - H\mu)^T V_y^{-1}(y - H\mu)\}$$

$$V_{\hat{\mu}} = \left(H^T V_y^{-1} H\right)^{-1}$$

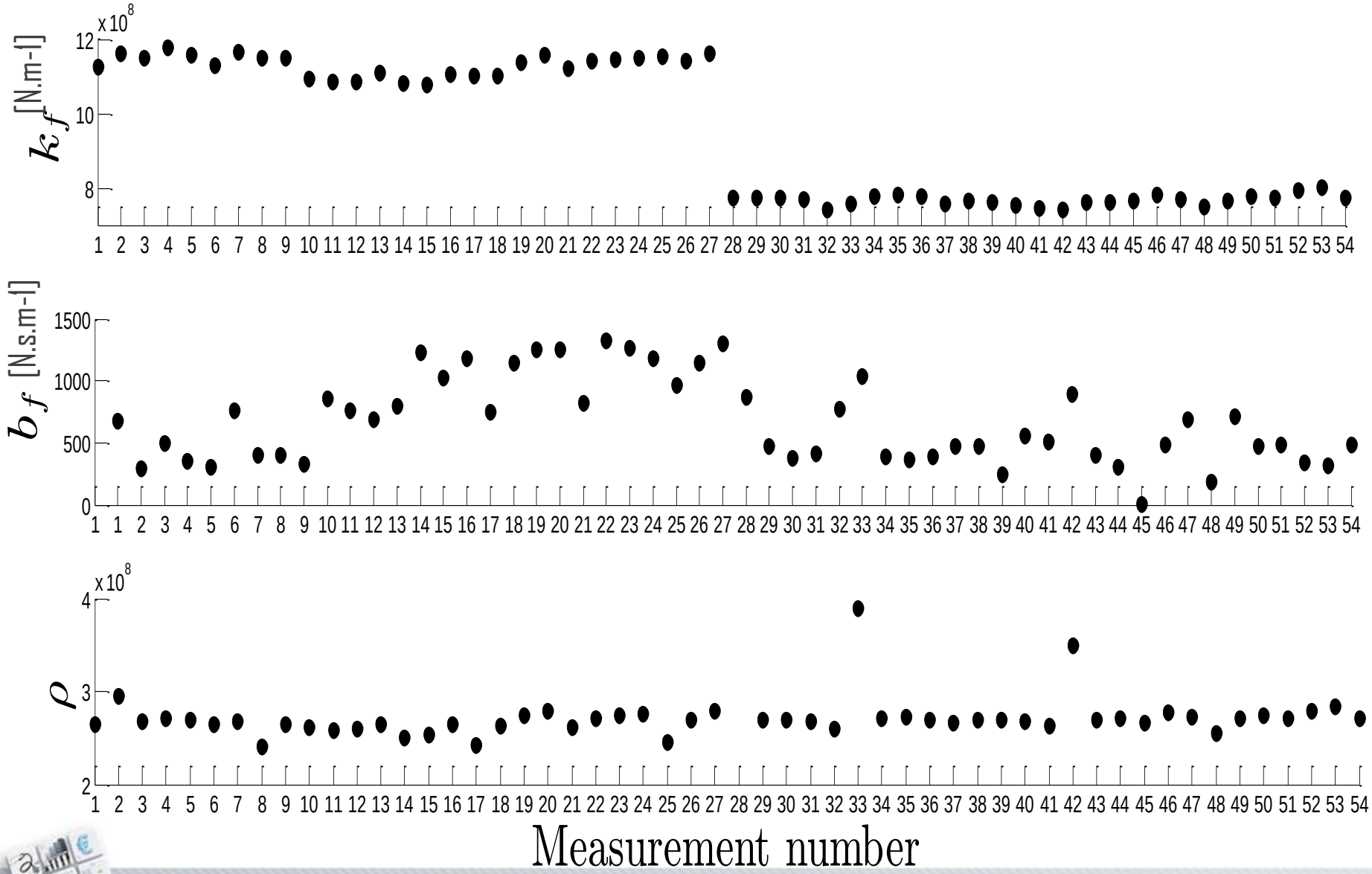
→ Parameters uncertainty for a given data set y

Uncertainty
Amplitude: 1.5%
Phase : 1° (radian)

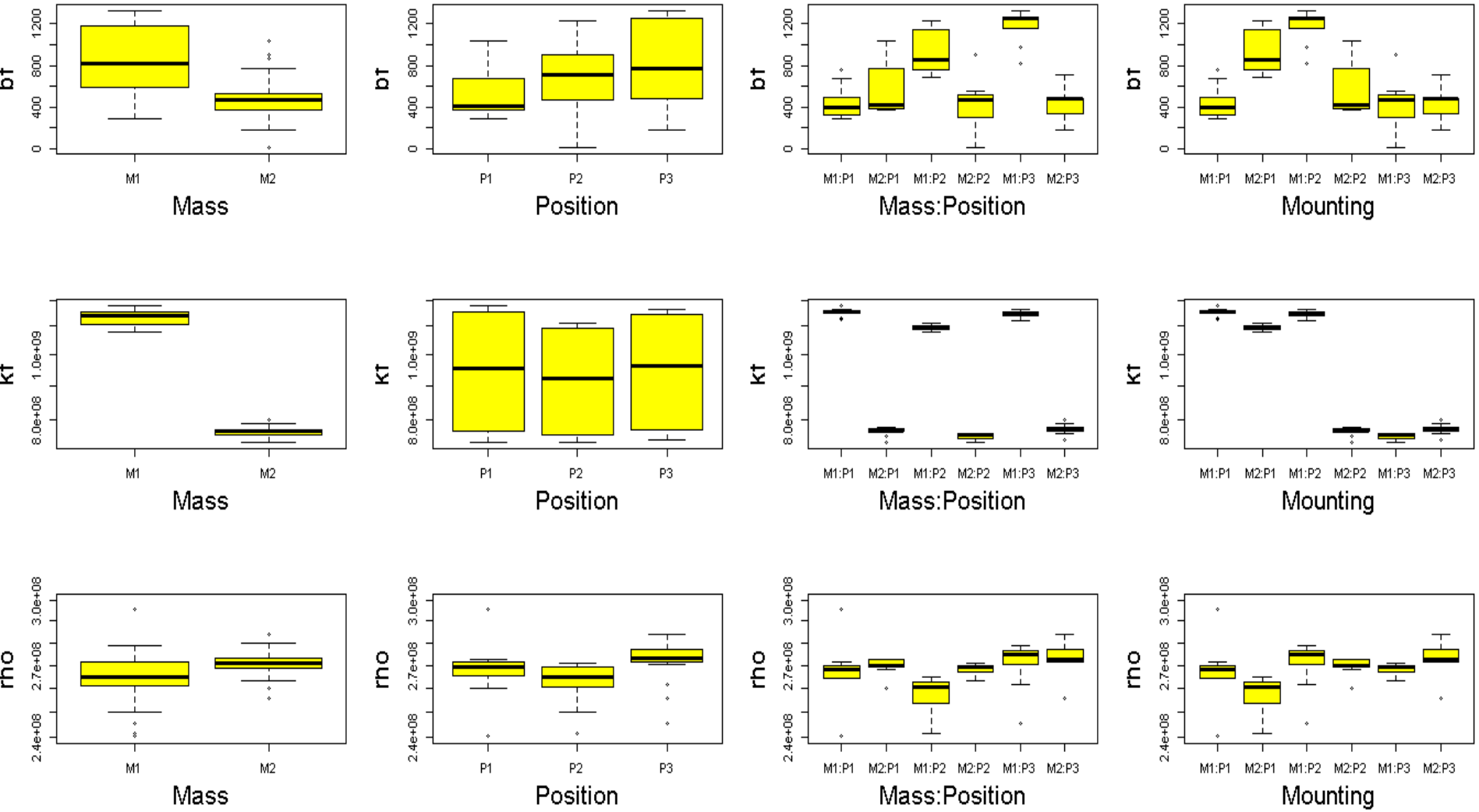
PARAMETER ESTIMATION SOFTWARE



PARAMETER ESTIMATION FOR ALL DATA SETS



VISUALIZATION OF THE EFFECTS USING BOXPLOTS



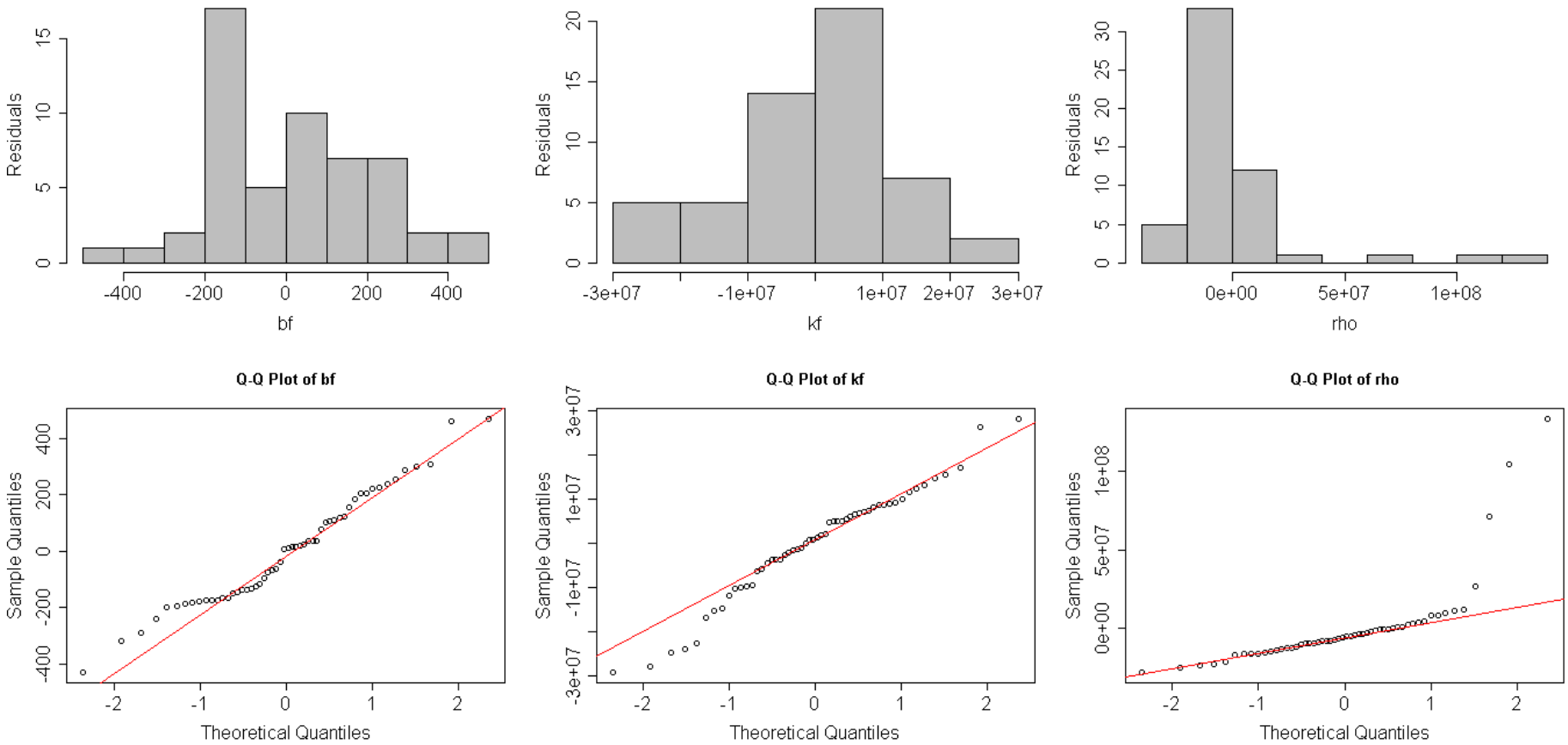
STATISTICAL MODELING

$$\theta_{ijk} = \mu + M_i + P_j + (MP)_{ij} + N_{k(ji)} + \epsilon_{ijk},$$

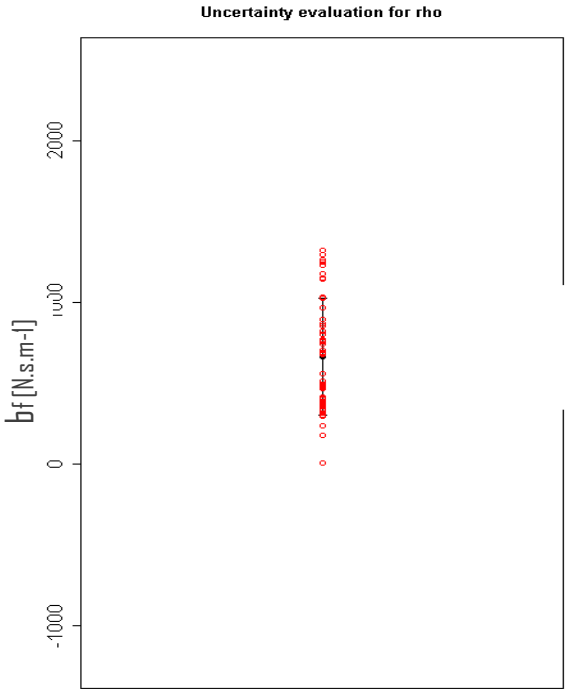
- μ mean over all measurements (54 data sets)
- $M_i \sim N(0, \sigma_M^2)$ random variables modeling the variations from mass to mass (2 masses)
- $P_j \sim N(0, \sigma_P^2)$ random variables modeling the variations from position to position (3 positions)
- $(MP)_{ij} \sim N(0, \sigma_{MP}^2)$ random variables modeling the variations due to the interaction of the mass with the position
- $N_{k(ji)} \sim N(0, \sigma_N^2)$ random variables modeling the variations from mounting to mounting for a fixed mass and a fixed position [nested within a mass and a position]
- $\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$ random variables modeling all over variations

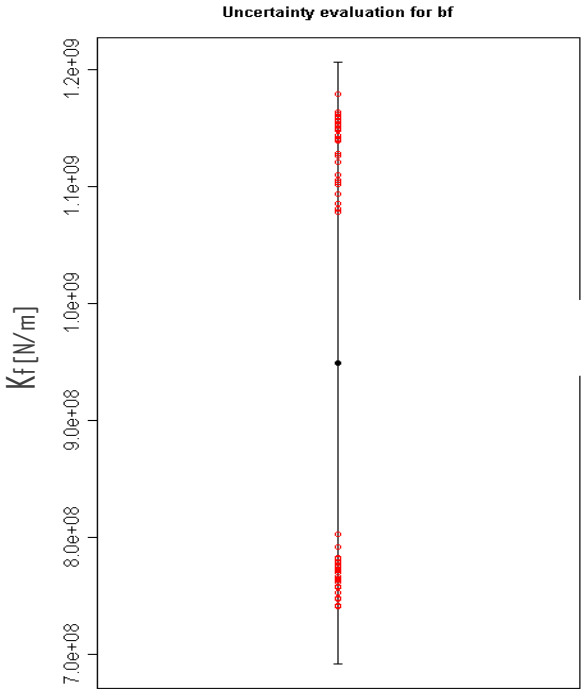
Estimation of the variances $\left(\sigma_M^2, \sigma_P^2, \sigma_{MP}^2, \sigma_N^2, \sigma_\epsilon^2\right)$ via Maximum Likelihood

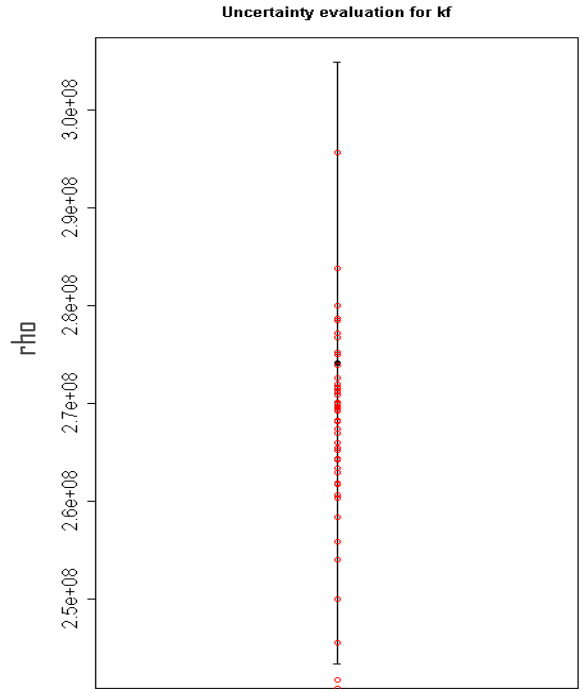
NORMALITY PLOTS ON THE RESIDUALS



UNCERTAINTY RESULTS







	$k_f[N.m^{-1}]$	ρ	$b_f[N.s.m^{-1}]$
σ_M/μ	**0.2699265**	0.0407914	**0.3096485**
σ_P/μ	0.0175827	0.0141691	-
σ_{MP}/μ	0.0121667	0.0068643	0.1898280
σ_N/μ	0.0020917	0.0000751	0.2578908
σ_ϵ/μ	0.0138433	**0.1034802**	0.3072550

CONCLUSION

- The statistical model is general so **results from PTB and CEM with other masses, positions and mounting protocol can be included** to provide end users with more accurate uncertainty evaluation

- Next step would be to perform parameter estimation with the **more complex model** (Scaled Nonlinear Least-Square fit with a Multiple Starting Point approach) **for HBM and Interface transducers**

* **The research leading to these results has received funding within an EMRP project IND09. The EMRP is jointly funded by the EMRP participating countries within EURAMET and the Europen Union**

THANK YOU FOR YOUR ATTENTION