Parameter identification for dynamic calibration of force transducers using chirp excitations and assessment of the associated uncertainty

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OVERVIEW OF THE PRESENTATION

Design of Experiment

Measurement

Frequency Response

Parameter Identification

Definition of the Statistical Model

Variations of the measurement conditions (torque, etc.):

Observed variations of the resonance frequency:

Dispersion of the estimated parameters:

Model assumptions:

Uncertainty Evaluation
SCHEMATIC SET UP

- Shakerhead
- Charge-Amplifier
- PC with AD-Card & Data-Analysis Software PULSE
- Function Generator
- Power Amplifier

Input Signal
FREQUENCY RESPONSE
Output Signal
SOURCES OF UNCERTAINTY

1. EFFECT OF THE MASS
2. EFFECT OF THE POSITION OF THE BASE ACCELEROMETER
3. EFFECT OF THE MOUNTING

The design of experiment is created to evaluate the sources
2 MASSES

- 400g
- 1kg
BASE ACCELEROMETER POSITIONS
(0°, 120°, 240°)
Mounting

- 2 masses (400g, 1kg) screwed on the top
  - 1 sensor Kistler 9175B
  - 1 screw with 2 diameters
    - 1 washer
  - 1 screw (to fix the sensor to the shaker table)
RESULTS FOR THE MASS OF 400g

AMPLITUDE RATIO BETWEEN THE FORCE TRANSDUCER OUTPUT AND THE BASE ACCELEROMETER

Variations due to the mounting and to the position of the base accelerometer on the shaker table
RESULTS FOR THE MASS OF 1kg

AMPLITUDE RATIO BETWEEN THE FORCE TRANSDUCER OUTPUT AND THE BASE ACCELEROMETER

Variations due to the mounting and to the position of the base accelerometer on the shaker table
**SYSTEM MODELING**

**TRANSFER FUNCTION**

**Ratio between the force transducer output and the acceleration at the top mass**

\[
H(s) = -\frac{\rho}{sb_f + k_f}
\]

**Ratio between the force transducer output and the acceleration at the base mass**

\[
H(s) = -\frac{\rho}{s^2 + \frac{b_f}{M} + \frac{k_f}{M}} \quad M = m_t + m_h
\]

\[
H(s) = -\frac{\rho A(s)}{s^2 A(s) + \frac{(sb_f + k_f)}{m_h} (A(s) - \frac{\mu_t}{m_h})}
\]

**3 parameters**

**5 parameters**
PARAMETER ESTIMATION

• Only Kistler 9175b transducer has been measured
  • Coupling between the loaded mass and the transducer is very stiff → Simple Model best-suited

• Weighted Least-Square Method: « System identification of force transducers for dynamic measurements », A. Link and al., IMEKO 2009

\[
\mu = \left( \frac{k_f \rho^{-1}}{M}, \frac{b_f \rho^{-1}}{M}, \rho^{-1} \right)
\]

\[
\hat{\mu} = \arg \min_{\mu} \{(y - H\mu)^T V_y^{-1} (y - H\mu)\}
\]

\[
V_{\hat{\mu}} = \left( H^T V_y^{-1} H \right)^{-1} \quad \text{Parameters uncertainty for a given data set } y
\]

Uncertainty
Amplitude: 1.5%
Phase: 1° (radian)
PARAMETER ESTIMATION FOR ALL DATA SETS

**Graphs:**
- $k_f [N.m^{-1}]$
- $b_f [N.m.s.m^{-1}]$
- $\rho$

**Axes:**
- Measurement number

**Data:**
- $k_f$: 10^{-8} to 12 \times 10^8
- $b_f$: 0 to 1500
- $\rho$: 2 to 4 \times 10^8

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Visualization of the effects using boxplots

Mass

Position

Mass:Position

Mounting

Mass

Position

Mass:Position

Mounting

Mass

Position

Mass:Position

Mounting

Mass

Position

Mass:Position

Mounting

Mass

Position

Mass:Position

Mounting

Mass

Position

Mass:Position

Mounting
\[ \theta_{ijk} = \mu + M_i + P_j + (MP)_{ij} + N_{k(ij)} + \epsilon_{ijk}, \]

- \( \mu \) mean over all measurements (54 data sets)

- \( M_i \sim N(0, \sigma_M^2) \) random variables modeling the variations from mass to mass (2 masses)

- \( P_j \sim N(0, \sigma_P^2) \) random variables modeling the variations from position to position (3 positions)

- \( (MP)_{ij} \sim N(0, \sigma_{MP}^2) \) random variables modeling the variations due to the interaction of the mass with the position

- \( N_{k(ij)} \sim N(0, \sigma_N^2) \) random variables modeling the variations from mounting to mounting for a fixed mass and a fixed position [nested within a mass and a position]

- \( \epsilon_{ijk} \sim N(0, \sigma_\epsilon^2) \) random variables modeling all over variations

Estimation of the variances \( (\sigma_M^2, \sigma_P^2, \sigma_{MP}^2, \sigma_N^2, \sigma_\epsilon^2) \) via Maximum Likelihood
NORMALITY PLOTS ON THE RESIDUALS

Q-Q Plot of bf

Q-Q Plot of kf

Q-Q Plot of rho

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UNCERTAINTY RESULTS

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<th>$\sigma_M/\mu$</th>
<th>$k_f [N.m^{-1}]$</th>
<th>$\rho$</th>
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CONCLUSION

• The statistical model is general so results from PTB and CEM with other masses, positions and mounting protocol can be included to provide end users with more accurate uncertainty evaluation.

• Next step would be to perform parameter estimation with the more complex model (Scaled Nonlinear Least-Square fit with a Multiple Starting Point approach) for HBM and Interface transducers.

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THANK YOU FOR YOUR ATTENTION