Metrology
Measurement Science and Technology

Dynamic, or Nondynamic – This is the Question.

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EURAMET
8th Workshop on Analysis of Dynamic Measurements
May 5th, 2014, INRIM Torino, Italy

Slides
www.mmm.ethz.ch/dok/e0001065.pdf
02.05.2014

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by everyday language

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Terminology

Terminology

Some keywords around the cardinal term

dynamic

- kinetic, variable, transient, drifting, unstable
- nondynamic, static, steady, stationary, constant, stable, time invariant
- time and space dependent, correlated
- update, correlate, convolve, stabilise, detrend, filter, deconvolve, reconstruct

What is the mathematical base?

Terminology

The main goals for consistent terms are

dynamic identification (calibration),
dynamic measurement,
dynamic reconstruction,

which directly impact

time dependent measurement errors
and
time dependent measurement uncertainties
in measurement results

Terminology

Some terms refer to signals

(models of real-world, physical or other quantities)

Some terms refer to systems

(models of real-world, physical or other processes).

So,

all terms should be well defined properties

of signals or systems,

which we describe mathematically

from the standpoint of

Signal and System Theory.

Terminology

What does the adjective

«dynamic»

really mean?

Terminology

A qualitative description by NPL

in everyday language:

"Dynamic measurement, …,

where

a physical quantity being measured

varies with time

and where this variation may have

significant effect on the measurement result

… and the associated uncertainty."

http://www.npl.co.uk/server.php?show=ConWebDoc.2131; T.E. (2008)

Terminology

A qualitative description by Rolls-Royce

in everyday language:

"We treat measurements as "dynamic", when the rate of change of the quantity value impacts the Metrology.

Therefore, a measurement system with infinite bandwidth, which faithfully reproduces amplitude and phase of the parameter, is not dynamic!

However, a system, measuring changes occurring over several hours may be dynamic, if it´s bandwidth is insufficient for the task"

Terminology

A qualitative description in everyday language:

A

dynamic system

is a system,

whose present state depends on both,

present and past values

and / or

present and past derivative values

(velocity value, gradient value)

of at least one of several signals.

Terminology

Accordingly, a

nondynamic system

is a system,

whose state merely depends on the

present values of all signals concerned.

Terminology

Involved participants concerning the definition "dynamic":

signals to be measured (?)

systems

measurement task (?)

measurement tools (?)

Terminology

The main player concerning the definition "dynamic":

System,

it´s

properties and behaviour

Let´s talk "System"

Dynamic System –
description and behaviour

A quantitative description:

If we are able to describe a process by
differential- (difference-) equations
in space and time,
we call the developed mathematical model
«**Dynamic System**».

Then, we assign the term
«**dynamic**»
to this modelled process.

Attention:

Differential (difference) equations

describe

relations between

defined

models of independent and dependent

quantities

of a process.

In this framework

they do not describe

physical objects directly.

Levels of empirical and / or analytical models

(e.g. linear, time invariant (LTI) dynamic system of second order)

Dependence of quantities:

$$\mathbf{u}(t) \rightarrow \mathbf{y}(t)$$

Symbolic mathematical model:

$$\mathbf{y}(t) = \mathbf{f}(\mathbf{u}(t), \mathbf{p}(t), t)$$

General mathematical model:

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t) \quad [\{y\}\, s^{-2}]$$

Dedicated mathematical model:

$$m\, \ddot{y}(t) + c\, \dot{y}(t) + \kappa\, y(t) = f(t) \quad [N]$$

Physical unit model:

$$Nm^{-1}s^{2}\, ms^{-2} \quad Nm^{-1}s\, ms^{-1} \quad Nm^{-1}m \quad = \quad N$$

Levels of
empirical and / or analytical models

(linear, time invariant (LTI), dynamic system of second order)

Particular mathematical model:

$$0.05\,\ddot{y}(t) + 0.32\,\dot{y}(t) + 1.07\,y(t) = f(t) \quad [N]$$

Solved mathematical model:

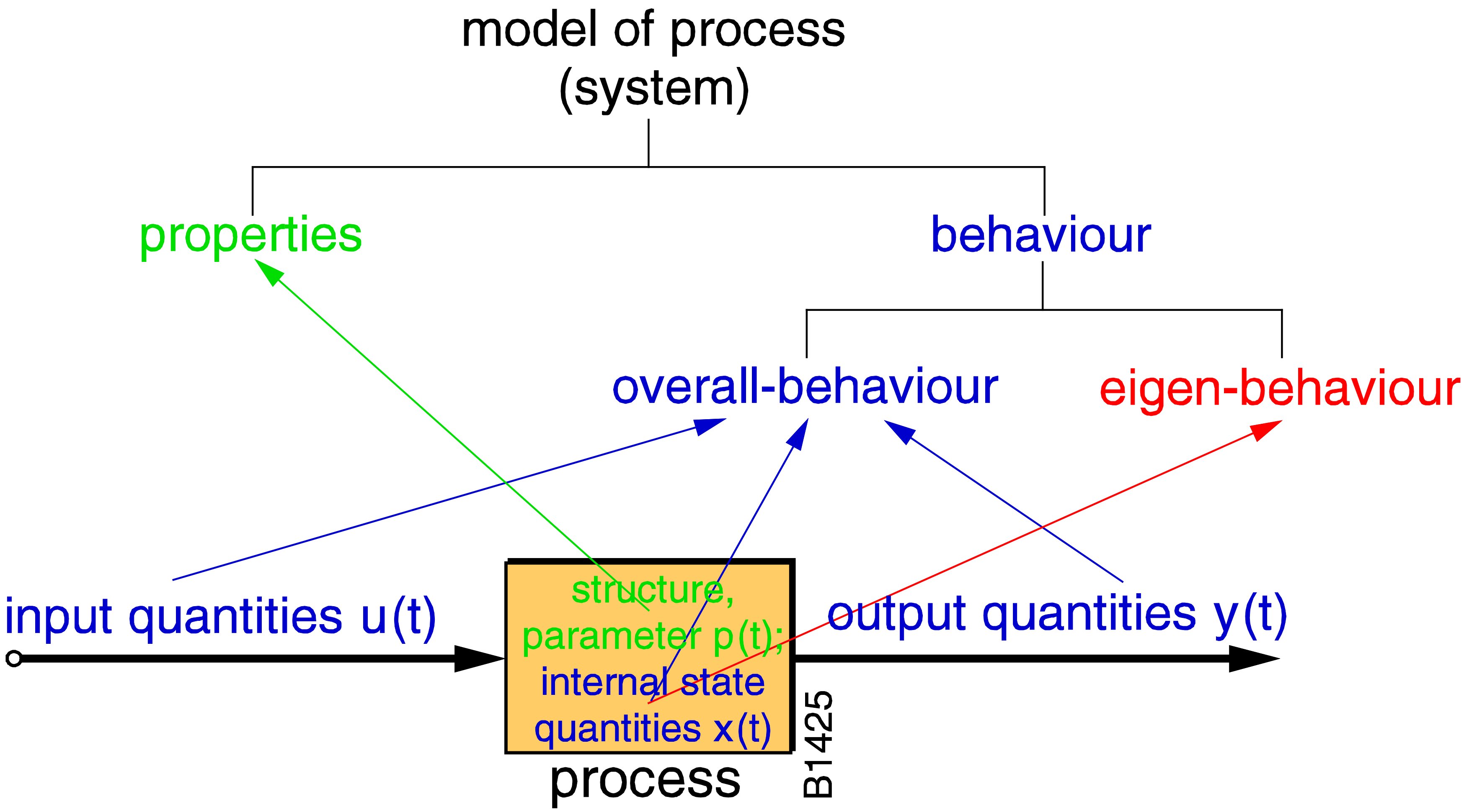
several analytical and numerical equations

as general solutions

Particular behaviour model:

for particular state signals and input signals

Description (Model) of the Dynamic Process



Each system will show a behaviour.

But what about signals?

Every signal originates from some source,
which we call system.

Signals are manifestations of systems.

Therefore Signals show

no genuine behaviour;

we could not justify it by mathematical reasoning.

Signals have individual **properties** only!

Description (Model) of the Dynamic Process

Differential equations of first and higher order are assembled in a set of

(N) coupled homogeneous differential equations of first order,

by taking the respective derivative terms of the equations as new signals $x_n(t)$, which we call state signals and which we unite in the state signal vector $\mathbf{x}(t)$:

State Space Description

Description (Model) of the Dynamic Process

The ***kernel*** of a nonlinear, dynamic system relates state signals and their derivatives:

$$\dot{\mathbf{x}}(t) = \mathbf{f}\{\mathbf{x}(t)\}$$

This intrinsic model of a dynamic process describes the **dynamic** *properties.*

(Whether the individual state signals $x_n(t)$ are of real or physical significance, is irrelevant.)

Description (Model) of the Dynamic Process

An important Property of a Dynamic System
is it´s
«Stability»

It is determined exclusively by this
kernel of the dynamic system.

$$\dot{\mathbf{x}}(t) = \mathbf{f}\{\mathbf{x}(t)\}$$

Instability and Drift

Are
Instability and **Drift**
Synonyms?

Not at all!

A system may be stable and it´s output signal
may nevertheless drift.

Again, there exists
no quantitative definition or mathematical formalism
for an
"instable signal"!

Instability Concerning a System

Citation 1

"A process is said to be **stable**,
when all of the response parameters that we use
to measure the process
have both constant means and constant variances
over time,
and also have a constant distribution."

Obviously, not stability is meant here, but stationarity.

NIST: Engineering Statistics Handbook
http://www.itl.nist.gov/div898/handbook/ppc/section4/ppc45.htm

Instability Concerning a System

Citation 2

"**Stability** of a measuring instrument:

property of a measuring instrument,
whereby its metrological properties remain
constant in time"

Obviously, not stability is meant here, but stationarity.

International Vocabulary of Metrology (VIM); Definition 4.19

Description (Model) of the kernel of the dynamic process

$$\dot{\mathbf{x}}(t) = \mathbf{f}\{\mathbf{x}(t)\}$$

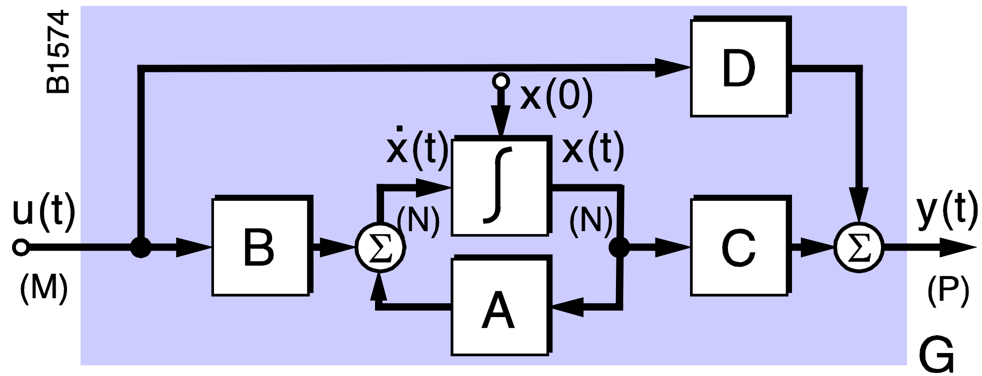
Extension:
If we want to describe
relations between signals and systems,
we expand
the kernel in the
input-state-output model

$$\dot{\mathbf{x}}(t) = \mathbf{f}\{\mathbf{x}(t), \mathbf{u}(t), t\}$$
$$\mathbf{y}(t) = \mathbf{g}\{\mathbf{x}(t), \mathbf{u}(t), t\}$$

Description (Model) of the Dynamic Process

Simplifications deliver the popular standard description of the
linear, time invariant (LTI) dynamic process of $(N)^{th}$ order:

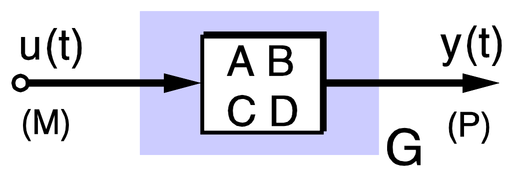
$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)$$



or in short

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \hline \mathbf{y}(t) \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{x}(t) \\ \hline \mathbf{u}(t) \end{bmatrix}$$

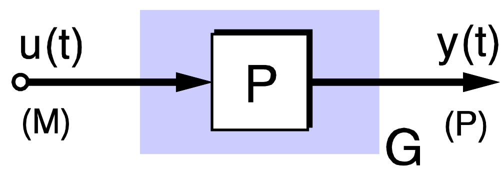
or



with the partitioned, time invariant (constant) parameter matrix **P**

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array}\right]$$

or



Behaviour of the Dynamic Process

The general analytical / numerical solution of a linear, time invariant (LTI) dynamic system delivers the

behaviour

in form of the output signal vector

$$\mathbf{y}(t) = \mathbf{C}\,e^{\mathbf{A}(t)}\,\mathbf{x}(0) + \mathbf{C}\int_0^t e^{\mathbf{A}(t-\tau)}\,\mathbf{B}\,\mathbf{u}(\tau)\,d\tau + \mathbf{D}\,\mathbf{u}(t)$$

which contains all parameter matrices **A**, **B**, **C**, **D** and the

matrix exponential function $\mathbf{\Phi}(t) = e^{\mathbf{A}(t)}$, called

state transition function matrix.

Description of the Dynamic Process

With time independent (constant) parameters **p**
we need only the

system **transfer** description
(transfer function model):
$\mathbf{u}(t) \rightarrow \mathbf{y}(t)$

With time dependent (variable) parameters **p**
we additionally need the

system **sensitivity** description
(sensitivity function model):
$\mathbf{p}(t) \rightarrow \mathbf{y}(t)$

State
of a Dynamic System

We say, as soon as input signals **u**(t) vary in time, a stable dynamic system evolves in time.

We say, the dynamic system is in a

«Transient State».

This transient state may be **stationary** or **nonstationary**

State
of a Dynamic System

A correct statement:

A Dynamic System
can be momentarily in a
Static State.

The antonym of
«Dynamic System»
is
«Nondynamic System»

and not

«Static System».

State of a Dynamic System

As soon as all input signals **u**(t) are constant,
all derivatives in the differential equations are zero,
and the set of differential equations has degraded
to a set of
algebraic and / or transcendent equations.

$$\dot{\mathbf{x}}(t) = \mathbf{f}\{\mathbf{x}(t); \mathbf{u}(t)\} \qquad \qquad \mathbf{0} = \mathbf{f}\{\mathbf{x}; \mathbf{u}\}$$
$$\mathbf{y}(t) = \mathbf{g}\{\mathbf{x}(t); \mathbf{u}(t)\} \longrightarrow\longrightarrow\longrightarrow\longrightarrow \mathbf{y} = \mathbf{g}\{\mathbf{x}; \mathbf{u}\}$$

We say, the dynamic system is in a
«Static State»
(steady state, equilibrium state, rest state).

This particular state is momentarily
constant.

Dynamic Error System –

a consistent and systematic model

We have to admit:

Every measurement procedure

is

error and uncertainty prone.

There are

measurement errors

and

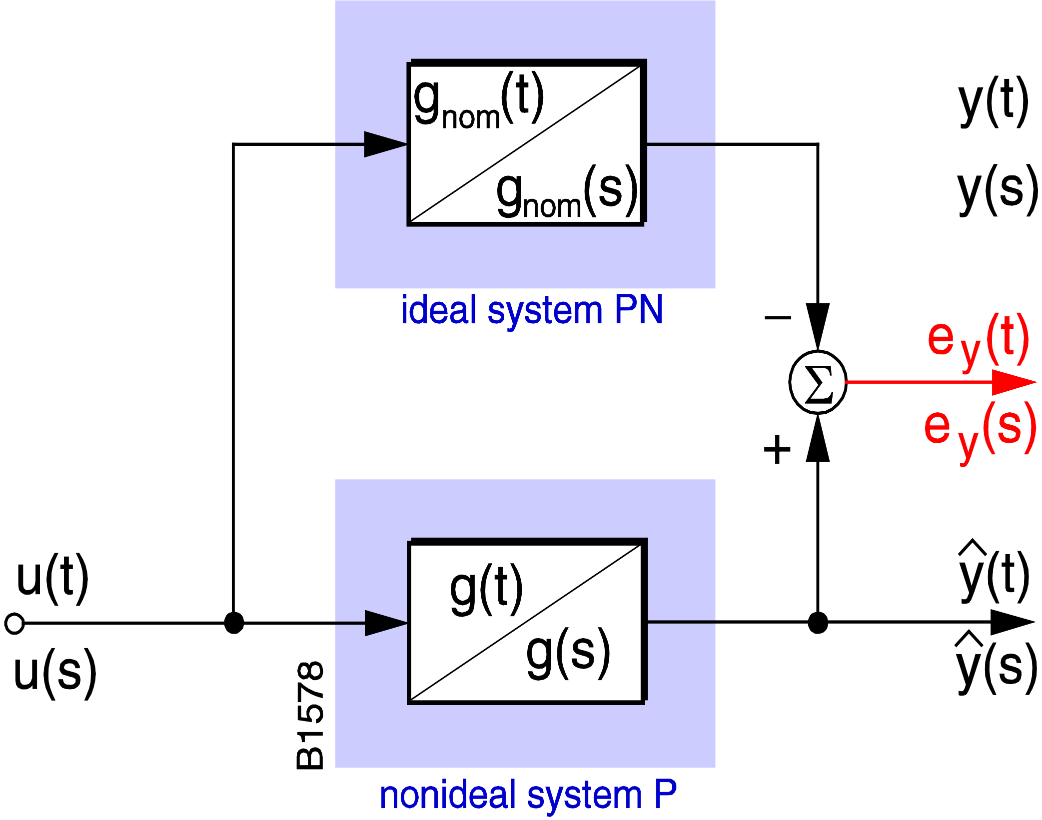
measurement uncertainties.

Time dependent errors

are handled as all other errors too:

1. *Error Description*

with respect to a defined ideal, *nominal system* PN

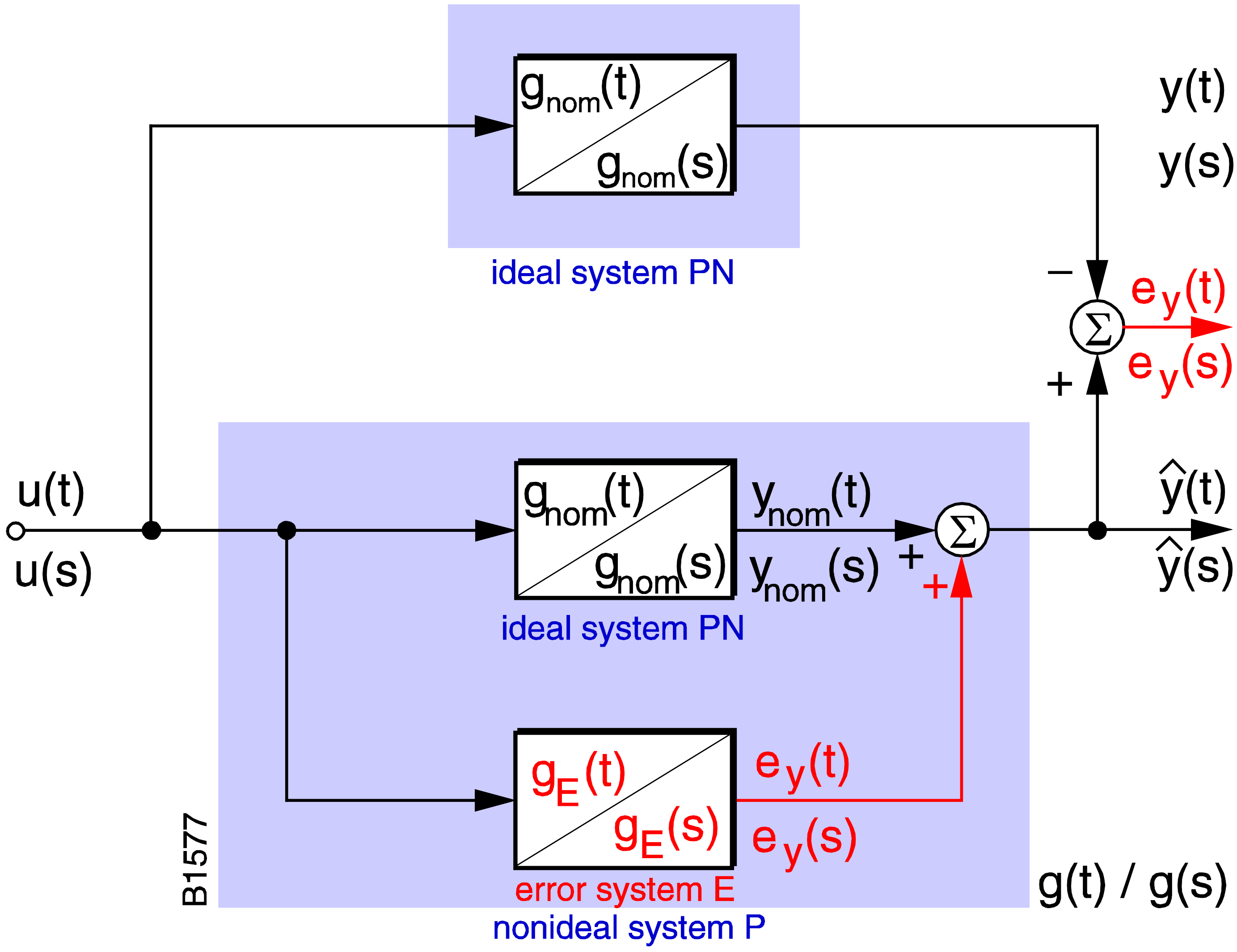


$$e_y(t) = \hat{y}(t) - y(t) \quad [\{y\}] \qquad / \qquad e_y(s) = \hat{y}(s) - y(s) \quad [\{y\}s^{-1}]$$

2. Error System

A nonideal system P

consists of an ideal, nominal system PN and an error system E



Example

Description of a
nonideal, dynamic system P of first order

in the time domain

$$T\frac{d}{dt}\hat{y}(t)+\hat{y}(t)=g_{nom}\,u(t)\quad[\{y\}]$$

or

$$\hat{y}(t)=g(t)*u(t)\quad[\{y\}]$$

in the frequency domain

$$\hat{y}(s)=g_{nom}\frac{1}{1+Ts}u(s)$$

or

$$\hat{y}(s)=g(s)\cdot u(s)\quad[\{y\}\,s^{-1}]$$

Example

Description of the

error system E

of a nonideal, dynamic system P of first order

in the time domain

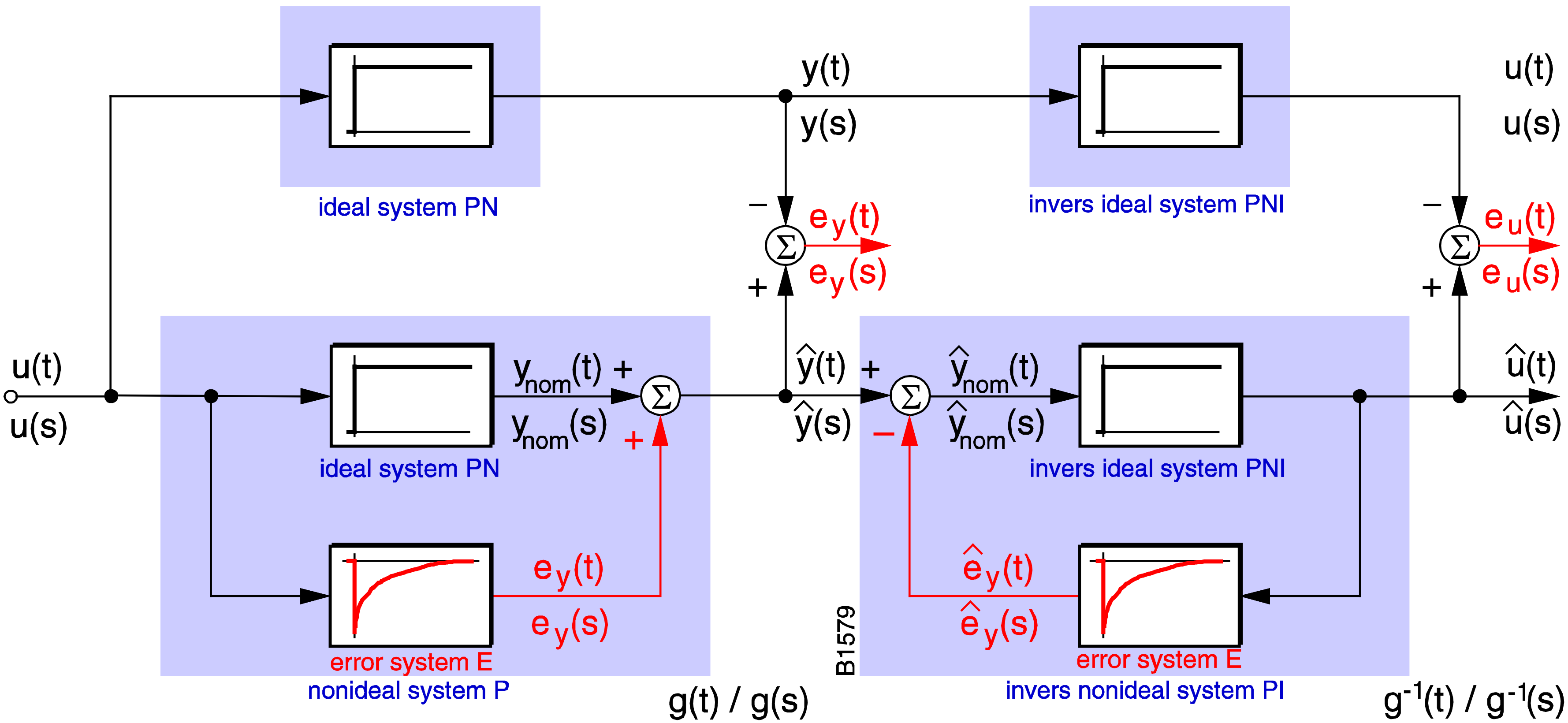
$$T\frac{d}{dt}e_y(t) + e_y(t) = -g_{nom}\,T\frac{d}{dt}u(t) \quad [\{y\}]$$

in the frequency domain

$$e_y(s) = g_{nom}\frac{-T\,s}{1+T\,s}u(s) \quad [\{y\}\,s^{-1}]$$

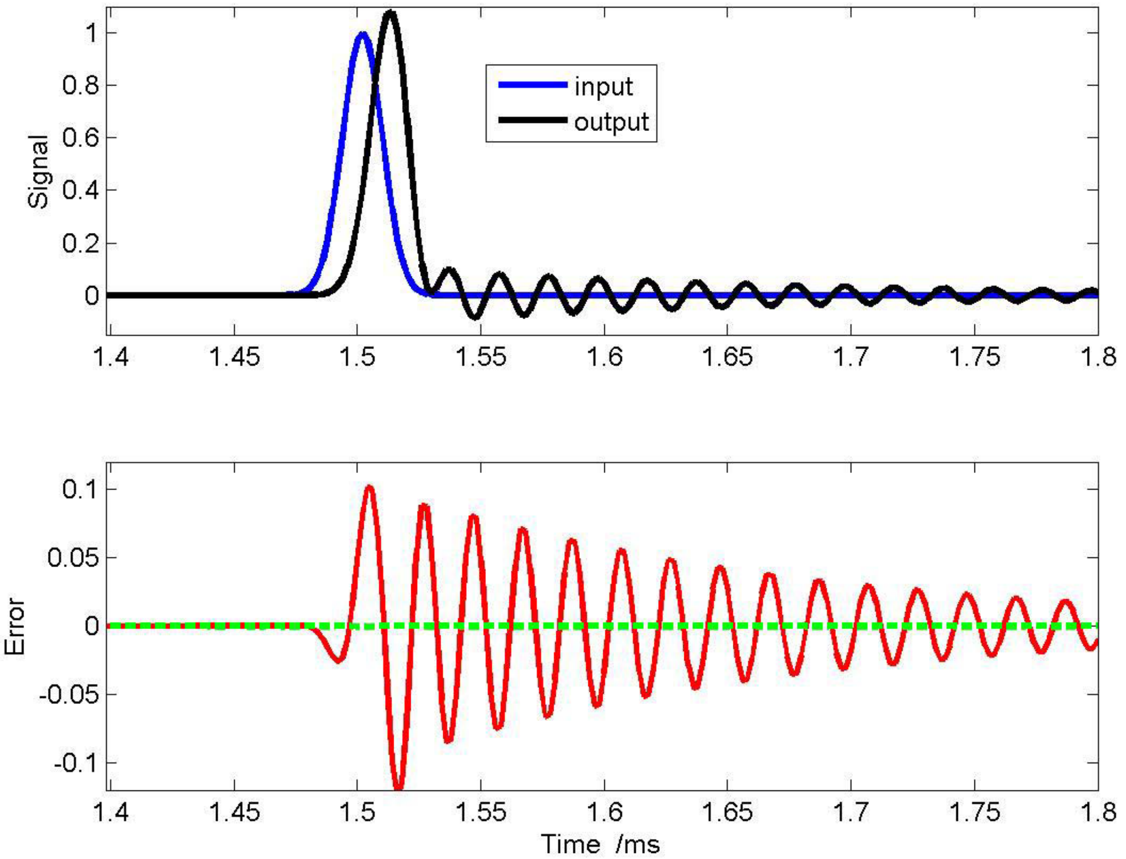
3. Error Correction

by a Reconstruction Process R
(Inversion Process, Deconvolution Process, Filter Process)
for a nonideal, dynamic system P of first order



3. Error Correction

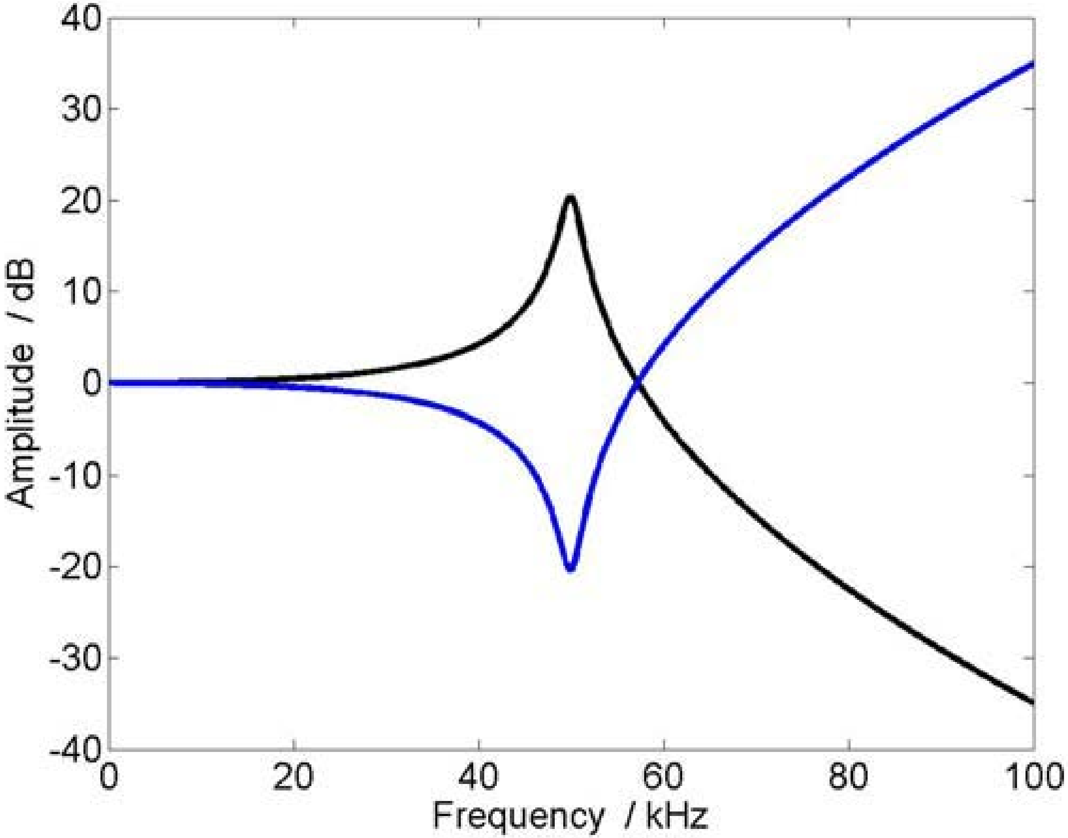
by a Reconstruction Process R
(Inversion Process, Deconvolution Process, Filter Process)
for a "nonideal", dynamic system P of second order
in the time domain



C. Elster and A. Link, AMCTM VIII, 2008

3. Error Correction

by a Reconstruction Process R
(Inversion Process, Deconvolution Process, Filter Process)
for a "nonideal", dynamic system P of second order
in the frequency domain



black:
nonideal System

blue:
inverse System

C. Elster and A. Link, AMCTM, 2008

Reconstruction System

Advantage

No inversion of the dynamic error system E is necessary,
which means, also
no linearization of the nonlinear error system E!

Disadvantage

The reconstruction task may be ill-posed and the
reconstruction system may become unstable
due to
structural and parametric constellations of the feedback loop.

Uncertainties

are,

like parameters and errors,

signals

too, and

are treated likewise

Terminology –
by Signal and System Theory

Terminology
by Signal and System Theory

Concerning Systems

(Error Systems, Uncertainty Systems)

- dynamic / nondynamic system
- dynamic system in steady state
- stable / unstable system
- linear / nonlinear system

Terminology
by Signal and System Theory

Concerning Signals (Parameters, Errors, Uncertainties)

- time and space dependent signal
- constant / transient signal
- stationary / nonstationary signal
- deterministic / random signal
- drifting signals

With signals,
combinations (superposition) of properties
are normal.

The described concepts are universally valid.

All measurement processes can be analysed like this.

The principle is simple,

but modelling is elaborate:

- optimal choice of model quantities
- analytic / empiric determination of structures
- experimental determination of the parameters
- uncertain results due to several reasons

Mathematical / numerical problems

Normally, they are solvable, at least approximately.

"Get the Physics right,

the rest is Mathematics."

Rudolf Kálmán (2005)

In everyday language,

Terminology

is not important,

however,

if we have to rely on it quantitatively in Science and Technology,

it is important.

Ambiguity and **Misunderstanding**

is avoided by

Systematic Structures

and

Consistent Terminology.

Unfortunately,
ideas in Metrology
are often heavily
instrumentation fixed
and seldom
context oriented.

Additionally,
there is no concise distinction between
(physical) reality
on the one hand and
(mental) models of the reality
on the other hand;
we mostly locate a confusing mix.

Supplements

Terminology

"dynamic"

According to our eigen-frequencies,

we

normally feel in steady state situations at ease

and

wait for transient procedures to fade.

Terminology

Summarising comments:

"The Metrology of Dynamic Systems is
not well understood or documented."

P. L., Rolls-Royce (2012)

"… tools cover the engineering aspects
but the requirements for Metrology are lacking."
"Scientific literature on the subject is scarce
and needs to be adapted for a wider audience."

B. H., NPL (2012)

Errors

"Treatment of errors in accordance with modern Estimation Theory forms an essential part of Measurement Science"

L. Finkelstein (1975) [1]

"The classical error theory is not a regular theory.

It *consists* of rather loosely connected definitions

of various kinds of errors

and principles of their determination and assessment.

It is *lacking* …

a system of basic concepts

and logically derived consistent principles

of the determination and assessment of errors."

R. Morawsky, et.al. (1988) [2]

Errors

"The treatment of systematic errors is often mishandled.
This is due to lack of understanding and education,
based on a fundamental ambiguity as to what is meant by the term."

Roger Barlow (2002) [3]

"The approach based on true value and error was questioned
as being based on unknowable quantities, i.e. *idealized concepts*.
The very terms were almost banned from the literature,
and whoever dared to use them was considered suspiciously
as a supporter of old ideas."

Walter Bich, INRIM (2012) [4]

"Should the term «Error» appear in the VIM or not?"

P. de Bièvre (2010) [5]

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[2] Morawsky, R. et. al.: "Theory of measurement in teaching metrology in engineering faculties", Measurement, 6 (1988) 2, pp. 63- 67

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[4] Bich, W.: "From Errors To Probability Functions. Evaluation of the Concept of Measurement Uncertainty". IEEE TIM, 8 (2012), p. 2153

[5] de Bièvre, P.: Roundtable Discussion on the "International Vocabulary of Basic and General Terms in Metrology" (VIM) IMEKO Joint Symposium, London (September, 2010)