# DROP-WEIGHT SYSTEM FOR DYNAMIC PRESSURE CALIBRATION

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# 1) Introduction

- Within the framework of the EMRP-project IND09 Dynamic - Traceable dynamic measurement of mechanical quantities Centre for Metrology and Accreditation (MIKES) is developing a measurement standard for dynamic pressure
- The new measurement standard is based on the drop weight principle and it provides valid traceability chain from 100 MPa to 500 MPa with the targeted relative measurement uncertainty in the order of 1%
- New primary dynamic pressure standard will provide traceability to the measurements in the different areas, e.g. hydraulic systems, engine, ammunition and firearms industries

# 2) Working principle of the measurement standard

- The device is manufactured by AVL
- Operating principle is based on drop weight system
- Generated pressure pulse is measured using up to three (3) pressure transducers simultaneously



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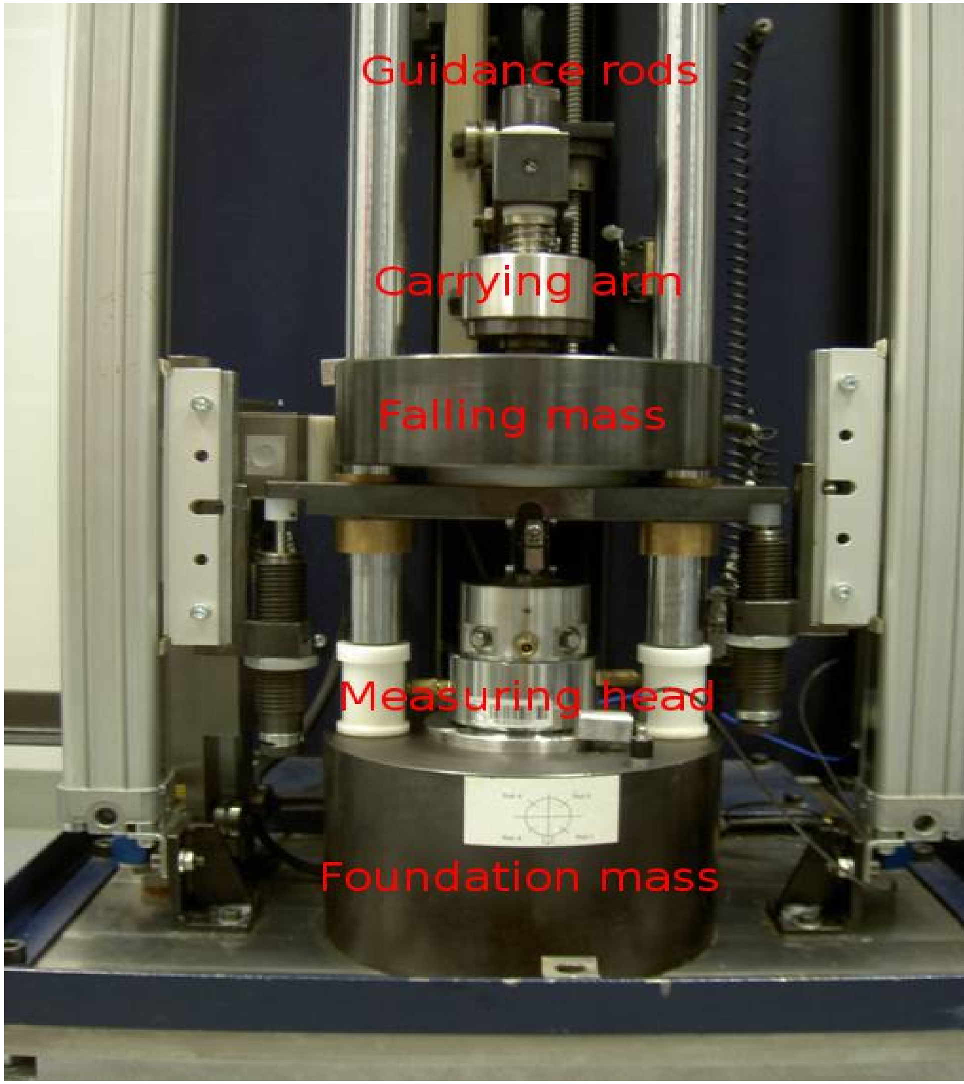
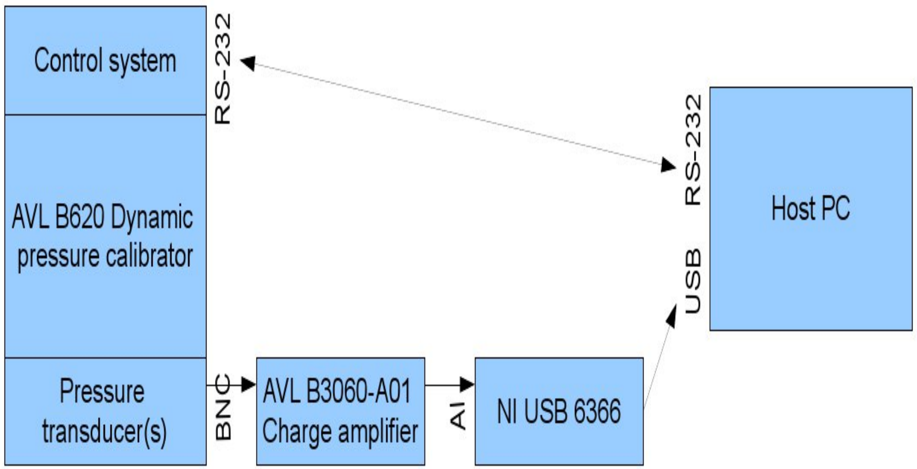
# 2) Working principle of the measurement standard

- Carrying arm and its electromagnet is used to lift a falling mass to the desired falling height
- After release the falling mass falls along guidance rods freely (friction is negligible) and collides with the piston
- Piston compresses glycerol to minimum volume, after that glycerol expands forcing the piston to move upwards
- Finally the falling mass is catched and lifted back to its initial position by the carrying arm

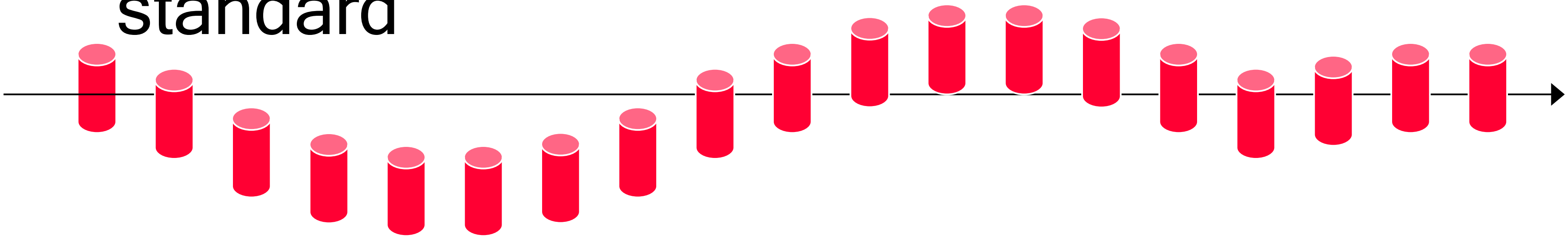


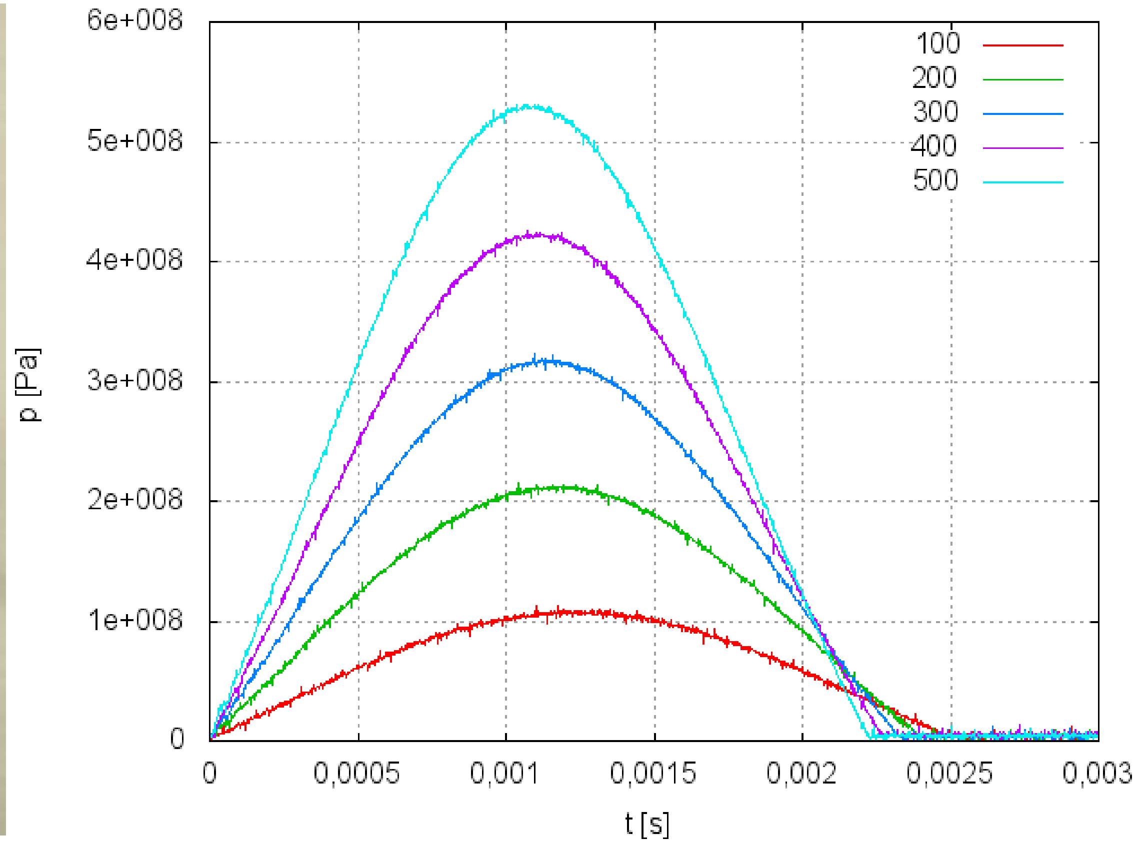
# 2) Working principle of the measurement standard

- Labview-program is used to control the device and to measure pressure pulses
- Communication between a host PC and the device is executed via serial port
- Charges produced by pressure transducers are transformed to voltages by a charge amplifier and digitized by a NI data acquisition device



# 2) Working principle of the measurement standard



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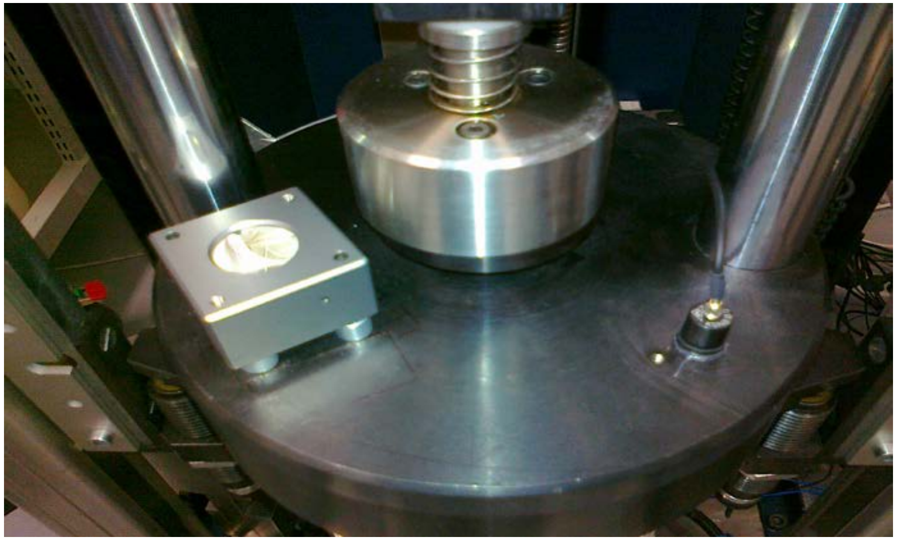
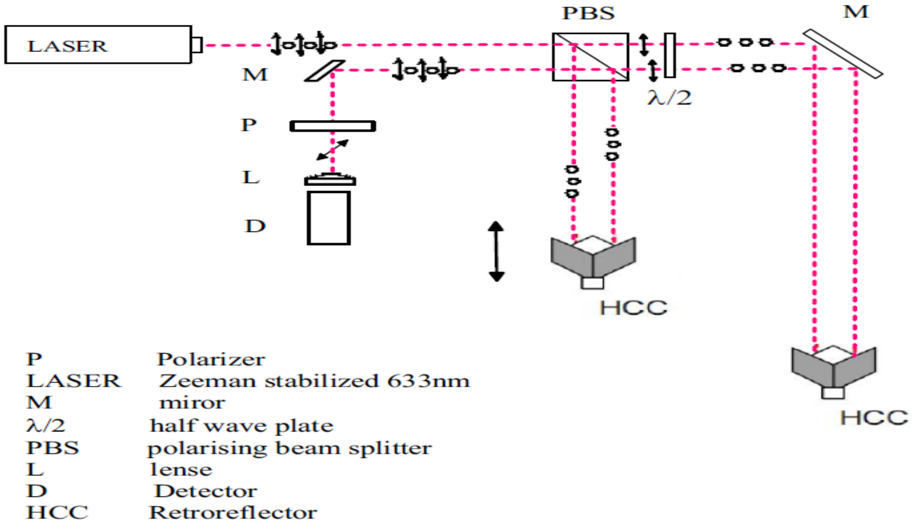
# 2) Working principle of the measurement standard

- Glycerol cylinder is like a spring with a mass (piston and falling weight) attached to it
- Due to inelastic collision between the weight and the piston the displacement of the weight and the displacement of the piston are equal
- Displacement is measured using accelerometer and laser interferometer
- Resulting measured displacements are compared







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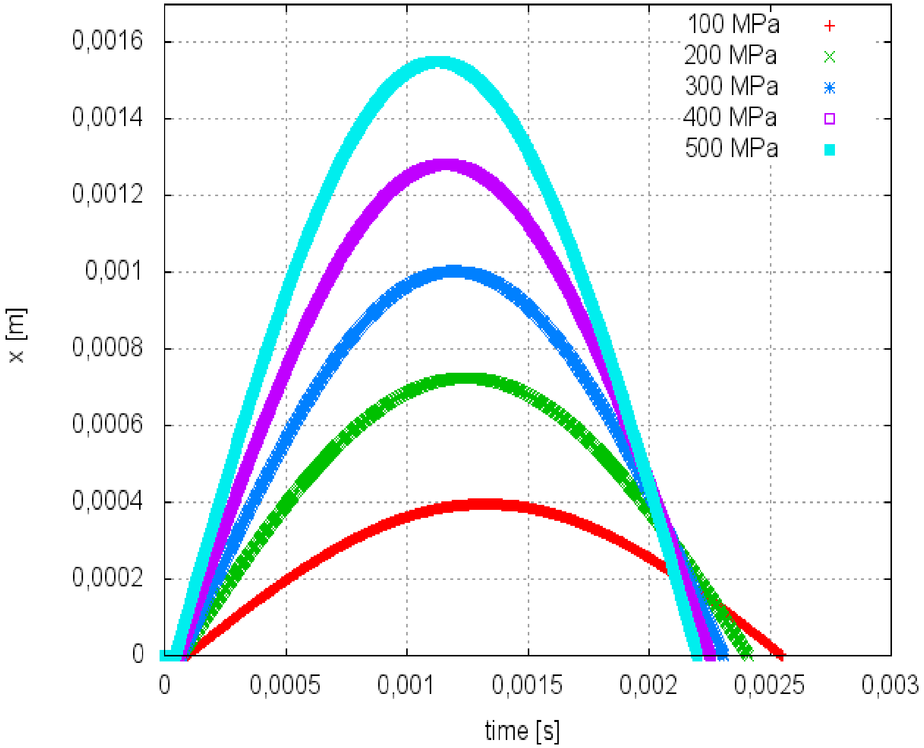
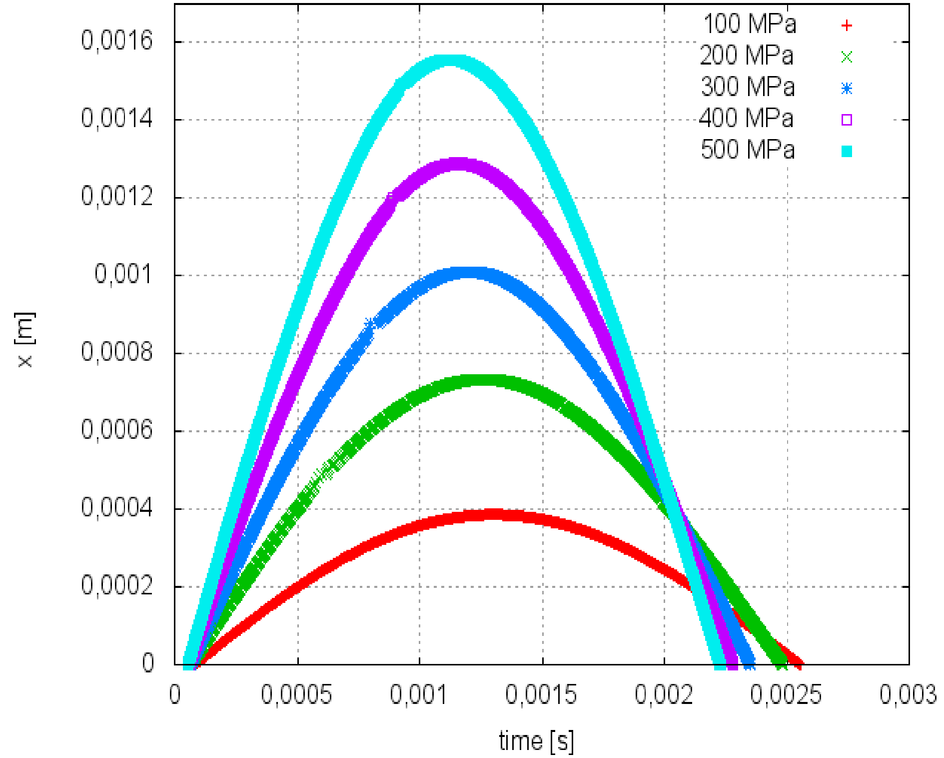
# 2) Working principle of the measurement standard




Accelerometer measurements




Interferometer measurements

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# 3) Modelling pressure pulse mathematically

- As glycerol behaviour during impact is a close match to mechanical spring, we are currently using two methods to describe the displacement of such spring mathematically:
  1. Damped harmonic oscillator method
  2. General Hooke's law method
- Damped harmonic oscillator method is one-dimensional model and very simple; it takes into account stresses and displacements in one direction
- General Hooke's law is multidimensional model and much more complex; simultaneous stresses and displacements in different directions are taken into account


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# Damped harmonic oscillator method

- Linear second order differential equation needs to be solved for displacement $x$
- Model advantages: very simple model, easy to use
- Model drawbacks: treats liquid glycerol as a solid material, fitting the solution of the differential equation to displacement measurements required to obtain parameters $A_0$, $k$ and $D$

$$\mathbf{F} = m\mathbf{a} = m\frac{d^2x}{dt^2} = -kx - D\frac{dx}{dt}$$

$$\Rightarrow \frac{d^2x}{dt^2} + D\frac{dx}{dt} + kx = 0$$

$m$ is total mass of piston and weight
$a$ is acceleration
$x$ is piston displacement
$k$ is spring constant
$D$ is damping parameter
$A_0$ is amplitude of motion

# Damped harmonic oscillator method

- Assuming damping to be relatively small, letting the oscillator to be at rest in $t = 0$ and renaming a constant of integration as $A_0$ leads to solution
- If the damping parameter $D$ is zero, the spring obeys Hooke's law $F = -kx$
- This is the case for our situation

Solution:

$$\Rightarrow x(t) = A_0 e^{-\frac{D}{2m}t} \sin\left(\sqrt{\frac{k}{m} - \frac{D^2}{4m^2}}\, t\right)$$

If $D = 0$, then

$$x(t) = A_0 \sin \sqrt{\frac{k}{m}}\, t$$

# General Hooke's law method

- Hooke's law can be formulated generally as a tensor product of a 4th order compliance tensor and a 2nd order stress tensor
- Model advantages: multidimensional stresses and strains are taken into account, no fit required
- Model drawbacks: material moduli values may be difficult to know as a function of pressure

$$\boldsymbol{\varepsilon} = -\mathbf{s}\boldsymbol{\sigma} \Rightarrow \varepsilon_{ij} = -\sum_{k=1}^{3}\sum_{l=1}^{3} s_{ijkl}\varepsilon_{kl}$$

In matrix form:

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & -\frac{\upsilon}{E} & -\frac{\upsilon}{E} & 0 & 0 & 0 \\ -\frac{\upsilon}{E} & \frac{1}{E} & -\frac{\upsilon}{E} & 0 & 0 & 0 \\ -\frac{\upsilon}{E} & -\frac{\upsilon}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix}$$

$\boldsymbol{\varepsilon}$ is the strain tensor
$\boldsymbol{s}$ is the compliance tensor
$\boldsymbol{\sigma}$ is the stress tensor
$E$ is Young's modulus
$G$ is Shear modulus
$\nu$ is Poisson's ratio

# General Hooke's law method

- 6x6 -matrix must be inverted to solve for stress vector

Inverting the matrix equation:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} \hat{E}(1-\upsilon) & \hat{E}\upsilon & \hat{E}\upsilon & 0 & 0 & 0 \\ \hat{E}\upsilon & \hat{E}(1-\upsilon) & \hat{E}\upsilon & 0 & 0 & 0 \\ \hat{E}\upsilon & \hat{E}\upsilon & \hat{E}(1-\upsilon) & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{pmatrix}$$

$\sigma_{ij}$ are normal stresses
$\varepsilon_{ij}$ are normal strains
$\tau_{ij}$ are shear stresses
$\gamma_{ij}$ are shear strains
$\hat{E}=E/[(1-2v)(1+v)]$



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# 4) Calculating pressure

- Pressure inside the glycerol chamber can be calculated starting from the definintion of pressure in harmonic oscillator method
- $A_0$ and $k$ ($D$ = 0) are obtained from fitting $x(t)$ to displacement data
- In general Hooke's law method the starting point is the definition of mechanical pressure
- If deformations in directions apart from the direction of piston displacement are negligible, the model becomes onedimensional

$$p = \frac{F}{A} = \frac{m\ddot{x}}{A} = \frac{A_0 k}{A}\sin\left(\sqrt{\frac{k}{m}}t\right)$$

$$p = -\sigma_{av} = -\frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right)$$

$$= \frac{E}{3(1-2\upsilon)}\varepsilon_v = K\varepsilon_v$$

If $\varepsilon_v = \varepsilon_{xx}$

$$p = K\varepsilon_{xx}$$

$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ is volumetric strain
$K$ is bulk modulus

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# 5) Uncertainty parameters

- Uncertainty of pressure depends on uncertainties of pressure model parameters
- Both models have measured displacement of the piston, *A* and *T* (temperature of glycerol) as common parameters
- In addition Harmonic oscillator model requires fitting parameters
- Initial volume of pressure chamber is required in general Hooke's law model due to dimensionless strain

| Model | Uncertainty parameters |
|---|---|
| Harmonic oscillator | $A_0$, $k$ $A$, $m$, $x_{meas}$, $T$ |
| General Hooke's law | $K$, $x_{meas}$, $A$, $V_{init}$, $T$ |

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# 6) Next steps

- Lots of measurements required to define the values of fitting parameters of Harmonic oscillator model in different conditions and normal strains of General Hooke's law model (different transducer combinations -> different volume -> different fitting parameters) and uncertainty analysis
- Comparison between different primary dynamic pressure standard systems, deadline June 2014

# 7) Summary

- The new measurement standard developed by MIKES provides calibration measurements for dynamic pressure transducers with a valid traceability chain from 100 MPa to 500 MPa
- Glycerol cylinder is treated as a spring with a mass (falling weight and piston) on top of it
- Spring displacement is measured using accelerometer and laser interferometer and the results are used as input in two separate mathematical models to calculate pressure inside the cylinder
- Key uncertainty parameters which can be measured are mass of the falling object, measured displacement, piston area, glycerol volume, glycerol temperature (in both models)
- Other model dependent uncertainty parameters can be significant

# The end.

## Thank You for Your attention

The research within the project has received funding from the European Union on the basis of Decision No 912/2009/EC. It is carried out in the European Research Programme (EMRP) project IND09 Traceable dynamic measurement of mechanical quantities.

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Mittatekniikan keskus