PTB 268. PTB-Seminar: **Berechnung der Messunsicherheit**

– Empfehlungen für die Praxis

Messunsicherheitsmodell und -bilanz bei der Kalibrierung von Durchflussmessgeräten

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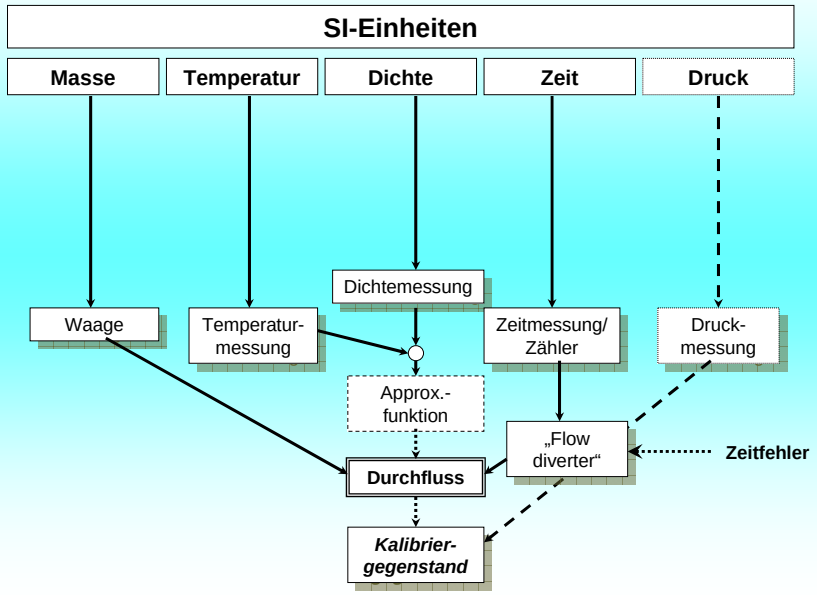
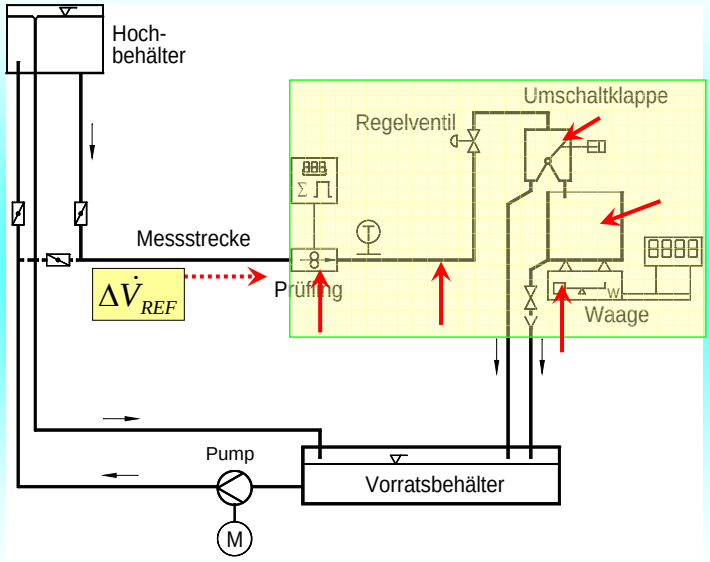
PTB **Durchflussmessung** – *Der Weg zum Messunsicherheitsmodell*

- Durchfluss-Normalmesseinrichtung
- Modell und Einflussgrößen
- Messunsicherheitsbilanz
- Beiträge der Einflussgrößen
- Dynamische Einflüsse

Komponentenweise Rückführkette: *statisch*

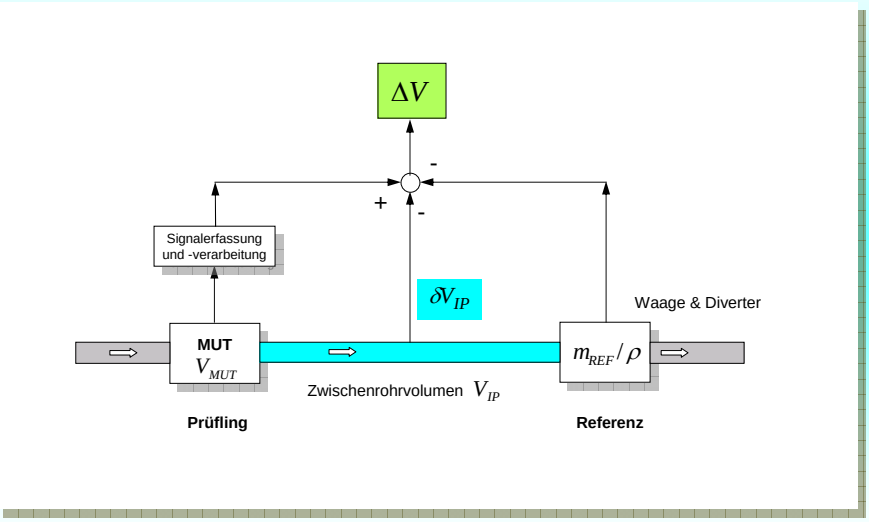


Funktionsprinzip: Gravimetrische Kalibriereinrichtung



Messunsicherheit: ***Einflussfaktoren***

Modell des Messprozesses: *Durchflusskalibrierung*



Modellgleichung des Messprozesses: *K-Faktor*

(Geräte-) **K-Faktor** (Impulszählung):

$$K_{Meter} = \frac{f_{Output}}{\dot{V}_{REF}}$$

Standardunsicherheit:

$$u^2_{K\_meter} = \left(\frac{\partial K_{Meter}}{\partial f_{Outout}} u_f\right)^2 + \left(\frac{\partial K_{Meter}}{\partial \dot{V}} \cdot \frac{\partial \dot{V}}{\partial m}\, u_m\right)^2 + \left(\frac{\partial K_{Meter}}{\partial \dot{V}} \cdot \frac{\partial \dot{V}}{\partial \rho_{Water}}\, u_\rho\right)^2 +$$
$$+ \left(\frac{\partial K_{Meter}}{\partial \dot{V}} \cdot \frac{\partial \dot{V}}{\partial (\Delta V_{IP})}\, u_{\Delta V}\right)^2 + \left(\frac{\partial K_{Meter}}{\partial \dot{V}} \cdot \frac{\partial \dot{V}}{\partial (\Delta V_{T\_Error})}\, u_{T\_Error}\right)^2 + \left(\frac{\partial K_{Meter}}{\partial \dot{V}} \cdot \frac{\partial \dot{V}}{\partial T_{MEAS}}\, u_T\right)^2$$

Relative Standardunsicherheit

$$\left(\frac{u_{K\_Meter}}{K_{Meter}}\right)^2 = \left(\frac{u_f}{f_{Output}}\right)^2 + \left(\frac{u_m}{m}\right)^2 + \left(\frac{u_\rho}{\rho_{Water}}\right)^2 + \left(\frac{u_{\Delta V}}{V_0}\right)^2 + \left(\frac{u_{T\_Error}}{V_0}\right)^2 + \left(\frac{u_T}{T_{MEAS}}\right)^2$$

Komponenten: *In-process-Dichtemessung*

Dichtemessgerät:

- Typ: DMA 500
  (Anton Paar GmbH., Austria)
- Sensor: vibrating U-tube

Charakteristika:

- *Bereich:* 0...3 g/cm³
- *Genauigkeit:* - Dichte: $5 \cdot 10^{-6}$ g/cm³
  - Temperatur: 0.01 °C

Pumpe zur Probennahme:
Probe: 1 ml

Probenbehälter

Messung der Wasserdichte (Funktionsprinzip)

Messung der Wasserdichte: *mittels Approximationsfunktion*

Eingangsgrößen der Dichte-Approximationsfunktion:

$$\rho_{Approx}(\vartheta_{Water}) = f(\vartheta_{Cal,i}, \rho_{Cal,i}, \vartheta_{Water})$$

Unsicherheitsbeiträge:

- Temperaturanzeige des Dichtemessgerätes:	± **10 mK**, standard uncertainty Type A
- Dichteanzeige:	**0.0085 kg/m³** (Type B: rectangular probability distribution)
- Numerische Approximation (3. Ordnung):	± **$2.0 \cdot 10^{-6}$** or **1,997 kg/m³** (rectangular probability function)

Messung der Wasserdichte: *mittels Approximationsfunktion*

Unsicherheitsbeitrag durch Anwendung der Approximationsfunktion:
(Sensitivitätskoeffizienten: numerische Berechnung)

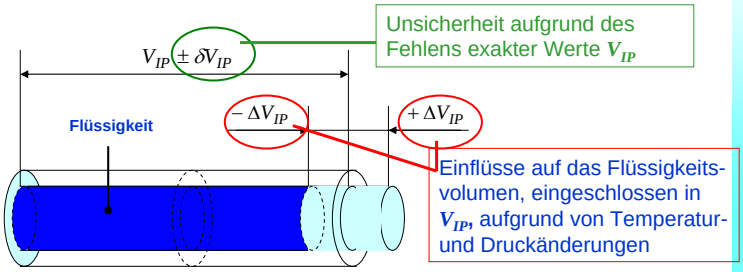
$$\left[U(\rho_{Approx})\right]^2 = \left[\frac{\partial \rho_{Approx}}{\partial \vartheta_{Cal,i}} \cdot u(\vartheta_{Cal,i})\right]^2 + \ldots$$

$$\ldots + \left[\frac{\partial \rho_{Approx}}{\partial \rho_{Cal,i}} \cdot u(\rho_{Cal,i})\right]^2 + \left[\frac{\partial \rho_{Approx}}{\partial \vartheta_{Process}} \cdot u(\vartheta_{Process})\right]^2$$

Unsicherheitsbeiträge (Einflussgrößen):

- Temperaturmessung:	Pt-100-Sensor (mit Elektronik)
- Temperaturanzeige:	DMA 5000, Dichtemessgerät
- Dichteanzeige:	DMA 5000, Dichtemessgerät

„Grob"- Model: *Zwischenrohr*



Einflüsse: *1) – Zwischenrohr-Volumen* | *2) – eingeschlossenes Luftvolumen*

$$\Delta V_{IP} = \left[\frac{\pi}{4} \cdot \left[d_i + \frac{(p_{finish} - p_0) \cdot d_i^{\,2}}{2 \cdot w \cdot E}\right]^2 \cdot l \cdot \left[1 + 3 \cdot \alpha \cdot (\vartheta_{finish} - \vartheta_0)\right] - V_{air,0} \cdot \frac{p_0}{p_{finish}} \cdot \frac{\vartheta_{finish} + 273.15}{\vartheta_0 + 273.15}\right] - \ldots.$$

$$\ldots. - \left[\frac{\pi}{4} \cdot \left[d_i + \frac{(p_{start} - p_0) \cdot d_i^{\,2}}{2 \cdot w \cdot E}\right]^2 \cdot l \cdot \left[1 + 3 \cdot \alpha \cdot (\vartheta_{start} - \vartheta_0)\right] - V_{air,0} \cdot \frac{p_0}{p_{start}} \cdot \frac{\vartheta_{start} + 273.15}{\vartheta_0 + 273.15}\right]$$

3) – eingeschlossenes Flüssigkeitsvolumen

CALCULATION OF UNCERTAINTY BUDGET

Test Rig: *3-tons balance*

Parameters:

Facility	Flowrate:	**72,0**	m³/h	
- Balance	Target weight:	**2500,0**	kg	
- Cal. weights	Density	**7845,0**	kg/m³	Carbon steel
	Diversion time:	125,1	s	
	Ref. Volume:	2,5030	m³	
- Ambient air	Air density	1,1800	kg/m³	
- Interc. piping	IP volume:	0,5008	m³	steel, plastics
	"effective" TC	1,7689E-04	1/K	
	delta Temp.:	**0,050**	K	During measm.
- Diverter	Timing error:	**20,0**	ms	Standard dev.
Fluid	Water density:	998,8173	kg/m³	(at 20 °C)
	Water temp.:	**20,000**	°C	
Test meter	Pulse frequenc	**209,00**	Hz	Turbine meter Example
Repeatability		**3,50E-05**		

Measurement quantity:

- Mass

Symbol	*Source of uncertainty*	*Value ±g*	*Probability distribution*	*Divisor*	c_i	$u_i(W_x)$ ±g	$u_i(W_x)$ ± %
W_S	Calibration of standard weights	2,3	normal	2,0	1,0	1,150	4,60E-05
S_D	Discrimination of balance	0,5	rectangular	1,73	1,0	0,289	1,16E-05
S_R	Repeatability of balance	104	normal	2,0	1,0	52,000	2,08E-03
S_{Drift}	Drift of balance	10	rectangular	1,73	1,0	5,780	2,31E-04
							2,093E-03
$u_c(W_x)$	Combined uncertainty		normal			52,3	**2,093E-03**
$U(W_x)$	Expanded uncertainty		normal(k = 2)			104,7	0,004

- Buoyancy

Symbol	*Source of uncertainty*	*Value ±kg/m³*	*Probability distribution*	*Divisor*	c_i	$u_i(W_x)$ ±kg	$u_i(W_x)$ ± %
D_{Air}	Ambient air density	5,90E-04	normal	2,0	8,758E-04	2,584E-07	2,19E-05
D_{Water}	Water density	3,31E-02	normal	2,0	1,920E-08	3,178E-10	3,18E-11
D_{Cal}	Density of calibration weights	1,0	normal	2,0	1,185E-06	5,927E-07	7,56E-09
							2,189E-05
$u_c(C_B)$	Combined uncertainty		normal			6,466E-07	**5,479E-08**
$U(C_B)$	Expanded uncertainty		normal(k = 2)			0,0	0,000

- Temperature

Symbol	*Source of uncertainty*	*Value ±K*	*Probability distribution*	*Divisor*	c_i	$u_i(T_x)$ ±K	$u_i(T_x)$ ± %
T_D	Discrimination of temp.measurement	0,025	rectangular	1,73	1,0	0,014	7,23E-02
T_C	Sensor/meter calibration	0,18	rectangular	1,73	1,0	0,104	5,20E-01
T_G	Temp. gradient in water (estimated)	0,5	rectangular	1,73	1,0	0,289	1,45E+00
							1,538E+00
$u_c(T_x)$	Combined uncertainty		normal			**0,308**	1,538E+00
$U(T_x)$	Expanded uncertainty		normal(k = 2)			0,615	3,075

Fortsetzung:

- Density (Aproximation function based on density measurement with type DMA 500 density meter)

Symbol	*Source of uncertainty*	*Value ±*	*Probability distribution*	*Divisor*	c_i	$u_i(D_x)$ ±kg/m³	$u_i(D_x)$ ± %
D_{T_approx}	Temp. Meas. for appr. [K]	0,010	normal	2,0	6,6435E-06	3,3218E-08	3,33E-09
D_{D_approx}	Density meas. for appr. [kg/m³]	0,0085	normal	2,0	1,9995	0,008	8,51E-04
D_P	Temp. downstream of MUT [K]	**0,308**	normal	2,0	2,08E-01	0,032	3,20E-03
D_S	Numeric approx. Error [kg/m³]	0,002	rectangular	1,73	1,0	0,001	1,16E-04
							3,315E-03
$u_c(D_x)$	Combined uncertainty		normal			0,033	**3,315E-03**
$U(D_x)$	Expanded uncertainty		normal(k = 2)			0,066	0,007

- Time (Measurement/Diversion)

Symbol	*Source of uncertainty*	*Value ± ms*	*Probability distribution*	*Divisor*	c_i	$u_i(T_x)$ s	$u_i(T_x)$ ± %
T_C	Calibration of Timer (5x10-9)	5,0E-12	normal	2,0	1,0	2,500E-15	2,00E-15
T_D	Discrimination of Timer display	0,001	rectangular	1,73	1,0	5,780E-07	4,62E-07
T_T	Discrmination of Diverter Time	0,05	rectangular	1,73	1,0	2,890E-05	2,31E-05
							2,310E-05
$u_c(T_x)$	Combined uncertainty		normal			2,891E-05	**2,3099E-05**
$U(T_x)$	Expanded uncertainty		normal(k = 2)			5,782E-05	4,620E-05

- Diverter timing error

Symbol	*Source of uncertainty*	*Value ±*	*Probability distribution*	*Divisor*	c_i	$u_i(T_{D\_Error})$ m³	$u_i(T_{D\_Error})$ ± %
T_C	Diverter timing error [ms]	**20,00**	normal				
F	Flowrate [m³/h]	72,00					
V_{Error}	Diversion error volume [m³]	4,0000E-04	normal	2,0	1,0	2,000E-04	7,9905E-03
$u_c(T_{Error})$	Combined uncertainty		normal			2,000E-04	**7,991E-03**
$U(T_{Error})$	Expanded uncertainty		normal(k = 2)			4,000E-04	1,598E-02

- Interconnecting piping volume error

Symbol	*Source of uncertainty*	*Value ±*	*Probability distribution*	*Divisor*	c_i	$u_i(V_{IP\_Error})$ m³	$u_i(V_{IP\_Error})$ ± %
delta T_{IP}	Temp. Variation during calibr. [K]	0,050	rectangular				
V_{IP_TC}	IP volume [m³]	0,501					
TC	effektive TC [1/K]	1,7689E-04					
	IP volume error	4,4294E-06	rectangular	1,73	1,0	2,560E-06	1,023E-04
$u_c(T_{Error})$	Combined uncertainty		normal			2,560E-06	**1,023E-04**
$U(T_{Error})$	Expanded uncertainty		normal(k = 2)			5,121E-06	2,046E-04

- Meter under test

Symbol	*Source of uncertainty*	*Value ±*	*Probability distribution*	*Divisor*	c_i	$u_i(f_x)$ Hz	$u_i(f_x)$ ± %
$T_D = 0{,}5*1/N_{Pulses}$	Discr. of frequency meas.	3,32E-05	rectangular	1,73	1,0	4,0141E-03	1,921E-03
$u_c(f_x)$	Combined uncertainty		normal			4,014E-03	**1,921E-03**
$U(f_x)$	Expanded uncertainty		normal(k = 2)			8,028E-03	0,004

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Fortsetzung 2:

Summarized Uncertainty - "*Flowrate measurement*"

Symbol	*Source of uncertainty*	*Value ± %*	*Probability distribution*	*u_i(FLOW) ± %*
u_c(m)	Combined uncertainty (MASS)	2,093E-03	normal	2,093E-03
$u_c(C_{Buoy})$	Combined uncertainty (BUOYANCY)	5,479E-08	normal	5,479E-08
u_c(RHO)	Combined uncertainty (DENSITY)	3,315E-03	normal	3,315E-03
$u_c(T_{MEAS})$	Combined uncertainty (TIME)	2,310E-05	normal	2,310E-05
$u_c(V_{IP})$	Combined uncertainty (IP volume)	1,023E-04	normal	1,023E-04
$u_c(T_{Error})$	Combined uncertainty (Time error)	7,991E-03	normal	7,991E-03
S(Repeatabil.)	Repeatability uncertainty	3,500E-03	normal	3,500E-03
u_c(FLOW)	Combined uncertainty (FLOW)		normal	0,00956
U(FLOW)	**Expanded uncertainty (FLOW)**		**normal(k = 2)**	**0,019**

Summarized Uncertainty - "*Quantity passed measurement*"

Symbol	*Source of uncertainty*	*Value ± %*	*Probability distribution*	*u_i(FLOW) ± %*
u_c(m)	Combined uncertainty (MASS)	2,093E-03	normal	2,093E-03
$u_c(C_{Buoy})$	Combined uncertainty (BUOYANCY)	5,479E-08	normal	5,479E-08
u_c(RHO)	Combined uncertainty (DENSITY)	3,315E-03	normal	3,315E-03
$u_c(T_{MEAS})$	Combined uncertainty (TIME)	2,310E-05	not effective	
$u_c(V_{IP})$	Combined uncertainty (IP volume)	1,023E-04	normal	1,023E-04
$u_c(T_{Error})$	Combined uncertainty (Time error)	7,991E-03	normal	7,991E-03
S(Repeatabil.)	Repeatability uncertainty	3,500E-03	normal	3,500E-03
u_c(QUANTITY)	Combined uncertainty (QUANTITY)		normal	0,00956
U(Quantity)	**Expanded uncertainty (QUANTITY)**		**normal(k = 2)**	**0,019**

Summarized Uncertainty - "*Meter K-factor determination*"

Symbol	*Source of uncertainty*	*Value ±%*	*Probability distribution*	*u_i(K) ± %*
u_c(m)	Combined uncertainty (MASS)	2,093E-03	normal	2,093E-03
$u_c(C_{Buoy})$	Combined uncertainty (BUOYANCY)	5,479E-08	normal	5,479E-08
u_c(RHO)	Combined uncertainty (DENSITY)	3,315E-03	normal	3,315E-03
$u_c(T_{MEAS})$	Combined uncertainty (TIME)	2,310E-05	normal	2,310E-05
$u_c(V_{IP})$	Combined uncertainty (IP volume)	1,023E-04	normal	1,023E-04
$u_c(T_{Error})$	Combined uncertainty (Time error)	7,991E-03	normal	7,991E-03
u_c(Frequence)	Combined uncertainty (Frequency)	1,921E-03	normal	1,921E-03
S(Repeatabil.)	Repeatability uncertainty	3,500E-03	normal	3,500E-03
u_c(K-FACTOR)	Combined uncertainty (K-FACTOR)		normal	0,00976
U(K-FACTOR)	**Expanded uncertainty (K-FACTOR)**		**normal(k = 2)**	**0,020**

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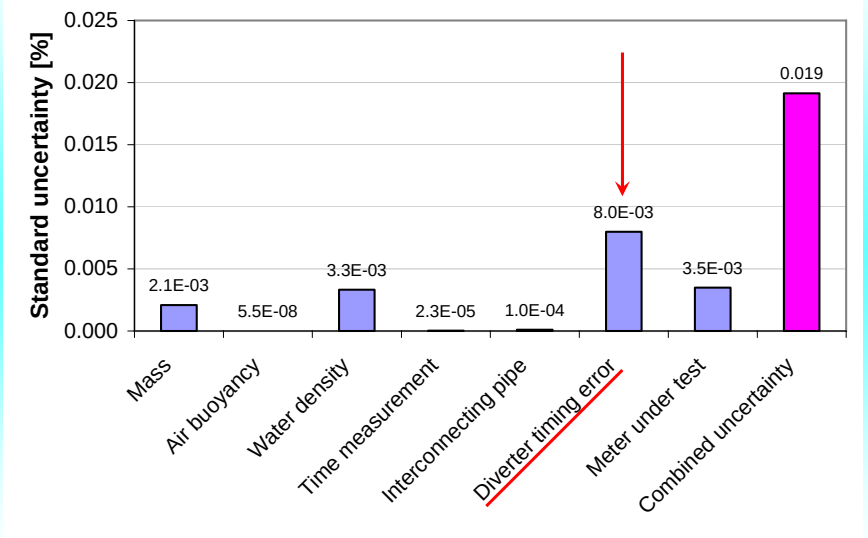
Erweiterte Messunsicherheit



Unsicherheitsbeiträge relevanter Funktionskomponenten

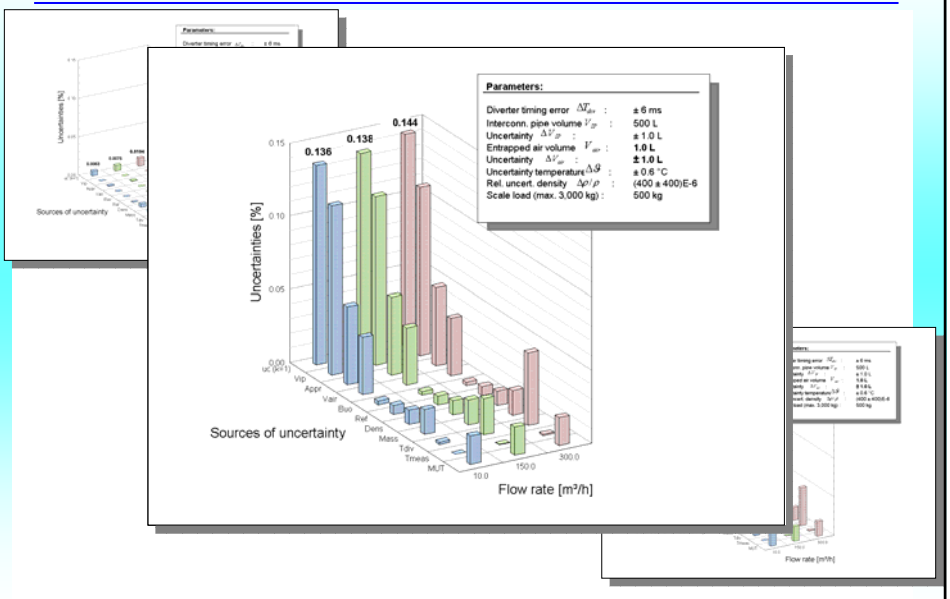
Gravimetrische Messeinrichtung: *Messunsicherheit*

	Sources of uncertainty	Relative standard uncertainty [%]			
$u(x_1)$	***Mass***				***2.09E-03***
	Calibration of standard weights			4.60E-05	
	Discrimination of balance			1.16E-05	
	Repeatability of balance			2.08E-03	
	Drift of balance			2.31E-04	
$u(x_2)$	***Bouyancy***				***2.19E-05***
	Ambient air density			2.19E-05	
	Water density			3.18E-11	
	Density of calibration weights			7.56E-09	
$u(x_3)$	***Temperature of fluid***		1.54E+00		
	Discrimination of temp.meas.	7.23E-02			
	Sensor/meter calibration	5.20E-01			
	Temp. gradient in water (estim.)	1.45E+00			
$u(x_4)$	***Density of fluid***				***3.31E-03***
	Temp. Meas. for appr. function			3.33E-09	
	Density meas. for appr. function			8.51E-04	
	Temp. downstream of MUT		1.54E+00	3.20E-03	
	Numeric approx. Error			1.16E-04	
$u(x_5)$	***Time (Measurement/Diversion)***				***2.31E-05***
	Calibration of Timer (5x10-9)			2.00E-15	
	Discrimination of Timer display			4.62E-07	
	Discrimination of Diverter Time			2.31E-05	
$u(x_6)$	***Diverter timing error***				***7.99E-03***
	Diverter timing error				
	Flowrate				
	Diversion error volume			7.99E-03	
$u(x_7)$	***Interconnecting piping volume***				***1.02E-04***
	Temp. Variation during calibr.		1.54E+00		
	IP volume				
	effektive TC				
	IP volume error			1.02E-04	
$u(x_8)$	***Meter under test (MUT)***				***1.92E-04***
	Discr. of frequency meas.			1.92E-04	
$u(x_9)$	***Repeatability uncertainty***				3.50E-03
	u_c(K-factor)	Combined uncertainty			***0.010***
	U(K-factor)	Expanded Uncertainty (k=2)			**0.019**

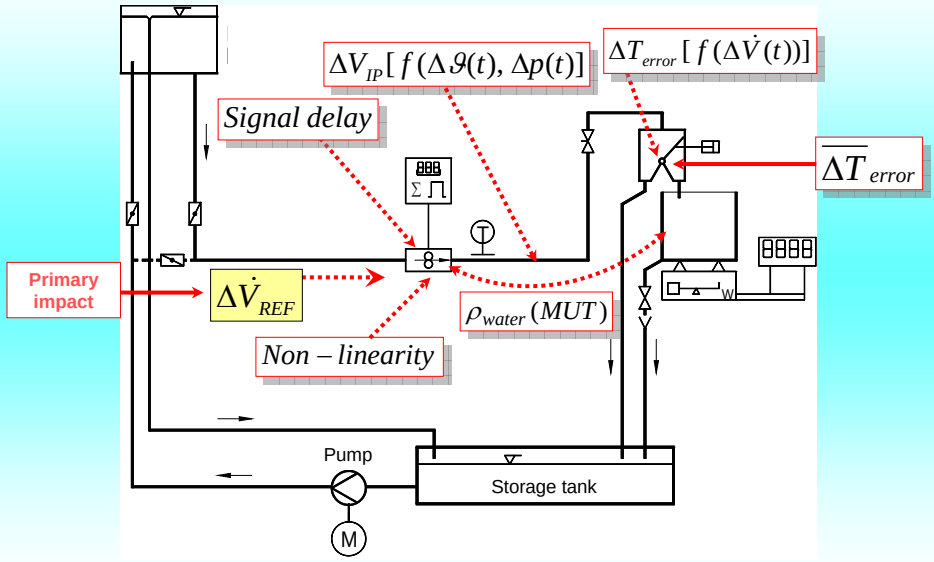
Korrelations-effekte

$u(y)$

Einflussfaktoren, die auf die Messunsicherheit wirken

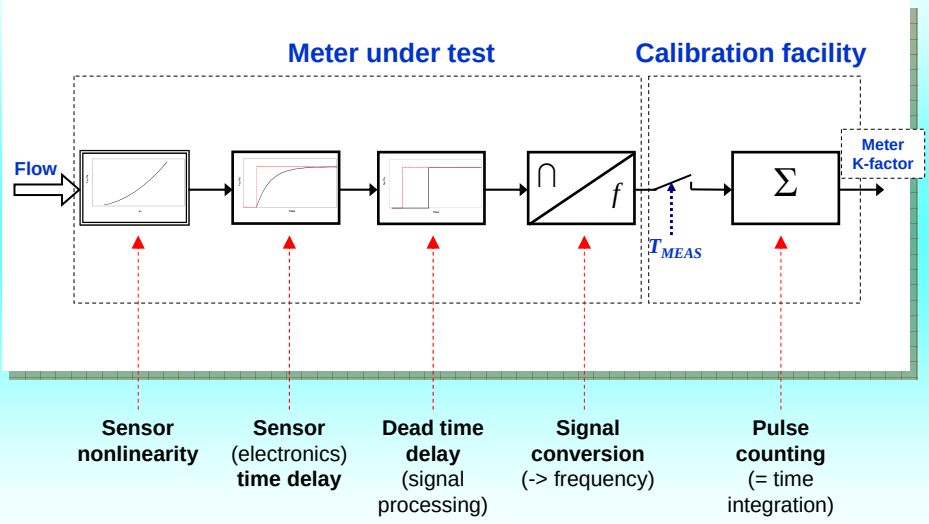


Gravimetric flow calibration facility

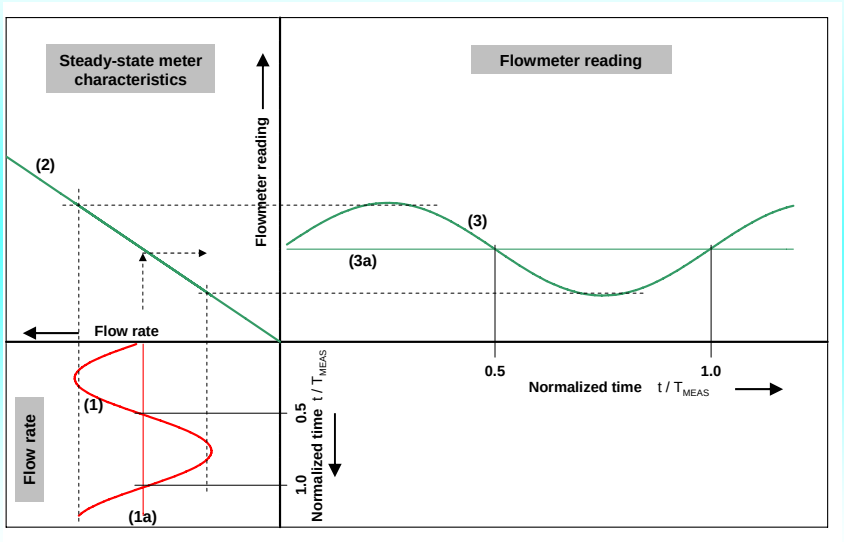


Element-by-element traceability: *Dynamic effects*

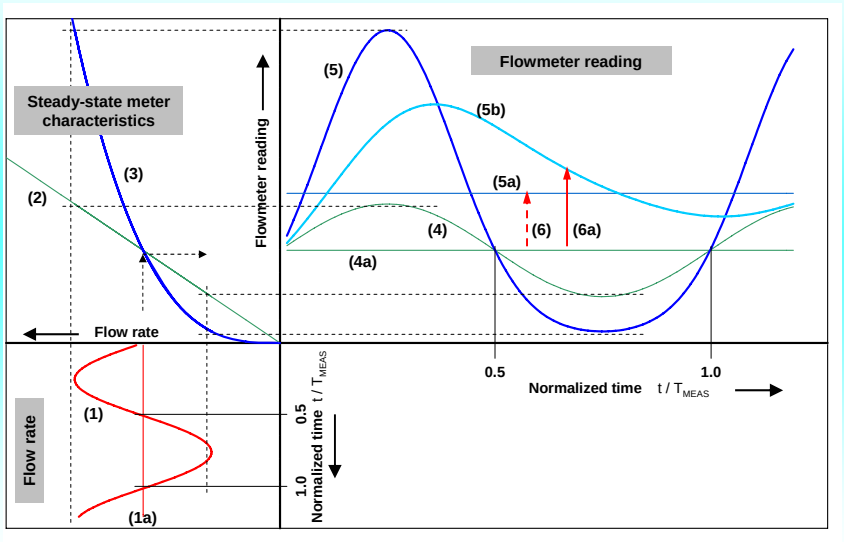
Flowmeter: *dynamic behavior*



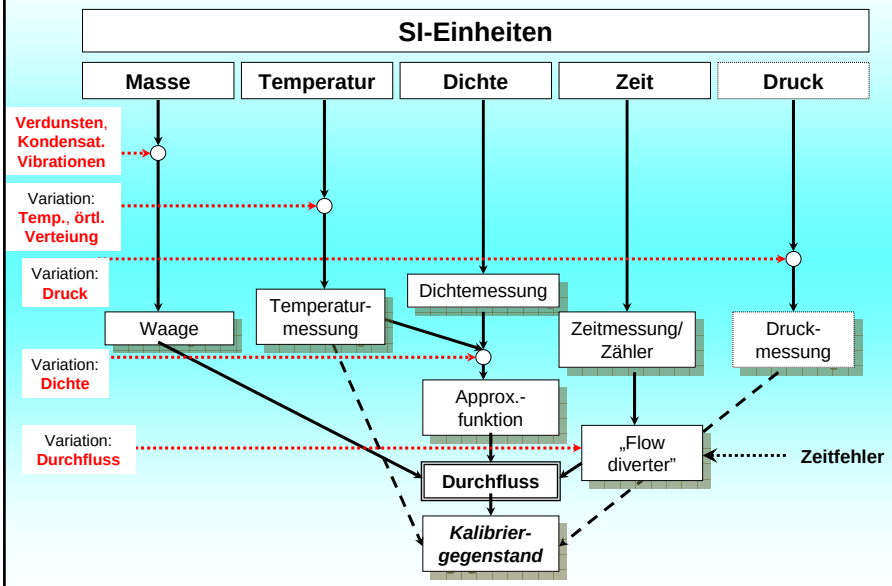
Meter errors caused by flow rate variations:
linear steady-state characteristics



Meter errors caused by flow rate variations:
due to nonlinear steady-state characteristics



Komponentenweise Rückführkette: *dynamische Effekte*



Unsicherheitsbudget – Erweiterung

Nur statische Einflüsse

Sources of uncertainty	Relative standard uncertainty [%]			
Mass				***2,09E-03***
Calibration of standard weights			4,60E-05	
Discrimination of balance			1,16E-05	
Repeatability of balance			2,08E-03	
Drift of balance			2,31E-04	
Bouyancy				***2,19E-05***
Ambient air density			2,19E-05	
Water density			3,18E-11	
Density of calibration weights			7,56E-09	
Temperature of fluid		#######		
Discrimination of temp.meas.	7,23E-02			
Sensor/meter calibration	5,20E-01			
Temp. gradient in water (estim.)	1,45E+00			
Density of fluid				***3,31E-03***
Temp. Meas. for appr. function			3,33E-09	
Density meas. for appr. function			8,51E-04	
Temp. downstream of MUT		1,54E+00	3,20E-03	
Numeric approx. Error			1,16E-04	
Time (Measurement/Diversion)				***2,31E-05***
Calibration of Timer (5x10-9)			2,00E-15	
Discrimination of Timer display			4,62E-07	
Discrimination of Diverter Time			2,31E-05	
Diverter timing error				***7,99E-03***
Diverter timing error				
Flowrate				
Diversion error volume			7,99E-03	
Interconnecting piping volume				***1,02E-04***
Temp. Variation during calibr.		1,54E+00		
IP volume				
effektive TC				
IP volume error			1,02E-04	
Meter under test (MUT)				***1,92E-04***
Discr. of frequency meas.			1,92E-04	
Repeatability of MUT				
Reproduceability of MUT				
Repeatability uncertainty				3,50E-03
u_c(K-factor)	Combined uncertainty			*0,010*
U(K-factor)	Expanded Uncertainty (k=2			0,019

Statische & dynamische Einflüsse

Sources of uncertain	Relative standard uncertainty [%]			
Mass				***2,09E-03***
Calibration of standard weights			4,60E-05	
Discrimination of balance			1,16E-05	
Repeatability of balance			2,08E-03	
Drift of balance			2,31E-04	
Dynamic impacts				
Bouyancy				***2,19E-05***
Ambient air density			2,19E-05	
Water density			3,18E-11	
Density of calibration weights			7,56E-09	
Temperature of fluid		1,54E+00		
Discrimination of temp.meas.	7,23E-02			
Sensor/meter calibration	5,20E-01			
Temp. gradient in water (estir	1,45E+00			
Dynamic impacts				
Density of fluid				***3,31E-03***
Temp. Meas. for appr. function			3,33E-09	
Density meas. for appr. function			8,51E-04	
Temp. downstream of MUT		1,54E+00	3,20E-03	
Numeric approx. Error			1,16E-04	
Dynamic impacts				
Time (Measurement/Diversion)				***2,31E-05***
Calibration of Timer (5x10-9)			2,00E-15	
Discrimination of Timer display			4,62E-07	
Discrimination of Diverter Time			2,31E-05	
Diverter timing error				***7,99E-03***
Diverter timing error				
Flowrate				
Diversion error volume			7,99E-03	
Dynamic impacts				
Interconnecting piping volume				***1,02E-04***
Temp. Variation during calibr.		1,54E+00		
IP volume				
effektive TC				
IP volume error			1,02E-04	
Dynamic impacts (pressure)				
Meter under test (MUT)				***1,92E-04***
Discr. of frequency meas.			1,92E-04	
Repeatability of MUT				
Reproduceability of MUT				
Dynamic impacts				
Repeatability uncertainty of facility				3,50E-03
u_c(K-factor)	Combined uncertainty			*0,010*
U(K-factor)	Expanded Uncertainty (k=2)			0,019

PTB

Vielen Dank für Ihre Aufmerksamkeit.

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