Annex C: Measurement uncertainty of the total height of profile of a depth-setting standard with the standard deviation of the groove depth as topography term

In this example, the uncertainty sources derived in general in chapter 2 of Annex A are transferred to the conditions of depth-setting standards. In the example a glass standard with a nominal value of 3 µm is assumed.

1. Calibration factor

1.1 Reference standard

According to the calibration certificate, the uncertainty of the reference standard (calibrated depth-setting standard) is, for example, U_n = 10 nm, at k = 2,

$$u^2(Pt_n) = \frac{U_n^2}{4} = 25\,\text{nm}^2$$

1.2 Difference measuring trace – calibration trace

$u^2(Pt_{my}) = \frac{a_y^2 \cdot G^2}{3}$. With a_y = 100 µm and G = 20 nm/mm (Annex A Figure 1),

$$u^2(Pt_{my}) = \frac{(100\,\mu\text{m})^2 \cdot (20\,\text{nm/mm})^2}{3} = 1{,}33\,\text{nm}^2$$

1.3 Repeatability

As estimated value for the uncertainty in the tracing of the reference standard, the experimental standard deviation of the mean value of Pt_{m} is taken from m_{w} repeat measurements performed at the same point:

$u^2(b) = \frac{s_w^2(Pt_m)}{m_w} = s_w^2(\overline{Pt_m})$. With a typical value for the experimental variance $s_w(\overline{Pt_m}) = 2nm$ is

$u^2(b) = 4\,\text{nm}^2$.

2. Topografy of the standard

In the direction of the longitudinal axis, the groove depth of the test object to be measured is usually not constant. A typical value is a variation width of 40 nm. This is why the groove depth is determined on m_t different points of the groove. As estimated value for the uncertainty component of the profile points, the standard deviation of the mean value from Pt_{m} is used: $s_t\left(\overline{Pt_{\text{m}}}\right) = \frac{s_t(Pt_{\text{m}})}{\sqrt{m_t}}$.

A prerequisite of this observation is that the groove depth varies randomly and not systematically. In the case of typically m_t=5 profile steps, Pt_{m} has an experimental standard deviation of 5 nm. This means that in the case of this observation, the less labour-intensive dissemination of the mean value of the groove depth is of main interest, and an exacter positioning of the measuring point is dispensed with.

$$u^2(z_e) = \frac{s_t^2(Pt_\mathrm{m})}{m_t} = 5\,\mathrm{nm}^2$$

3. Straightness deviation of the reference

For the guidance devaitions in the guiding area used for the device, $Wt_0 \leq 20$ nm is assumed. The variance contribution is

$$u^2(z_{ref}) = \frac{Wt_0^2}{12} = \frac{(20\,\mathrm{nm})^2}{12} = 33\,\mathrm{nm}^2 .$$

4. Background noise

The background noise of an optical flat be Rz_0 = 20 nm.

$$u^2(z_{pl}) = \frac{Rz_0^2}{12} = 33\,\mathrm{nm}^2 .$$

5. Plastic deformation

For a glass standard, the plastic deformation is negligibly small.

6. Uncertainty of the points of the total profile

From the sum of these uncertainties, the variance of the points of the total profile is obtained as follows:

$$u^2(z_g) = \frac{U_n^2}{4} + \frac{a_y^2 \cdot G^2}{3} + \frac{s_w^2(Pt_m)}{m_w} + \frac{s_t^2(Pt_m)}{m_t} + \frac{Wt_0^2}{12} + \frac{Rz_0^2}{12}$$

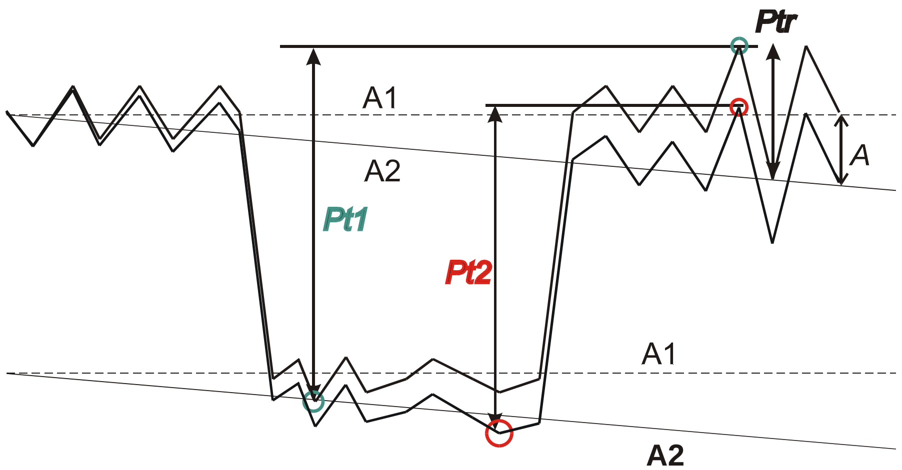
The first three contain the uncertainty of the traceabilty and the last three the uncertainty contributions from the measuring process. The summands of this equation are listed in the table in chapter 9. With the exemplary numerical values from chapters 1 to 4 the following is obtained:

$u\,(z_g)$ = 10.1 nm.

7. Parameter function

7.1 Uncertainty of measurement of the total height of profile *Pt*

In the aligned profile, the total height of profile *Pt* is the difference between the highest z-value on the reference plane (z_h) and the lowest z-value in the tread of the groove (z_l).
When the alignment is made, the profile points are lifted differently and an alignment deviation occurs due to roughness and flatness errors. By this, the difference between the highest and the lowest point varies by A at most, as a function of where these lie. Taking the average of several measurements, the expectation value is zero. The model for *Pt* is valid:
$Pt = z_h - z_l + A.$



A1: reference lines @ leveling 1
A2: reference lines @ leveling 2
A: leveling deviation
Pt1: *Pt* @ leveling 1
Pt2: *Pt* @ leveling 2
Ptr : roughness on reference plane

Figure 7: Uncertainty of *Pt* due to alignment deviation

The following is valid for the uncertainty of the total height of profile *Pt*:

$$u^2(Pt) = (\frac{\partial Pt}{\partial z_h})^2 \cdot u^2(z_h) + (\frac{\partial Pt}{\partial z_l)})^2 \cdot u^2(z_l) + (\frac{\partial Pt}{\partial A})^2 \cdot u^2(A).$$

The two first sensitivity coefficients are 1. The third describes in principle the cosinue factor which changes with the alignment. Due to the smaller angles and their changes, it is equal to one with sufficient accuracy. The uncertainty of the z-values $u(z_h)$ and $u(z_l)$ is equal to that of the points of the overall profile $u(z_{g)})$.. The peak value Pt_r of the roughness or the flatness deviation of the profile components in the reference line sections included in the evaluation serves as an estimated value for the variability of the alignment deviation *u(A)*. Within Pt_r , a rectangular distribution is assumed.

$$u^2(Pt) = 2 \cdot u^2(z_g) + \frac{1}{12} \cdot (Pt_r)^2 .$$

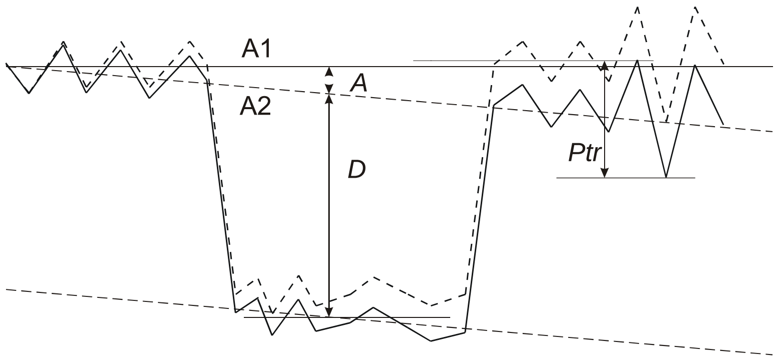
On depth-setting standards, Pt_r= 10 nm is a usual value. The variance of *Pt* thus is determined as

$$u^2(Pt) = 2 \cdot (10{,}1\ \text{nm})^2 + \frac{1}{12} \cdot (10\ \text{nm})^2,$$

and the standard uncertainty of *Pt* is $u(Pt) = 14{,}6$ nm

With the coverage factor k=2, an expanded uncertainty of measurement of *U* (*Pt*) = 29.2 nm is obtained.

7.2 Measurement uncertainty of the groove depth *D*



A1: reference line with levelling error
A2: assumed correct reference line
A: levelling error
Ptr : roughness on reference plane
D: depth of groove

Figure 8: Uncertainty of D due to alignment deviation

As an approximation for the calculation of the groove depth *D* in accordance withto DIN EN ISO 5436-1, the following model is assumed. On the aligned profile, the difference of the mean values of the "top profile sections" and the "bottom profile section" are formed (Annex A Figure 2). If the reference line sections are aligned with the alignment deviation *A*, the profile will be distorted and *D* will be changed. The model: For the mean of various measurements, the expected value of A is zero.

Figure 8: Uncertainty of D by alignment deviation

$D = \frac{1}{n_h} \cdot \sum_{i=1}^{n_h} z_{ghi} - \frac{1}{n_l} \cdot \sum_{i=1}^{n_l} z_{gli} + A$. For the uncertainty, the sum rule is valid

$$u^2(D) = \sum_{i=1}^{n_h} \left(\frac{\partial D}{\partial z_{ghi}}\right)^2 \cdot u^2(z_{ghi}) + \sum_{i=1}^{n_l} \left(\frac{\partial D}{\partial z_{gli}}\right)^2 \cdot u^2(z_{gli}) + \left(\frac{\partial D}{\partial A}\right)^2 \cdot u^2(A).$$

$\frac{\partial D}{\partial z_{ghi}} = \frac{1}{n_h}$ for i = 1 to n_h, $\frac{\partial D}{\partial z_{gli}} = \frac{1}{n_l}$ for i=1 to n_l , $u(z_{ghi}), u(z_{gli}) = u(z_g)$.

For the gvien small angles and their changes, the cosine factor $\frac{\partial D}{\partial A}$ is equal to 1 with sufficient accuracy.

In contrast to the analysis of the uncertainty of *Pt*, the following observation is valid as estimated value for *u*(*A*): Due to the definition of D (sampling in the centre and on the edge of the profile section), the affect of the alignment deviation only amounts to half the contribution of Pt_r.

$$u^2(D) = (\frac{1}{n_h} + \frac{1}{n_l}) \cdot u^2(z_g) + \frac{1}{12} \cdot \left(\frac{Pt_r}{2}\right)^2 .$$

Averaging over the profile points only acts on the random deviations in z_g so that the standard uncertainty

$$u^2(D) = \frac{U_n^2}{4} + \frac{a_y^2 \cdot G^2}{3} + \frac{s_w^2(D_m)}{m_w} + \frac{s_t^2(Pt_m)}{m_t} + \frac{Wt_0^2}{12} + \frac{1}{12} \cdot (\frac{Pt_r}{2})^2 + (\frac{1}{n_h} + \frac{1}{n_l}) \cdot [\frac{\overline{Rz_0}^2}{12}]$$

u(*D*) = 8.5 nm.

With the coverage factor k = 2, an expanded uncertainty of measurement of *U*(*D*) = 17 nm is obtained.

8. Evaluation with filtering

For the evaluation with *λs*, the uncertainty of the points of the primary profile is varied by the smoothing factor of the filter function f_s as a function of the the short-wave low-pass filter *λs* and the spacing of the measuring points *Δx* (Annex A, chapter 4).

λs /µm	Δx /µm	f_s
2.5	0.5	0.55
8	1.5	0.53
8	0.5	0.31

Table 1: Filter factors for different low-pass filter wavelengths.

The filter factor f_s only affects the currently measured random quantities so that the following is valid for the uncertainty of the points of the primary profile:

$$u^2(z_s) = \frac{U_n^2}{4} + \frac{a_y^2 \cdot G^2}{3} + \frac{Wt_0^2}{12} + f_s^2 \cdot [\frac{s_t^2(Pt_m)}{m_t} \frac{s_w^2(Pt_m)}{m_w} + \frac{\overline{Rz_0}^2}{12}]$$

If the same values as in chapters 1 to 4 are used for the input quantities, the following is obtained for filtering with λs = 8 µm and Δx = 0.5 µm:

$u(z_s)$ = 8.5 nm.

In the practical calculation, the nature of the input quantities is to be taken into account: If the estimated values for the input quantities (Pt_m, Pt_r, Wt_0, Rz_0) stem from profile data already filtered, they include already the effect of the short-wave low-pass filter. If the filtered values are taken for the calculation of $u(z_s)$ in accordance with the above equation, the filter factor f_s thus must not be applied again. The calculation scheme then becomes the same as in chapter 6, only with the values of the filtered input quantities.

8.1 Uncertainty of the total height of profile *Pt*

From the uncertainty of the profile points and the alignment deviation

$$u^2(Pt) = 2 \cdot u^2(z_s) + \frac{1}{12} \cdot (Pt_r)^2 =$$

$$2 \cdot \{ \frac{1}{4} \cdot U_n^2 + \frac{a_y^2 \cdot G^2}{3} + \frac{1}{12} \cdot Wt_0^2 + f_s^2 \cdot (\frac{s_t^2(Pt_m)}{m_t} + \frac{s_w^2(Pt_m)}{m_w} + \frac{1}{12} \cdot \overline{Rz_0}^2)\} + \frac{1}{12} \cdot Pt_r^2$$

and $u(Pt)$ = 11.7 nm.

8.2 Uncertainty of the groove depth *D*

As in chapter 7.2, the algorithm of *D* averages section by section over the filtered profile points.

$$u^2(D) = (\frac{1}{n_h} + \frac{1}{n_l}) \cdot u^2(z_s) + \frac{1}{12} \cdot (\frac{Pt_r}{2})^2$$

The averaging only affects the random deviations in z_s so that

$$u^2(D) = \frac{U_n^2}{4} + \frac{a_y^2 \cdot G^2}{3} + \frac{Wt_0^2}{12} + \frac{1}{12} \cdot (\frac{Pt_r}{2})^2 + f_s^2 \cdot [\frac{s_t^2(Pt_m)}{m_t} + \frac{s_w^2(D_m)}{m_w} + (\frac{1}{n_h} + \frac{1}{n_l}) \cdot (\frac{\overline{Rz_0}^2}{12})]$$

and $u(D)$ = 8.1 nm

9. Summary of the measurement uncertainty of total height of profile *Pt* and groove depth *D*

For the exemplary values, a depth-setting standard with the nominal value *Pt* = 3 µm was assumed and for the input quantities, typical values were used.

Total height of profile *Pt*, evaluation without filtering

Capter	Input quantity catchword	Calculation of input quantity	Exemplary value	Sensi-tivity coeff.	Method of determination, distribution	Variance /nm²
1.1	Reference standard	$\frac{U_n^2}{4}$	U_n=10 nm	1	B Gaussian	25
1.2	Difference measuring point – calibration point	$\frac{a_y^2 \cdot G^2}{3}$	a_y=100 µm G=20nm/mm	1	B Rectangular	1.33
1.3	Repeatability	$\frac{s_w^2(Pt_m)}{m_w}$	$s_w(Pt_m) = 5$ nm $m_w = 5$	1	A Gaussian	5
2	Topography over groove length	$\frac{s_t^2(Pt_m)}{m_t}$	$s_t(Pt_m) = 5$ nm m_t=5	1	A Gaussian	5
3	Guidance deviation	$\frac{Wt_0^2}{12}$	$Wt_0 = 20$ nm	1	B Rectangular	33
4	Background noise	$\frac{\overline{Rz_0}^2}{12}$	$\overline{Rz_0} = 20$ nm	1	A Rectangular	33
6	Variance of the profile points	Sum of this column			$u^2(z_g)$	102.33
						Uncer-tainty /nm
6	Uncertainty of the profile points				$u(z_g)$	10.1
7.1	Uncertainty of total height of profile	$[2 \cdot u^2(z_g) + \frac{1}{12} \cdot Pt_r^2]^{1/2}$	Pt_r = 10 nm,		$u(Pt)$	14.6
	Expanded uncertainty of total height of profile *Pt*	$k \cdot u(Pt)$			$U(Pt)$	29.2

Groove depth *D*, evaluation without filtering

Capter	Input quantity catchword	Calculation of input quantity	Exemplary value	Sensi-tivity coeff.	Method of determination, distribution	Variance /nm²
1.1	Reference standard	$\frac{U_n^2}{4}$	U_n=10 nm	1	B Gaussian	25
1.2	Difference measuring point – calibration point	$\frac{a_y^2 \cdot G^2}{3}$	a_y=100 µm G=20nm/mm	1	B Rectangular	1.33
1.3	Repeatability	$\frac{s_w^2(D_m)}{m_w}$	$s_w(D_m) = 5$ nm $m_w = 5$	1	A Gaussian	5
2	Topography over groove length	$\frac{s_t^2(Pt_m)}{m_t}$	$s_t(Pt_m) = 5$ nm m_t=5	1	A Gaussian	5
3	Guidance deviation	$\frac{Wt_0^2}{12}$	$Wt_0 = 20$ nm	1	B Rectangular	33
4	Background noise	$(\frac{1}{n_h} + \frac{1}{n_l}) \cdot \frac{\overline{Rz_0}^2}{12}$	$\overline{Rz_0} = 20$ nm n_h, n_l = 100	1	A Rectangular	0.7
7.2	Profile alignment	$\frac{1}{12} \cdot (\frac{Pt_r}{2})^2$	Pt_r = 10 nm	1	A Rectangular	2.1
7.2	Variance of the groove depth	Sum of this column			$u^2(D)$	72.13
						Uncer-tainty /nm
7.2	Uncertainty of the groove depth				$u(D)$	8.5
	Expanded uncertainty of groove depth *D*	$k \cdot u(D)$			$U(D)$	17

Total height of profile *Pt*, evaluation with filtering λs = 8 µm

Chapter	Input quantity catchword	Calculation of input quantity	Exemplary value	Sensi-tivity coeff.	Method of determination, distribution	Variance /nm²
1.1	Reference standard	$\frac{U_n^2}{4}$	U_n=10 nm	1	B Gaussian	25
1.2	Difference measuring point – calibration point	$\frac{a_y^2 \cdot G^2}{3}$	a_y=100 µm G=20nm/mm	1	B Rectangular	1.33
1.3	Repeatability	$f_s^2 \cdot \frac{s_w^2(Pt_m)}{\mathrm{m_w}}$	$s_w(Pt_m) = 5$ nm m_w = 5	1	A Gaussian	0.5
2	Topography over groove length	$f_s^2 \cdot \frac{s_t^2(Pt_m)}{\mathrm{m_t}}$	$s_t(Pt_m) = 5$ nm m_t=5	1	A Gaussian	5
3	Guidance deviation	$\frac{Wt_0^2}{12}$	$Wt_0 = 20$ nm	1	B Rectangular	33
4	Background noise	$f_s^2 \cdot \frac{\overline{Rz_0}^2}{12}$	$\overline{Rz_0} = 20$ nm	1	A Rectangular	3.2
8	Variance of the profile points	Sum of this column			$u^2(z_s)$	68.03
						Uncer-tainty /nm
8	Uncertainty of the profile points				$u(z_s)$	8.2
8.1	Uncertainty of total height of profile *Pt*	$[2 \cdot u^2(z_g) + \frac{1}{12} \cdot Pt_r^2]^{1/2}$	Pt_r = 10 nm,		$u(Pt)$	11.7
	Expanded uncertainty of total height of profile *Pt*	$k \cdot u(Pt)$			$U(Pt)$	23.4

Groove depth D, evaluation with filtering λs = 8 µm

Chapter	Input quantity catchword	Calculation of input quantity	Exemplary value	Sensi-tivity coeff.	Method of determination, distribution	Variance /nm²
1.1	Reference standard	$\frac{U_n^2}{4}$	U_n=10 nm	1	B Gaussian	25
1.2	Difference measuring point – calibration point	$\frac{a_y^2 \cdot G^2}{3}$	a_y=100 µm G=20nm/mm	1	B Rectangular	1.33
1.3	Repeatability	$f_s^2 \cdot \frac{s_w^2(D_m)}{\mathrm{m_w}}$	$s_w(D_m) = 5\,\mathrm{nm}$ m_w = 5	1	A Gaussian	0.5
2	Topograph over groove length	$f_s^2 \cdot \frac{s_t^2(Pt_m)}{\mathrm{m_t}}$	$s_t(Pt_m) = 5\,\mathrm{nm}$ m_t=5	1	A Gaussian	5
3	Guidance deviation	$\frac{Wt_0^2}{12}$	$Wt_0 = 20\,\mathrm{nm}$	1	B Rectangular	33
4	Background noise	$f_s^2 \cdot (\frac{1}{n_h} + \frac{1}{n_l}) \cdot \frac{\overline{Rz_0}^2}{12}$	$\overline{Rz_0} = 20\,\mathrm{nm}$ n_h, n_l = 100	1	A Rectangular	0.06
8.2	Profile alignment	$\frac{1}{12} \cdot (\frac{Pt_r}{2})^2$	Pt_r = 10 nm	1	A Rectangular	0.2
8.2	Variance of the groove depth	Sum of this column			$u^2(D)$	65.09
						Uncer-tainty /nm
8.2	Uncertainty of the groove depth				$u(D)$	8.1
	Expanded Uncertainty of groove depth D	$k \cdot u(D)$			$U(D)$	16.2