Annex B: Uncertainty of measurement of the total height of profile of a depth-setting standard

In this example, the uncertainty sources derived in general in chapter 2 of Annex A are transferred to the conditions of depth-setting standards. In the example a glass standard with a nominal value of 3 µm is assumed.

1 Calibration factor

1.1 Reference standard

According to the calibration certificate, the uncertainty of the reference standard (calibrated depth-setting standard) is, for example, U_n = 10 nm, at k = 2,

$$u^2(Pt_n) = \frac{U_n^2}{4} = 25\,\text{nm}^2$$

1.2 Deviation measuring trace – calibration trace

$u^2(Pt_{my}) = \frac{a_y^2 \cdot G^2}{3}$. With a_y = 100 µm and G = 20 nm/mm (Annex A Figure 3),

$$u^2(Pt_{my}) = \frac{(100\,\mu\text{m})^2 \cdot (20\,\text{nm/mm})^2}{3} = 1{,}33\,\text{nm}^2$$

1.3 Repeatability

As estimated value for the repeatability in the tracing of a reference standard, the empirical variance from m_w repeat measurements is taken:

$u^2(b) = \frac{s_w^2(Pt_m)}{m_w} = s_w^2(\overline{Pt_m})$. With a typical value for the empirical variance $s_w(\overline{Pt_m}) = 2nm$,

$u^2(b) = 4\,\text{nm}^2$.

2 Topography of standard

2.1 Roughness of reference plane

The reference and measuring areas of the test object have a residual roughness. As estimated value for the uncertainty of the profile points $s(\overline{Pt_r}) = \frac{s(Pt_r)}{\sqrt{m_t}}$, the standard deviation of the mean value of Pt_r is taken. In the example, the mean value of Pt_r for five measurements has an empirical standard deviation of 5 nm.

$$u^2(z_e) = \frac{s_t^2(Pt_r)}{m_t} = 5\,\text{nm}^2$$

2.2 Localisation of traces

The depth of the groove is generally not constant over the whole length. Caused by an uncertain position of the trace the measured height is uncertain. The influencing variables and the sensitivity coefficient are the same as they are for the calibration procedure on the reference standard used for the determination of the calibration factor. (Cf. Annex B, chapter1.2).

$$u_{top}^2(z_e) = \frac{a_y^2 \cdot G^2}{3}$$

In this case the gradient G of the test object is considered. In the example the same values as in chapter 1.2 are applied.

$$u_{top}^2(z_e) = \frac{a_y^2 \cdot G^2}{3} = \frac{(100\,\mu\text{m})^2 \cdot (20\,\text{nm/mm})^2}{3} = 1{,}33\,\text{nm}^2 .$$

Note:
This term has to be applied only for comparisons at the same level of hierarchy. Each partner has to consider with this term the uncertain position of his measurement.
On the contrary, during the dissemination of the groove depth value at the higher level during the calibration of the depth setting standard the trace position is assumed to be free of error. The uncertainty by positioning deviations is considered at the lower level during the calibration step, by which the calibration factor is determined (Cf. Annex B, chapter1.2).

3 Straightness deviation of reference

For the straightness deviations of reference profile used for the device, $Wt_0 \leq 20$ nm is assumed. The variance contribution is, rectangular distribution assumed

$$u^2(z_{ref}) = \frac{Wt_0^2}{12} = \frac{(20\,\text{nm})^2}{12} = 33\,\text{nm}^2 .$$

4 Background noise

The background noise of an optical flat be Rz_0 = 20 nm, rectangular distribution assumed.

$$u^2(z_{pl}) = \frac{\left(\overline{Rz_0}\right)^2}{12} = 33\,\text{nm}^2 .$$

5 Plastic deformation

For a glass standard the plastic deformation is negligibly small.

6 Uncertainty of points of total profile

From the sum of these uncertainties the variance of the points of the total profile is obtained as follows:

$$u^2(z_g) = \frac{1}{4} \cdot U_n^2 + \frac{a_y^2 \cdot G^2}{3} + \frac{s_w^2(Pt_m)}{m_w} + \frac{s_t^2(Pt_r)}{m_t} + \frac{a_y^2 \cdot G^2}{3} + \frac{1}{12} \cdot Wt_0^2 + \frac{1}{12} \cdot \overline{Rz_0}^2$$

The first three contain the uncertainty of the dissemination process and the last three the uncertainty contribution from the measuring process on the test object. The terms of this equation are listed in the table in chapter 9. With the exemplary numerical values from chapters 1 to 4 the uncertainty for each point of profile is obtained:

$u\ (z_g)$ = 11 nm.

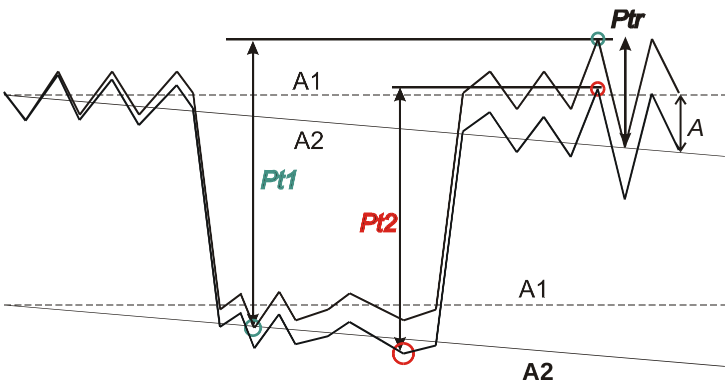
7 Parameter function

7.1 Uncertainty of measurement of total height of profile *Pt*

In the aligned profile, the total height of profile *Pt* is the difference between the highest z-value on the reference plane (z_h) and the lowest z-value in the tread of the groove (z_l). As a result of roughness and flatness deviations, an alignment deviation *A* occurs.

When the alignment is made, the profile points are lifted differently so that the difference between highest and lowest point varies by A at most in dependence on where these lie. Taking the average of several measurements, their expectation value is zero. The model for *Pt* is valid:

$Pt = z_h - z_l + A.$



A1: reference lines @ leveling 1
A2: reference lines @ leveling 2
A: leveling deviation
Pt1: *Pt* @ leveling 1
Pt2: *Pt* @ leveling 2
Ptr : roughness on reference plane

Figure 7: Uncertainty of *Pt* due to alignment deviation

The following is valid for the uncertainty of the profile height *Pt*:

$$u^2(Pt) = (\frac{\partial Pt}{\partial z_h})^2 \cdot u^2(z_h) + (\frac{\partial Pt}{\partial z_l})^2 \cdot u^2(z_l) + (\frac{\partial Pt}{\partial A})^2 \cdot u^2(A).$$

The two first sensitivity coefficients are 1. The third describes in principle the cosinue factor which changes with the alignment. Due to the smaller angles and their changes, it is equal to one with sufficient accuracy. The uncertainty of the z-values $u(z_h)$ and $u(z_l)$ is equal to that of the points of the overall profile $u(z_g)$.. The peak value Pt_r of the roughness or the flatness deviation of the profile components in the reference line sections included in the evaluation serves as an estimated value for the variability of the alignment deviation *u(A)*. Within Pt_r , a rectangular distribution is assumed.

$$u^2(Pt) = 2 \cdot u^2(z_g) + \frac{1}{12} \cdot (Pt_r)^2 .$$

$$u^2(Pt) = 2 \cdot \{ \frac{1}{4} \cdot U_n^2 + \frac{a_y^2 \cdot G^2}{3} + \frac{s_w^2(Pt_m)}{m_w} + \frac{s_t^2(Pt_r)}{m_t} + \frac{a_y^2 \cdot G^2}{3} + \frac{1}{12} \cdot Wt_0^2 + \frac{1}{12} \cdot \overline{Rz_0}^2 \} + \frac{1}{12} \cdot Pt_r^2$$

On depth-setting standards, Pt_r= 10 nm is a usual value. The variance of *Pt* thus is determined as

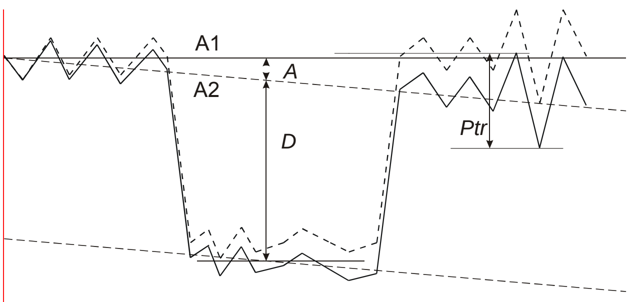
$$u^2(Pt) = 2 \cdot (10{,}1\text{ nm})^2 + \frac{1}{12} \cdot (10\text{ nm})^2 ,$$

and the standard uncertainty of *Pt* is $u(Pt) = 14{,}6\text{ nm}$

7.2 Uncertainty of measurement of groove depth *D*

As an approximation for the calculation of the groove depth *D* according to DIN EN ISO 5436-1 the following model is assumed. On the aligned profile the difference of the mean values of the "top profile sections z_h " and the "bottom profile section z_l " are formed (Annex A Figure 2). If the reference line section are aligned with the alignment deviation *A*, the profile will be distorted and *D* will be changed. For the mean of various measurements, the expected value of A is zero. The model for D:

$$D = \overline{z_h} - \overline{z_l} + A$$



A1: reference line with levelling error
A2: assumed correct reference line
A: levelling error
Ptr : roughness on reference plane
D: depth of groove

Figure 8: Uncertainty of D due to alignment deviation

Kommentar [KS1]: Bild mit englischer caption

$D = \frac{1}{n_h} \cdot \sum_{i=1}^{n_h} z_{ghi} - \frac{1}{n_l} \cdot \sum_{i=1}^{n_l} z_{gli} + A$. For the uncertainty, the sum rule is valid

$$u^2(D) = \sum_{i=1}^{n_h} (\frac{\partial D}{\partial z_{ghi}})^2 \cdot u^2(z_{ghi}) + \sum_{i=1}^{n_l} (\frac{\partial D}{\partial z_{gli}})^2 \cdot u^2(z_{gli}) + (\frac{\partial D}{\partial A})^2 \cdot u^2(A).$$

$\frac{\partial D}{\partial z_{ghi}} = \frac{1}{n_h}$ für i = 1 bis n_h, $\frac{\partial D}{\partial z_{gli}} = \frac{1}{n_l}$ für i=1 bis n_l, $u(z_{ghi}), u(z_{gli}) = u(z_g)$.

For the gvien small angles and their changes, the cosine factor $\frac{\partial D}{\partial A}$ is equal to 1 with sufficient accuracy.

In contrast to the analysis of the uncertainty of *Pt*, the following observation is valid as estimated value for *u*(*A*): Due to the definition of D (sampling in the centre and on the edge of the profile section), the affect of the alignment deviation only amounts to half the contribution of Pt_r.

$$u^2(D) = (\frac{1}{n_h} + \frac{1}{n_l}) \cdot u^2(z_g) + \frac{1}{12} \cdot \left(\frac{Pt_r}{2} \right)^2.$$

Averaging over the profile points only acts on the random deviations in z_g so that the standard uncertainty

$$u^2(D) = \left(\frac{1}{n_h} + \frac{1}{n_l} \right) \cdot \frac{1}{12} \cdot \overline{Rz_0}^2 + \frac{1}{4} \cdot U_n^2 + \frac{a_y^2 \cdot G^2}{3} + \frac{s_w^2(D_m)}{m_w} + \frac{s_t^2(Pt_r)}{m_t} + \frac{a_y^2 \cdot G^2}{3} + \frac{1}{12} \cdot Wt_0^2 + \frac{1}{12} \cdot \left(\frac{Pt_r}{2} \right)^2$$

u(*D*) = 8.6 nm.

With the coverage factor k = 2, an expanded uncertainty of measurement of *U*(*D*) = 17 nm is obtained.

8 Evaluation with filtering

For the evaluation with *λs* the uncertainty of the points of the primary profile is varied by the smoothing factor of the filter function f_s in dependence on the short-wave low-pass filter *λs* and the sampling distance *Δx* (Annex A, chapter 4).

λs / µm	Δx /µm	f_s
2,5	0,5	0,55
8	1,5	0,53
8	0,5	0,31

Table 1: Filter factors for different short wavelength cutoffs

The filter factor has an effect only on the currently measured quantities so that the following is valid for the uncertainty of the points of the primary profile:

$$u^2(z_s) = \frac{U_n^2}{4} + \frac{a_y^2 \cdot G^2}{3} + \frac{a_y^2 \cdot G^2}{3} + \frac{Wt_0^2}{12} + f_s^2 \cdot [\frac{s_t^2(Pt_r)}{m_t} + \frac{s_w^2(Pt_m)}{m_w} + \frac{\overline{Rz_0}^2}{12}]$$

If the same values for the input quantities as in chapters 1 to 4 are used, the following is obtained for filtering with λs = 8 µm and Δx = 0,5 µm:

$u(z_s)$ = 8,1 nm.

In the practical calculation the nature of the input quantities is to be taken into account: If the estimated values for the input quantities (Pt_m, Pt_r, Wt_0, Rz_0) stem from profile data already filtered, they include already the effect of the short-wave low-pass filter. If the filtered values are taken for the calculation of $u(z_s)$ according to the above equation, the filter factor f_s thus must not be applied once again. The calculation scheme then becomes the same as in chapter 6, only with the values of the filtered input quantities.

8.1 Uncertainty of total height of profile *Pt*

If in this filtering the roughness in the area of the reference line sections is measured with Pt_r = 6 nm and $u(Pt_n)$ = 8 nm,

$$u^2(Pt) = 2 \cdot u^2(z_s) + \frac{1}{12} \cdot (Pt_r)^2$$

$$u^2(Pt) = 2 \cdot \{ \frac{1}{4} \cdot U_n^2 + \frac{a_y^2 \cdot G^2}{3} + \frac{1}{12} \cdot Wt_0^2 + \frac{a_y^2 \cdot G^2}{3} + f_s^2 \cdot (\frac{s_t^2(Pt_r)}{m_t} + \frac{s_w^2(Pt_m)}{m_w} + \frac{1}{12} \cdot \overline{Rz_0}^2)\} + \frac{1}{12} \cdot Pt_r^2$$

$u(Pt)$ = 11,4 nm

8.2 Uncertainty of groove depth *D*

As in section 7.2, the algorithm of *D* averages section by section over the filtered profile points so that

$$u^2(D) = (\frac{1}{n_h} + \frac{1}{n_l}) \cdot u^2(z_s) + \frac{1}{12} \cdot (\frac{Pt_r}{2})^2$$

The averaging only affects the random deviations in z_s so that

$$u^2(D) = \left(\frac{1}{n_h} + \frac{1}{n_l}\right) \cdot f_s^2 \cdot \frac{1}{12} \cdot \overline{Rz_0}^2 + \frac{1}{4} \cdot U_n^2 + \frac{a_y^2 \cdot G^2}{3} + \frac{1}{12} \cdot Wt_0^2 + \frac{a_y^2 \cdot G^2}{3} + f_s^2 \cdot (\frac{s_t^2(Pt_r)}{m_t} + \frac{s_w^2(D_m)}{m_w} +)$$

$$+ \frac{1}{12} \cdot \left(\frac{Pt_r}{2}\right)^2$$

With Pt_r = 6nm and $u(D_n)$= 5 nm is

$u^2(D) = \frac{1}{50} \cdot (6{,}3\ \text{nm})^2 + (5\ \text{nm})^2 + \frac{1}{3} \cdot \left(\frac{6\ \text{nm}}{2}\right)^2$ and $u(D)$ = 5,3 nm.

9 Summary of uncertainty of measurement of total height of profile *Pt* and groove depth *D*

The exemplary values are calculated for a depth-setting standard with the nominal value *Pt* = 3 μm, evaluation without filtering

Chapter	Input quantity catchword	Calculation of input quantity	Exemplary value	Sensi-tivity coeff.	Method of determination, distribution	Variance / nm²
1.1	Reference standard	$\frac{U_n^2}{4}$	U_n=10 nm	1	B Gaussian	25
1.2	Difference measuring point – calibration point	$\frac{a_y^2 \cdot G^2}{3}$	a_y=100 μm G=20nm/mm	1	B uniform	1,33
1.3	Repeatability	$\frac{s_w^2(Pt_m)}{m_w}$	$s(\overline{Pt_m}) = 2$ nm	1	A Gaussian	4
2	Topography	$\frac{s_t^2(Pt_r)}{m_t}$	$s(\overline{Pt_r}) = 5$ nm	1	A Gaussian	25
3	Straightness deviations	$\frac{Wt_0^2}{12}$	$Wt_0 = 20$ nm	1	B uniform	33
4	Background noise	$\frac{\overline{Rz_0}^2}{12}$	$\overline{Rz_0} = 20$ nm	1	A uniform	33
6	$u^2(z_g)$	Σ				121,33
						Uncer-tainty /nm
6	$u(z_g)$					11,1
7.1	Uncertainty of total height of profile $u(Pt)$	$[2 \cdot u^2(z_g) + \frac{1}{12} \cdot Pt_r^2]^{1/2}$	Pt_r = 10 nm, $u(Pt_n)$= 8 nm			14
7.2	Uncertainty of groove depth $u(D)$	$[(\frac{1}{n_o} + \frac{1}{n_u}) \cdot u^2(z_g) + \frac{1}{12} \cdot (\frac{Pt_r}{2})^2]^{1/2}$	Pt_r = 10 nm $u(D_n)$= 5 nm			6

Evaluation with filtering λs = 8 μm, sampling distance Δx = 0,5 μm:

Chapter	Input quantity catchword	Calculation of input quantity	Exemplary value	Sensi-tivity coeff.	Method of determination, distribution	Variance / nm²
8	Reference standard	$\frac{U_n^2}{4}$	U_n=10 nm	1	B Gaussian	25
8	Difference measuring point – calibration point	$\frac{a_y^2 \cdot G^2}{3}$	a_y=100 µm G=20 nm/mm	1	B uniform	1,33
8	Repeatability	$\frac{s_w^2(Pt_m)}{m_w} \cdot f_s^2$	$s(\overline{Pt_m}) = 2$ nm	1	A Gaussian	4
8	Topography	$\frac{s_t^2(Pt_r)}{m_t} \cdot f_s^2$	$s(\overline{Pt_r}) = 5$ nm	1	A Gaussian	2,4
8	Straightness deviation	$\frac{Wt_0^2}{12} \cdot f_s^2$	$Wt_0 = 20$ nm	1	B uniform	3,2
8	Background noise	$\frac{1}{12} \cdot \overline{Rz_0}^2 \cdot f_s^2$	$\overline{Rz_0} = 20$ nm	1	A uniform	3,2
8	$u^2(z_s)$	Σ				39,13
						Uncer-tainty / nm
	$u(z_s)$					6,3
8.1	Uncertainty of total height of profile $u(Pt)$	$[2 \cdot u^2(z_s) + \frac{1}{12} \cdot Pt_r^2]^{1/2}$	$Pt_r = 6$ nm $u(Pt_n)$= 8 nm			10,3
8.2	Uncertainty of groove depth $u(D)$	$[(\frac{1}{n_o} + \frac{1}{n_u}) \cdot u^2(z_s) + \frac{1}{12} \cdot \left(\frac{Pt_r}{2}\right)^2]^{1/2}$	$Pt_r = 6$ nm $u(D_n)$= 5 nm			5,3