Annex B: Uncertainty of measurement of the total height of profile of a depth-setting standard

In this example, the uncertainty sources derived in general in chapter 2 of Annex A are transferred to the conditions of depth-setting standards. In the example a glass standard with a nominal value of 3 µm is assumed.

1 Calibration factor
1.1 Reference standard
According to the calibration certificate, the uncertainty of the reference standard (calibrated depth-setting standard) is, for example, \( U_n = 10 \text{ nm, at } k = 2 \),

\[
u^2(P_{n}) = \frac{U_n^2}{4} = 25 \text{ nm}^2\]

1.2 Deviation measuring trace – calibration trace

\[
u^2(P_{my}) = \left( \frac{a_r \cdot G}{3} \right)^2 \text{. With } a_r = 100 \text{ µm and } G = 20 \text{ nm/mm (Annex A Figure 3)},
u^2(P_{my}) = \left( \frac{(100 \text{ µm})^2 \cdot (20 \text{ nm/mm})^2}{3} \right) = 1.33 \text{ nm}^2\]

1.3 Repeatability

As estimated value for the repeatability in the tracing of a reference standard, the empirical variance from \( m_r \) repeat measurements is taken:

\[
u^2(b) = \frac{s^2(P_{m})}{m_r} = s^2(P_{n}) \text{. With a typical value for the empirical variance } s(P_{n}) = 2 \text{ nm},
u^2(b) = 4 \text{ nm}^2\]

2 Topography of standard
2.1 Roughness of reference plane

The reference and measuring areas of the test object have a residual roughness. As estimated value for the uncertainty of the profile points \( s(P_{r}) = \frac{s(P_{n})}{\sqrt{m_r}} \), the standard deviation of the mean value of \( P_{r} \) is taken. In the example, the mean value of \( P_{r} \) for five measurements has an empirical standard deviation of 5 nm.

\[
u^2(z_r) = \frac{s^2(P_{r})}{m_r} = 5 \text{ nm}^2\]
2.2 Localisation of traces

The depth of the groove is generally not constant over the whole length. Caused by an uncertain position of the trace the measured height is uncertain. The influencing variables and the sensitivity coefficient are the same as they are for the calibration procedure on the reference standard used for the determination of the calibration factor. (Cf. Annex B, chapter 1.2).

\[ u_{\text{local}}(z_g) = \frac{a_g^2 \cdot G^2}{3} \]

In this case the gradient \( G \) of the test object is considered. In the example the same values as in chapter 1.2 are applied.

\[ u_{\text{local}}(z_g) = \frac{a_g^2 \cdot G^2}{3} = \frac{(100 \mu\text{m})^2 \cdot (20 \text{ nm/mm})^2}{3} = 1.33 \text{ nm}^2. \]

Note:

This term has to be applied only for comparisons at the same level of hierarchy. Each partner has to consider with this term the uncertain position of his measurement.

On the contrary, during the dissemination of the groove depth value at the higher level during the calibration of the depth setting standard the trace position is assumed to be free of error. The uncertainty by positioning deviations is considered at the lower level during the calibration step, by which the calibration factor is determined (Cf. Annex B, chapter 1.2).

3 Straightness deviation of reference

For the straightness deviations of reference profile used for the device, \( Wt_0 \leq 20 \text{ nm} \) is assumed. The variance contribution is, rectangular distribution assumed

\[ u^2(z_{\text{ref}}) = \frac{Wt_0^2}{12} = \frac{(20 \text{ nm})^2}{12} = 33 \text{ nm}^2. \]

4 Background noise

The background noise of an optical flat be \( Rz_0 = 20 \text{ nm} \), rectangular distribution assumed.

\[ u^2(z_{\text{nu}}) = \frac{(Rz_0)^2}{12} = 33 \text{ nm}^2. \]

5 Plastic deformation

For a glass standard the plastic deformation is negligibly small.

6 Uncertainty of points of total profile

From the sum of these uncertainties the variance of the points of the total profile is obtained as follows:

\[ u^2(z_g) = \frac{1}{4} u^2 + \frac{a_g^2 \cdot G^2}{3} + \frac{s_l^2 (P_t)}{m_t} + \frac{s_l^2 (P_t)}{m_t} + \frac{a_g^2 \cdot G^2}{3} + \frac{1}{12} Wt + \frac{1}{12} Rz_0^2 \]
The first three contain the uncertainty of the dissemination process and the last three the uncertainty contribution from the measuring process on the test object. The terms of this equation are listed in the table in chapter 9. With the exemplary numerical values from chapters 1 to 4 the uncertainty for each point of profile is obtained:

\[ u(z_0) = 11 \text{ nm}. \]

7 Parameter function

7.1 Uncertainty of measurement of total height of profile \( P_T \)

In the aligned profile, the total height of profile \( P_T \) is the difference between the highest z-value on the reference plane \( (z_h) \) and the lowest z-value in the tread of the groove \( (z_l) \). As a result of roughness and flatness deviations, an alignment deviation \( A \) occurs.

When the alignment is made, the profile points are lifted differently so that the difference between highest and lowest point varies by \( A \) at most in dependence on where these lie. Taking the average of several measurements, their expectation value is zero. The model for \( P_T \) is valid:

\[ P_T = z_h - z_l + A. \]

Figure 7: Uncertainty of \( P_T \) due to alignment deviation

The following is valid for the uncertainty of the profile height \( P_T \):

\[ u^2(P_T) = \left( \frac{\partial P_T}{\partial z_h} \right)^2 \cdot u^2(z_h) + \left( \frac{\partial P_T}{\partial z_l} \right)^2 \cdot u^2(z_l) + \left( \frac{\partial P_T}{\partial A} \right)^2 \cdot u^2(A). \]
The two first sensitivity coefficients are 1. The third describes in principle the cosine factor which changes with the alignment. Due to the smaller angles and their changes, it is equal to one with sufficient accuracy. The uncertainty of the z-values $u(z_h)$ and $u(z_l)$ is equal to that of the points of the overall profile $u(z_0)$. The peak value $P_t$ of the roughness or the flatness deviation of the profile components in the reference line sections included in the evaluation serves as an estimated value for the variability of the alignment deviation $u(A)$. Within $P_t$, a rectangular distribution is assumed.

$$u^2(P_t) = 2 \cdot u^2(z_h) + \frac{1}{12} (P_t)^2.$$  

$$u^2(P_t) = 2 \cdot \left( \frac{1}{4} U_s^2 + \frac{a_1^2 \cdot G^2}{m} + \frac{s_1^2 (P_{t_{ru}})}{m} + \frac{a_1^2 \cdot G^2}{3} + \frac{1}{12} W_{G_s}^2 + \frac{1}{12} R_{z_s}^2 \right) + \frac{1}{12} P_t^2$$

On depth-setting standards, $P_t = 10 \text{ nm}$ is a usual value. The variance of $P_t$ thus is determined as

$$u^2(P_t) = 2 \cdot (10,1 \text{ nm})^2 + \frac{1}{12} (10 \text{ nm})^2,$$

and the standard uncertainty of $P_t$ is $u(P_t) = 14,6 \text{ nm}$

### 7.2 Uncertainty of measurement of groove depth $D$

As an approximation for the calculation of the groove depth $D$ according to DIN EN ISO 5436-1 the following model is assumed. On the aligned profile the difference of the mean values of the "top profile sections $z_h$" and the "bottom profile section $z_l$" are formed (Annex A Figure 2). If the reference line section are aligned with the alignment deviation $A$, the profile will be distorted and $D$ will be changed. For the mean of various measurements, the expected value of $A$ is zero. The model for $D$:

$$D = z_h - z_l + A$$

![Diagram](image.png)

**Figure 8:** Uncertainty of $D$ due to alignment deviation

Kommentar [KS1]: Bild mit englischer caption
For the uncertainty, the sum rule is valid
\[
\sum_{i=1}^{n} \left( \frac{\partial D}{\partial z_{yi}} \right)^2 \cdot u^2(z_{yi}) + \sum_{i=1}^{n} \left( \frac{\partial D}{\partial A} \right)^2 \cdot u^2(A).
\]

\[
\frac{\partial D}{\partial z_{yi}} = \frac{1}{n_i} \quad \text{für } i = 1 \text{ bis } n_i, \quad \frac{\partial D}{\partial A} = \frac{1}{n_i} \quad \text{für } i = 1 \text{ bis } n_i, \quad u(z_{yi}) = u(z_{yi}).
\]

For the given small angles and their changes, the cosine factor \( \frac{\partial D}{\partial A} \) is equal to 1 with sufficient accuracy.

In contrast to the analysis of the uncertainty of \( Pt \), the following observation is valid as estimated value for \( u(A) \): Due to the definition of \( D \) (sampling in the centre and on the edge of the profile section), the affect of the alignment deviation only amounts to half the contribution of \( Pt \).

\[
u^2(D) = \left( \frac{1}{n_b} + \frac{1}{n_t} \right) \cdot u^2(z_g) + \frac{1}{12} \left( \frac{Pt}{2} \right)^2.
\]

Averaging over the profile points only acts on the random deviations in \( z_g \) so that the standard uncertainty

\[
u^2(D) = \left( \frac{1}{n_b} + \frac{1}{n_t} \right) \cdot \frac{1}{12} \cdot \frac{Pt}{2} + \frac{1}{4} U^2 + \frac{a^2 \cdot G^2}{3} + \frac{s^2(D_m)}{m_e} + \frac{s^2(Pt)}{m_t} + \frac{a^2 \cdot G^2}{3} + \frac{1}{12} W_{zg}^2 + \frac{1}{12} \left( \frac{Pt}{2} \right)^2
\]

\( u(D) = 8.6 \, \text{nm} \).

With the coverage factor \( k = 2 \), an expanded uncertainty of measurement of \( U(D) = 17 \, \text{nm} \) is obtained.

8 Evaluation with filtering

For the evaluation with \( \lambda s \) the uncertainty of the points of the primary profile is varied by the smoothing factor of the filter function \( f_s \) in dependence on the short-wave low-pass filter \( \lambda s \) and the sampling distance \( \Delta x \) (Annex A, chapter 4).

<table>
<thead>
<tr>
<th>( \lambda_s ) / ( \mu \text{m} )</th>
<th>( \Delta x ) / ( \mu \text{m} )</th>
<th>( f_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
<td>0.53</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 1: Filter factors for different short wavelength cutoffs

The filter factor has an effect only on the currently measured quantities so that the following is valid for the uncertainty of the points of the primary profile:
If the same values for the input quantities as in chapters 1 to 4 are used, the following is obtained for filtering with $\lambda_s = 8 \mu m$ and $\Delta x = 0.5 \mu m$:

$$u(z_s) = 8.1 \text{ nm}.$$  

In the practical calculation the nature of the input quantities is to be taken into account: If the estimated values for the input quantities ($P_t m$, $P_t r$, $W t_0$, $R z_0$) stem from profile data already filtered, they include already the effect of the short-wave low-pass filter. If the filtered values are taken for the calculation of $u(z_s)$ according to the above equation, the filter factor $f_s$ thus must not be applied once again. The calculation scheme then becomes the same as in chapter 6, only with the values of the filtered input quantities.

### 8.1 Uncertainty of total height of profile $P t$

If in this filtering the roughness in the area of the reference line sections is measured with $P t_r = 6 \text{ nm}$ and $u(P t_n) = 8 \text{ nm}$, 

$$u^2(P t) = 2 \cdot u^2(z_s) + \frac{1}{12} (P t_r)^2$$

$$u^2(P t) = 2 \cdot \left\{ \frac{1}{4} U z_s^2 + \frac{a z_s^2 \cdot G z_s^2}{3} + \frac{1}{12} \cdot W t_s^2 + \frac{a z_s^2 \cdot G z_s^2}{3} + f_s^2 \cdot \left( \frac{s^2(P t_r)}{m_r} + \frac{s^2(P t_n)}{m_n} + \frac{1}{12} \overline{R z_0} \right) \right\} + \frac{1}{12} P t_r^2$$

$$u(P t) = 11.4 \text{ nm}$$

### 8.2 Uncertainty of groove depth $D$

As in section 7.2, the algorithm of $D$ averages section by section over the filtered profile points so that 

$$u^2(D) = \left( \frac{1}{n_h} + \frac{1}{n_l} \right) \cdot u^2(z_s) + \frac{1}{12} \left( \frac{P t_r}{2} \right)^2$$

The averaging only affects the random deviations in $z_s$ so that 

$$u^2(D) = \left( \frac{1}{n_h} + \frac{1}{n_l} \right) \cdot f_s^2 \cdot \frac{1}{12} \overline{R z_0}^2 + \frac{1}{4} U z_s^2 + \frac{a z_s^2 \cdot G z_s^2}{3} + \frac{1}{12} \cdot W t_s^2 + \frac{a z_s^2 \cdot G z_s^2}{3} + f_s^2 \cdot \left( \frac{s^2(P t_r)}{m_r} + \frac{s^2(D_n)}{m_n} \right)$$

$$+ \frac{1}{12} \left( \frac{P t_r}{2} \right)^2$$

With $P t_r = 6 \text{ nm}$ and $u(D_n) = 5 \text{ nm}$ is
\[ u^2(D) = \frac{1}{50} \cdot (6.3 \, \text{nm})^2 + (5 \, \text{nm})^2 + \frac{1}{3} \left( \frac{6 \, \text{nm}}{2} \right)^2 \] and \( u(D) = 5.3 \, \text{nm} \).
9 Summary of uncertainty of measurement of total height of profile \( Pt \) and groove depth \( D \)

The exemplary values are calculated for a depth-setting standard with the nominal value \( Pt = 3 \) µm, evaluation without filtering.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Input quantity catchword</th>
<th>Calculation of input quantity</th>
<th>Exemplary value</th>
<th>Sensitivity coeff.</th>
<th>Method of determination, distribution</th>
<th>Variance / ( \text{nm}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Reference standard</td>
<td>( \frac{U_n^2}{4} )</td>
<td>( U_n = 10 ) nm</td>
<td>1</td>
<td>B Gaussian</td>
<td>25</td>
</tr>
<tr>
<td>1.2</td>
<td>Difference measuring point – calibration point</td>
<td>( \frac{a_r^2 \cdot G^2}{3} )</td>
<td>( a_r = 100 ) µm, ( G = 20 \text{nm/mm} )</td>
<td>1</td>
<td>B uniform</td>
<td>1,33</td>
</tr>
<tr>
<td>1.3</td>
<td>Repeatability</td>
<td>( \frac{s_r^2(P_{m})}{m_r} )</td>
<td>( s(P_{m}) = 2 ) nm</td>
<td>1</td>
<td>A Gaussian</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Topography</td>
<td>( \frac{s_t^2(P_{t})}{m_t} )</td>
<td>( s(P_{t}) = 5 ) nm</td>
<td>1</td>
<td>A Gaussian</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>Straightness deviations</td>
<td>( W_{t_0} ) ( \frac{12}{2} )</td>
<td>( W_{t_0} = 20 ) nm</td>
<td>1</td>
<td>B uniform</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>Background noise</td>
<td>( \frac{R_z_{t_0}^2}{12} )</td>
<td>( R_z_{t_0} = 20 ) nm</td>
<td>1</td>
<td>A uniform</td>
<td>33</td>
</tr>
</tbody>
</table>

\[
\Sigma = \left( \frac{2 \cdot u(z_g)^2}{12} \right)^{1/2} + \frac{1}{12} \cdot Pt_r^2
\]

\[
u(Pt) = 10 \text{ nm}, \quad u(Pt_n) = 8 \text{ nm}
\]

\[
u(D) = 10 \text{ nm}, \quad u(D_n) = 5 \text{ nm}
\]

Evaluation with filtering \( \lambda_s = 8 \) µm, sampling distance \( \Delta x = 0,5 \) µm:
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Input quantity catchword</th>
<th>Calculation of input quantity</th>
<th>Exemplary value</th>
<th>Sensitivity coeff.</th>
<th>Method of determination, distribution</th>
<th>Variance / nm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Reference standard</td>
<td>$U_n^2 / 4$</td>
<td>$U_n=10$ nm</td>
<td>1</td>
<td>B Gaussian</td>
<td>25</td>
</tr>
</tbody>
</table>
| 8       | Difference measuring point – calibration point | $a_r^2 \cdot G^2 / 3$ | $a_r=100$ µm  
$G=20$ nm/mm | 1                  | B uniform                             | 1.33           |
| 8       | Repeatability            | $s^2(Pr_n) \cdot f_s^2$       | $s(Pr_m) = 2$ nm | 1                  | A Gaussian                             | 4              |
| 8       | Topography               | $s^2(Pr) \cdot f_s^2$         | $s(Pr) = 5$ nm  | 1                  | A Gaussian                             | 2.4            |
| 8       | Straightness deviation   | $Wt_z^2 / 12$                 | $Wt_n = 20$ nm  | 1                  | B uniform                             | 3.2            |
| 8       | Background noise         | $1 / 12 \cdot RZ_u^2 \cdot f_s^2$ | $RZ_u = 20$ nm | 1                  | A uniform                             | 3.2            |
| 8       | $u^2(z_s)$               | $\Sigma$                     |                 |                    | Uncertainty / nm                       | 39.13          |
|         | $u(z_s)$                 |                               |                 |                    |                                        | 6.3            |
| 8.1     | Uncertainty of total height of profile $u(Pr)$ | $[2 \cdot u^2(z_s) + 1 / 12 \cdot Pr_n^2]^{1/2}$ | $Pr = 6$ nm  
$u(Pr) = 8$ nm |                        |                                        | 10.3           |
| 8.2     | Uncertainty of groove depth $u(D)$ | $[(1 / n_u + 1 / n_n) \cdot u^2(z_s) + 1 / 12 \cdot \left( \frac{Pr_n^2}{2} \right)]^{1/2}$ | $Pr = 6$ nm  
$u(D) = 5$ nm |                        |                                        | 5.3            |