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Fundamental constants and units and the search for temporal variations

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This article reviews two aspects of the present research on fundamental constants: their use in a universal and precisely realizable system of units for metrology and the search for a conceivable temporal drift of the constants that may open an experimental window to new physics.

1. INTRODUCTION

These lectures attempt to cover two active topics of research in the field of the fundamental constants whose motivations may seem to be disconnected or even opposed. On one hand there is a successful program in metrology to link the realization of the base units as closely as possible to the values of fundamental constants like the speed of light c, the elementary charge e etc., because such an approach promises to provide a universal and precise system for all measurements of physical quantities. On the other hand, the universality of the constants may be questioned because the search for a theory of grand unification or quantum gravity inevitably seems to call for violations of Einstein's equivalence principle and therefore may imply spatial and temporal dependencies of the fundamental coupling constants. The experimental search for a temporal variation of fundamental constants is motivated by the idea that this may provide a window to new physics that may then guide or constrain the routes towards a deeper theoretical understanding. The two topics are linked by the fact that both of them benefit from the same striving for experimental precision in metrology. Improved precision will allow more reliable measurements in the applied sciences and it will also allow to look for tiny effects that may have gone unnoticed so far but that may provide hints to a more complete understanding of the foundations of physics.

The first sections of the article will give a brief overview of the present system of units and of the ongoing discussions on how to improve it. We will mainly focus on two examples: the unit of mass that is presently realized via an artefact that shall be replaced by a quantum definition based on fundamental constants and the unit of time, which is exceptional in the accuracy to which time and frequencies can be measured with atomic clocks. The second part will give a brief motivation for the search for variations of constants, will review some observations in geophysics and in astrophysics and will finally describe laboratory experiments that make use of a new generation of highly precise atomic clocks.

Obviously, the field of constants, units and their possible variations is much too vast to be covered in this article in its totality and I refer the interested reader to recent collections of papers originating from workshops and conferences that were devoted to exactly this range of topics [1–4].

2. THE INTERNATIONAL SYSTEM OF UNITS SI

The *Système Internationale d'Unités* SI [5] consists of 7 base units and 22 derived units that can be expressed in terms of the base units but that carry a given name, like newton, joule, ohm etc. The base units are: the metre for length, the kilogram for mass, the second for time, the ampere for electrical current, the kelvin for temperature, the mole for the amount of substance and the candela for luminous intensity. The definition and use of the candela may not be very familiar to physicists in general, but it is a base unit of practical importance because illumination

is one of the major uses of electrical energy. The example shows that the development of the SI is not exclusively driven by fundamental research but very much by applications and technology. The present definitions of the base units were laid down between 1889 (the kilogram) and 1983 (the metre) and they present a rather heterogeneous structure. Let us illustrate this by looking at the three definitions for metre, kilogram and second:
The metre is the length of the path travelled by light in vacuum during a time interval of (1/299 792 458) *of a second.*
This definition was chosen after it became possible in the 1970ies to measure absolute frequencies of selected optical reference wavelengths to a relative uncertainty of about 10^{-10} [6]. This is about the limit of laser interferometric length measurements and corresponds to the resolution of less than 1/100 of an interferometer fringe over a 10 m distance. At that time the metre was defined via the wavelength of a radiation from ^{86}Kr atoms and the measurements of λ and ν therefore represented a measurement of the speed of light $c = \lambda\nu$ in independent units metre and second. It was then decided to define the metre via a fixed value of the speed of light $c = 299\,792\,458$ m/s. Since then, the metre is not an independent unit any more, but it is linked to the second.
The kilogram is the unit of mass and it is equal to the mass of the international prototype.
The prototpye is a cylinder made from a platinum-iridium alloy [7]. It is kept at the international office for weights and measures BIPM in Sèvres near Paris and is at the top level of a hierarchy of copies that are compared periodically and are used to distribute the realization of the unit of mass to the national metrology institutes. While intrinsically the definition of the kg is without uncertainty, the necessary mass comparisons achieve a relative uncertainty of 10^{-8}, i.e. 10 μg/kg. Apart from the worries associated with having to rely on an artefact that may be lost or damaged, there seems to be an unexplained drift of about 50 μg over 100 years between the prototype and the next level of copies that are also kept at the BIPM. Consequently, alternative realizations of the unit of mass are the subject of intense research activities and may lead to a new definition of the kg in the near future (see section 4).
The second is the duration of 9 192 631 770 *periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the* ^{133}*Cs atom.*
This definition of the unit of time was adopted in 1967. Again, the definition is based on fixing the numerical value of a natural constant, $\nu_{\mathrm{CsHFS}} = 9\,192\,631\,770$ Hz, though it is not a fundamental constant in this case. The adoption of the atomic definition of the second followed the development of atomic clocks that had started in the 1950ies and that had demonstrated that atomic transition frequencies provided much higher stability than those observable in periodic astrophysical phenomena. In addition, transportable Cs-clocks were readily available and lended themselves immediately to applications in navigation, telecommunication etc. The adopted value of ν_{CsHFS} was the result of a measurement of the astronomical ephemeris second with the first operative cesium clock and the uncertainty of that measurement (about 10^{-9}) was limited by the telescopic observations. As we will see in the following, the measurement of time and frequency occupies a central position in metrology. The second is the base unit that can be realized with the lowest relative uncertainty. A level of less than 10^{-15} is now achieved with Cs fountain clocks that use laser cooled atoms (see section 5).

3. UNITS BASED ON FUNDAMENTAL CONSTANTS

The idea to use units based on fundamental constants or on quantum properties is not new but was quite clearly expressed by J. C. Maxwell as early as 1870 when he stated [8]: "*If, then, we wish to obtain standards of length, time, and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules.*" In more modern terms, the statement that molecules and atoms are "imperishable, unalterable and perfectly similar" is implicitly con-

Table 1
The essential relations between units and fundamental constants

unit	relation	device or effect
second	$\nu = \Delta E/h$	atomic clock (Cs hyperfine structure)
metre	$\lambda = c/\nu$	optical interferometer
volt	$U = nh\nu/e$	Josephson junction
ampere	$I = e\nu$	single electron transistor
ohm	$R_K = h/e^2$	quantum Hall effect
kelvin	$T = E/k_B$	Boltzmann's constant as a conversion factor

tained in Einstein's equivalence principle and well established in quantum statistics. Looking at the present status of the SI, we see that Maxwell's intention is now realized for length and time, but not for mass and the electrical units.

The basic idea of a greater reform of the SI [3,4] is to extend the method applied for the unit of length and to fix the values of a set of fundamental constants: The metre is already defined via the speed of light c and the second. Planck's constant h and the elementary charge e may be used to define the volt by linking it to a frequency in a Josephson junction. The ohm will be realized via the quantum Hall effect, also linking it to h and e via von Klitzing's constant $R_K = h/e^2$. These definitions of volt and ohm would fix the ampere, that alternatively or for consistency may also be realized by counting electrons in a single electron transistor. The Boltzmann constant k_B will act as a conversion factor linking the unit of temperature, the kelvin, to the joule. The Avogadro number N_A will define the mole. Unfortunately, N_A seems to be too big to allow a determination via counting of atoms up to a weighable quantity, but in section 4 an alternative way to measure N_A will be described. Either N_A or h could be used to provide a new definition of the kilogram (see section 4). Table 1 lists the essential physical relations and effects that will be used in such realizations of the units. The only "free" parameter that would remain is a suitably chosen atomic transition frequency ν that will define the unit of time. It appears in four of the six relations in table 1, thus underlining the importance of time and frequency measurements. Conceptually, it would be attractive to derive ν from a calculable quantum system like from a transition frequency in atomic hydrogen or positronium. In practice, it has turned out, however, that more complex atomic systems (like the ground state hyperfine structure in atomic cesium) allow better control of systematic frequency shifts and are therefore the favored systems for atomic clocks.

Note that this system of units involves h, c and e, but neither Newton's gravitational constant G nor masses of elementary particles. In this way they differ from Planck's units (using h, c and G), Stoney's units (using e, c and G, and a factor $1/\sqrt{\alpha}$ smaller than Planck's units; α: fine structure constant), and Dirac's units (using e, c and the electron mass m_e) [9]. These systems – and Planck's units in particular – are impractical because they differ by many orders of magnitude from the atomic units that are better adapted to describe the physics of our technical applications. In addition, the present relative uncertainty in the value of Newtons's constant G is $1 \cdot 10^{-4}$, about four orders of magnitude bigger than those of h and e.

The main experimental challenge on the way towards such a reform of the SI is to measure precise values of the constants and to provide a consistent link to the established realizations in order to assure the consistency, continuity and traceability of measurements.

4. ROUTES TOWARDS A NEW DEFINITION OF THE KILOGRAM

As mentioned above, there is a strong motivation to find a new definition of the base unit of mass that will replace the kilogram prototype in Sèvres. Two different methods are pursued and are now close to reaching the point of a conclusive evaluation: the watt balance and the Avogadro projects.

The watt balance has been proposed by B. Kibble from the British NPL in 1975 [10]. It is a device that compares mechanical and electrical power and relates mass to Planck's constant h. A simplified picture may be described like this: Imagine a balance where one arm holds a test mass m under the influence of Earth's gravity (acceleration of fall g). The second arm holds a coil that is placed in a radial magnetic field B and that can carry an electrical current I. The measurements will proceed as follows: In a static mode the current I is adjusted so that the Lorentz force acting on the coil is equal to the gravitational force acting on the test mass:

$$k_C BI = mg, \tag{1}$$

where k_C is an instrumental constant that contains the geometric properties of the coil. In a dynamic mode the lever of the balance is moved so that the coil and test mass move at velocity v. Since the coil moves in the magnetic field, a voltage

$$U = k_C Bv \tag{2}$$

is induced. If k_C and B are kept constant in both modes of operation, electrical and mechanical power can be equated:

$$UI = mgv. \tag{3}$$

The velocity v and acceleration g are measured with laser interferometers on the balance itself and with a free falling test body nearby, respectively. The values of U and I are measured with electrical quantum standards as outlined in section 3. This relates the macroscopic mass m with Planck's constant h, because U is measured in units proportional to h/e and I in units proportional to e. If the mass is known, the electromechanical watt balance therefore is a device to measure h. If this does not seem intuitive at first sight, one has be keep in mind that the quantum aspect enters into the measurements of the electrical quantitites. The most precise measurement of this kind has been performed at the US national institute NIST and it has succeeded in determining h with a relative uncertainty of $8 \cdot 10^{-8}$ [11].

The watt balance route towards a new definition of the kilogram would consist in fixing the value of h and using the watt balance to find the corresponding mass. The wording of the kilogram definition could be:
The kilogram is defined such that the Planck constant is equal to $6.62606896 \cdot 10^{-34}$ *Js.*
Or, alternatively, making use of the fixed values of h and of c:
The kilogram is the mass of a body at rest whose equivalent energy equals the energy of a photon of frequency $1.35639274 \cdot 10^{50}$ *Hz.*

The actual values mentioned here may still change according to the forthcoming evaluations of the measurements of fundamental constants [12]. This type of kilogram definition has the advantage that by fixing the value of h it provides a link to the electrical units that are of immense practical importance. It should be seen in conjunction with new quantum definitions of ampere, volt and ohm. Presently, the SI ampere is still defined via the force between two infinite parallel wires, which is unsuitable for the realization of the unit. A practical difficulty with this type of kilogram definition may be that it defines a quantity of everybody's daily experience in terms of abstract quantum effects. Suitable pictures would have to be found to explain this to school children and the general public.

The Avogadro project follows a different approach towards a new kilogram definition in that it seeks to precisely measure the number of atoms within a macroscopic mass [13,14]. In the SI, mass and amount of substance are independent quantities and the measurement of Avogadro's number N_A is a realization of the mole. Since N_A atoms of ^{12}C present a mass of exactly 12 g of isotopically pure carbon ^{12}C according to the definition of the atomic mass, measuring and fixing N_A will allow to relate the macroscopic to the atomic mass unit, i.e. to define the kilogram via an atomic mass. This program is pursued in a multinational cooperative effort that relies on recent developments in a number of precision measurement techniques: The cyclotron frequencies of single ions in Penning traps can be measured at a precision of 10^{-10} [15]. For an absolute mass measurement the value of the magnetic field would have to be determined, which is not possible at this level of precision. Mass ratios, however, can be determined by storing two ions of different species in the same trap. In this way the ratio of the masses of ^{28}Si and ^{12}C can be measured, i.e. the mass of ^{28}Si in atomic mass units. Thanks to developments for the semiconductor industry, silicon can be prepared in high purity and single crystals of a mass of several kilogram can be grown. To reduce the uncertainty contribution from the isotopic composition, the Avogadro project uses isotopically highly enriched ($> 99.99\%$) crystals of ^{28}Si. The lattice constant of the crystal, or equivalently the number density of the silicon atoms, can be determined from X-ray diffraction. To measure the total volume, the crystal is polished to a sphere (approximately 100 mm diameter for 1 kg, with $\approx$ 30 nm deviation from perfect roundness at best) and the shape is controlled and the diameter measured with optical interferometry. Finally, the mass of the sphere will be measured, allowing to determine the number of ^{28}Si atoms per kg, and, using the ^{28}Si molar mass from the ^{28}Si/^{12}C comparison, the number of atoms per mole N_A.

At present, two spheres of highly enriched ^{28}Si have been produced and the Avogadro project is in the final stages of their characterisation. The anticipated uncertainty budget gives a value in the low 10^{-8}, with about equal contributions from the measurements of the unit cell volume, the molar mass, the sphere volume and from the contribution of surface oxide layers.

The ensuing definitions of the kilogram and the mole from the Avogadro route may be formulated like:
The kilogram is the mass of $5.01845149 \cdot 10^{25}$ *free* ^{12}C *atoms at rest and in their ground state.*
The mole is the amount of substance of a system that contains $6.02214179 \cdot 10^{23}$ *specified entities.*
Again, the actual values may change according to the forthcoming evaluations of the measurements.

While these definitions are conceptually clear and very simple, the underlying effort that goes into the production of isotopically pure single crystals is substantial. The proponents of the watt balance approach state that fixing the value of h without uncertainty will be more beneficial for the combined adjustment of the values of fundamentals constants than doing so with N_A [16]. In the present discussion about the kilogram redefinition, both approaches, the watt balance and the Avogadro projects strive to reduce their uncertainties. Though the kilogram prototype may seem like an outdated concept, it has provided the stability of the mass definition over more than a century within an uncertainty of a few parts in 10^8, i.e. at a level of accuracy that is similar to what shall be achieved in the present measurements of h and N_A.

5. A NEW GENERATION OF ATOMIC CLOCKS

The concept of an atomic clock is to realize the frequency of an unperturbed atomic transition and to build a time scale by counting the periods of oscillation of this frequency. The oscillator of the clock produces an electromagnetic wave – microwaves generated from a crystal oscillator or light from a laser – that is used to probe a sample of atoms. The response of the atoms is measured (either in absorption, fluorescence or a more elaborate method) and used to derive an error signal so that the frequency of the oscillator can be locked to the atomic resonance. Finally, a counter and clockwork are needed to produce 1-s-ticks after a preset number of periods and other usable reference frequencies.

Application of the methods of laser cooling and trapping has led to significant improvements in the precision of atomic clocks and frequency standards over the last years [17,18]: Primary cesium clocks based on atomic fountains [19] realize the SI second with a relative uncertainty below $1 \cdot 10^{-15}$. Research towards *optical* clocks with trapped ions and atoms [21,18] has resulted in several systems that have demonstrated an even higher reproducibility. A significant advantage of an optical clock lies in the higher frequency of the oscillator, that allows one to perform precise frequency measurements in a shorter averaging time. The lowest instabilities that have been demonstrated are a few parts in 10^{15} in 1 s only, improving further like the square root of the averaging time.

Cesium clocks are the primary standards for time and frequency [17,18] as a direct realization of the SI definition. The second is defined via the hyperfine splitting frequency between the $F = 3$ and $F = 4$ levels of the $^2S_{1/2}$ ground state of the ^{133}Cs atom at approximately 9.192 GHz. In a so-called cesium fountain [19] (see Fig. 1), a dilute cloud of laser-cooled cesium atoms at a temperature of about 1 μK is launched upwards to initiate a free parabolic flight with an apogee at about 1 m above the cooling zone. A microwave cavity is mounted near the lower endpoints of the parabola and is traversed by the atoms twice – once during ascent, once during descent – so that Ramsey's method of interrogation with separated oscillatory fields [20] can be realized. The total interrogation time being on the order of 0.5 s, a resonance linewidth of 1 Hz is achieved, about a factor of 100 narrower than in the original devices developed since 1955 with a thermal atomic beam from an oven. Selection and detection of the hyperfine state is performed via optical pumping and laser induced resonance fluorescence. With the best cesium fountain clocks a relative uncertainty below $1 \cdot 10^{-15}$ is reached in the realization of the resonance frequency of the unperturbed Cs atom. The averaging time that is required to reach a statistical uncertainty at this level is on the order of 10^4 s. Several effects contribute to the uncertainty budget. In particular, cold cesium atoms show a relatively large density-dependent frequency shift because of long-range molecular states close to the continuum.

If an atom or ion is held in a trap, several problems that are encountered in atomic fountains can be eliminated: The interaction time is not limited by the movement of the atom through the finite interaction region and narrow resonances can be obtained at the limits set by the radiative lifetime of the excited state or by the linewidth of the interrogating oscillator. Ions are especially well suited because they carry electric charge and can be trapped in radiofrequency ion traps (Paul traps [22]) that provide confinement around a field-free saddle point of an oscillating electric quadrupole potential. This ensures that the internal level structure is only minimally perturbed by the trap. Combined with laser cooling it is possible to reach the so-called Lamb–Dicke regime: the motion of the ion is constrained to a region of a size that is smaller than the wavelength of the transition that is being probed. In this case, a recoil-free line is obtained and the linear Doppler shift is eliminated. A single ion, trapped in an ultrahigh vacuum is conceptually a very simple system that allows good control of systematic frequency shifts. The use of the much higher, optical reference frequency allows one to obtain a stability that is superior to a cesium clock. This is possible even though only a single ion is used to obtain a correction signal for the reference laser.

These advantages were first pointed out by Dehmelt in the 1970ies when he proposed the *mono-ion oscillator* [23] and predicted that it should be possible to reach an accuracy of 10^{-18} with an optical clock based on a dipole-forbidden,

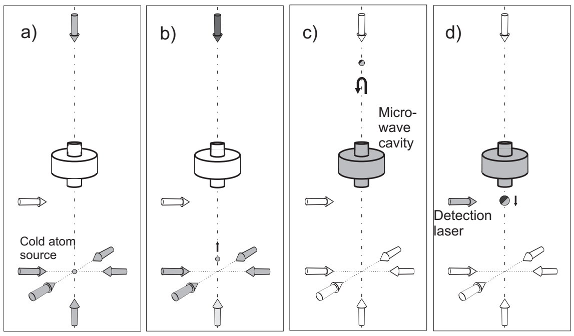

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Figure 1. Sequence of operation of a cesium fountain clock: (a) A cloud of cold atoms is loaded. (b) The atoms are launched upwards. (c) In free parabolic flight the atoms pass the microwave cavity twice where they are subjected to the radiation inducing the hyperfine transition. (d) State detection via laser excitation and fluorescence detection. For each step of the sequence, the active radiation fields are indicated in grey shading.

narrow-linewidth transition in a single, laser-cooled and trapped ion. To detect the excitation on the forbidden optical transition a double-resonance scheme is used (see Fig. 2): Both, a dipole-allowed transition and the forbidden narrow reference transition of the optical clock are driven by two different laser frequencies from the ground state. The dipole transition is used for laser cooling and the resulting resonance fluorescence is used for the optical detection of the ion. If the second laser excites the ion to the metastable upper level of the reference transition, the fluorescence disappears and will only reappear after the decay of the metastable state. Every excitation of the reference transition thus suppresses the subsequent scattering of a large number of photons on the cooling transition and can be detected with unity efficiency. A number of different atomic ions present suitable reference transitions with natural linewidths of the order of 1 Hz or below and several groups pursue research along these lines [21]. Some recent results from this field will be presented in section 8.

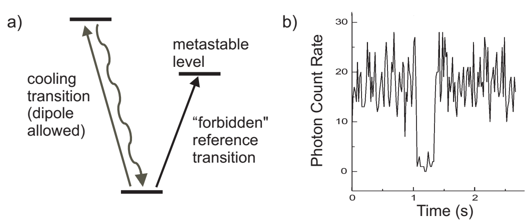

Figure 2. a) Simplified level scheme of an ion used in an optical clock: a strong dipole-allowed cooling transition and the forbidden and narrow reference transition orginate from the ground state. (b) Experimental quantum jump signal of a single $^{115}In^+$ ion: the fluorescence on the cooling transition shows a dark interval due to an excitation of the metastable level.

Optical frequency standards based on laser-cooled neutral atoms have been developed in the configuration of the so-called optical lattice clock. A neutral atom can be trapped using the optical dipole force (equivalent to the quadratic Stark effect) that is attractive towards the intensity maximum of a laser beam that is detuned below a strong atomic resonance line. An array of interference maxima produced by several intersecting laser beams produces the optical lattice that can trap one or several atoms in each lattice site. The detuning of the trapping laser is chosen such that the light shift it produces in the ground and excited state of the reference transition are equal so that no net shift of the reference frequency appears [24]. This approach is especially well suited to the alkaline-earth-like atoms with two valence electrons and is applied to the very narrow (mHz natural linewidth) $^1S_0 \rightarrow {}^3P_0$ transitions in neutral strontium, ytterbium and mercury. An important advantage of the optical lattice clock over the single-ion clock is that the signal can be derived from many (typically 10^5–10^6) atoms and that the short-term stability is therefore higher. The associated disadvantage is that – like in a cesium clock – the atoms may interact and perturb each other, and that optical traps are not as deep and stable as ion traps.

The progress in stability and accuracy of optical frequency standards in recent years has been so impressive that it is conceivable that in the future an optical transition will be used to define the unit of time. The problem of the precise conversion of an optical frequency to the microwave domain, where frequencies can be counted electronically in order to establish a time scale or can easily be compared in a phase coherent way was solved with the so-called femtosecond laser frequency comb generator [25,6] (see Fig. 3). Briefly, a mode-locked femtosecond laser emits a very regular temporal sequence of pulses that each have a duration of a few optical cycles only. In the frequency domain, this produces a comb of equally spaced optical frequencies f_m that can be written as $f_m = mf_r + f_{\rm ceo}$ (with $f_{\rm ceo} < f_r$), where f_r is the pulse repetition rate of the laser, the mode number m is a large integer (of order 10^5), and $f_{\rm ceo}$ (carrier-envelope-offset frequency) is a shift of the whole comb that is produced by group velocity dispersion in the laser. The repetition rate f_r can easily be measured with a fast photodiode. In order to determine $f_{\rm ceo}$, the comb is broadened in a nonlinear medium so that it covers at least one octave. Now the second harmonic of mode m from the "red" wing of the spectrum, at frequency $2(mf_r + f_{\rm ceo})$, can be mixed with mode $2m$ from the "blue" wing, at frequency $2mf_r + f_{\rm ceo}$, and $f_{\rm ceo}$ is obtained as a difference frequency. In this way, the precise relation between the two microwave frequencies f_r and $f_{\rm ceo}$ and the numerous optical frequencies f_m is known. This setup can be used for an absolute optical frequency measurement by referencing f_r and $f_{\rm ceo}$ to a microwave frequency standard and recording the beat note between an arbitrary optical frequency $f_{\rm o}$ and the closest comb frequency f_m. Vice versa, the setup may work as an optical clockwork, for example, by adjusting $f_{\rm ceo}$ to zero and by stabilizing one comb line f_m to $f_{\rm o}$ so that f_r is now an exact subharmonic to order m of f_o. The precision of these transfer schemes has been investigated and was found to be so high that it will not limit the performance of optical clocks for the foreseeable future.

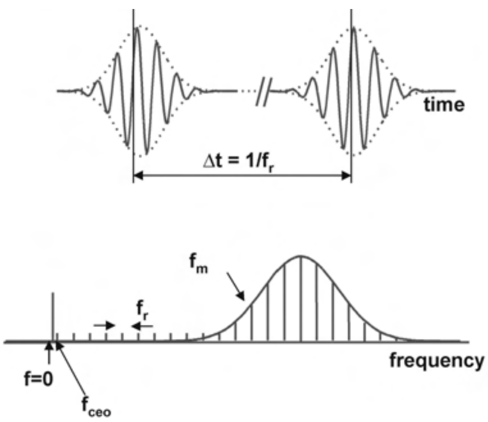

Figure 3. Temporal sequence of pulses from a femtosecond laser (top) and the generated optical frequency comb in the Fourier domain (bottom).

6. VARIATIONS OF FUNDAMENTAL CONSTANTS: MOTIVATION

While the first part of this review has shown how to make use of the universality of the fundamental constants for the practical purposes of metrology, we will now question exactly this postulate by looking at a possible time variability. In fact, some of the fundamental constants may be neither fundamental nor constant. The term fundamental describes that at the limit of our present understanding we cannot derive their values from any deeper principle, but this may change if knowledge progresses. The constancy of the constants seems very well established: it is implicit in the equivalence principle, in conservation laws and in the indistinguishability of quantum particles, that do not seem to carry a memory of when and where they were produced. On the other hand, candidate theories of quantum gravity predict violations of the equivalence principle. The fundamental coupling constants are known to be energy dependent. The inflationary model of the universe states that in a very early epoch the universe experienced a phase transition which changed a vacuum average of the Higgs field which determines the electron mass. The latter was zero before this transition and reached a value close or equal to the present value after the transition. Adiabatic changes of particle properties would still be possible in later phases of cosmological evolution without breaking their indistinguishability. The search for variations of the constants is therefore well motivated as a test of an important postulate of metrology and as a search for new physics. Over the past years, interest in this field has been quite intense and the subject has been addressed in various areas of physics [26–29,1,3]. The main part of the non-theoretical activities takes place in two areas: observational astrophysics and laboratory experiments with atomic clocks and frequency standards.

Early proposals of variable constants by Zwicky and Chalmers date back to the 1930ies and appear as attempts of alternative explanations for Hubble's observation of the distance-dependent redshift. An idea that attracted some attention is Dirac's large number hypothesis [30]. Dirac constructed two dimensionless ratios and noticed that they are of similar magnitude $10^{39} - 10^{40}$: The ratio of the electric and the gravitational force between electron and proton

$$R_1 \equiv \frac{e^2}{4\pi\epsilon_o m_p m_e\, G} \approx 2 \times 10^{39} \tag{4}$$

and the age of the universe in Dirac's "electronic units"

$$R_2 \equiv \frac{4\pi\epsilon_o m_e c^3}{e^2\, H_0} \approx 4 \times 10^{40}, \tag{5}$$

using the present value of the Hubble constant H_0. Dirac assumed an age of the universe of only $2 \cdot 10^9$ years and therefore the coincidence seemed more striking to him. His hypothesis was to set $R_1 = R_2$ and since H_0 changes with the age of the universe, he proposed that G would be time dependent, $G \propto H_0 \propto 1/t$ with a relative rate of change given by

$$\left|\frac{1}{G}\frac{dG}{dt}\right| = \left|\frac{d\ln G}{dt}\right| = H_0 \approx 6 \times 10^{-11}/\text{year}. \tag{6}$$

This is an order of magnitude that would be expected in a most simple model of cosmological variations, but it was soon understood that such a change of G would be too rapid to be consistent with established models of stellar evolution. Later, the result $d\ln G/dt = (1.0 \pm 2.3) \cdot 10^{-11}$/yr was inferred for the present epoch from the timing data of the binary pulsar PSR 1913+16 [31]. The most most recent result on dG/dt stems from the analysis of lunar laser ranging data. As a legacy of lunar missions between 1969 and 1973, retro reflectors have been left on the moon and have since been used to monitor the distance between earth and moon with an uncertainty finally reaching the cm-range – a remarkable precision experiment that allows tests of general relativity in our planetary system. At present, the constancy of G is established from this data as $d\ln G/dt = (2 \pm 7) \cdot 10^{-13}$/yr [32].

The most important test cases in the search for variations are dimensionless constants. This is because if variations in a dimensional quantity (c, G, etc.) would be detected, there would be no clear way to distinguish between a change of the quantity and a change of the unit. If the search for variations is motivated by the idea of unification of the fundamental interactions, it is not plausible to restrict the analysis by allowing only variations of a *single* constant but we have to allow *all* of them (or at least several) to vary, possibly in a correlated fashion. Most experiments and observations therefore concentrate on Sommerfeld's fine structure constant $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ and the proton-to-electron mass ratio m_p/m_e. While α is the coupling constant of the electromagnetic interaction, m_p/m_e also depends on the strength of the strong force and on the quark masses via the proton mass. Results on these numbers can be interpreted independently from the conventions of a specific system of units. Quite conveniently, both constants are ubiquitous in atomic and molecular spectra: α appears in atomic fine structure splittings and other relativistic contributions to level energies, and m_p/m_e in molecular vibration and rotation frequencies as well as in hyperfine structure.

7. SEARCH IN GEO- AND ASTROPHYSICS

Because of the strong motivation mentioned above, searches for variations of constants have been performed in various fields like astrophysics, geophysics, atomic and nuclear physics, etc. [26]. Two directions of research can be distinguished: Astrophysical or geophysical observations are used to analyze variations on a cosmological time scale. Alternatively, precision laboratory experiments can be undertaken to measure non-zero temporal derivatives of fundamental constants in the present epoch. In this section we will present two prominent examples for the first approach. Laboratory experiments with atomic clocks will be treated in the following section.

A frequently discussed geophysical analysis was performed using isotope ratios from material found in fossil nuclear reactors that were active in the Oklo region about 1.8 billion years ago. This unique geophysical phenomenon has been discovered in 1972 in Gabon in western Africa in uranium mining. It was noticed that the ore from some regions of the mine showed a significant deficiency of U-235. This could be explained by fission chain reactions that had been triggered spontaneously in the remote past within the uranium deposit so that parts of the deposit had behaved like a nuclear reactor. Of the two principal isotopes of uranium, the fissile U-235 has a shorter half-life than U-238. Consequently, the concentration of U-235 in natural uranium is decreasing steadily and at the time of activity of the Oklo reactors it was much higher than it is now (3.65% against 0.72%). In regions of high concentration of uranium, in the absence of strong neutron absorbers and with water as a moderator, self-sustained fission chain-reactions took place. Altogether, several tons of U-235 fissioned over a period of several 10^5 years. In this system, an analysis for a change of α was first performed by Shlyakhter [33], making use of a neutron capture resonance in Sm-149 at the very low energy of 0.1 eV:

$$n +^{149}_{62} \mathrm{Sm} \rightarrow^{150}_{62} \mathrm{Sm} + \gamma. \tag{7}$$

From the measured samarium isotope ratios in the ore and the modeled neutron flux in the reactor zones it was inferred that the cross section for this process did not change significantly in the $1.8 \cdot 10^9$ years since the activity. Since the position of a low-energy nuclear resonance will depend critically on the values of the coupling strengths, this was used to obtain a very stringent upper limit on a change of α in the range of 10^{-17} per year [33,34]. One may note, however, that several aspects of this analysis are model-dependent. In fact, contrasting conclusions have been drawn from the same data and conditions like the temperature of the reactor may have a critical influence [35]. In addition, the nuclear resonance energy would be influenced not only by changes of α (i.e. via the electromagnetic contribution to the nuclear level structure) but also by changes in the strength of the strong interaction. It is very difficult to clearly separate both influences.

A clearer view into the cosmological past over large scales can be obtained from the spectra of remote astrophysical sources. A group of astrophysicists led by J. Webb at the University of New South Wales in Sydney has measured the wavelengths of metal absorption lines produced by interstellar gas clouds in the continuous spectrum of the light from distant quasars [36]. In the so called many multiplet method, they analyze resonances from Mg, Mg^+, Fe^+, Cr^+ and other elements. The value of α enters into the fine structure splitting (whence the name) which is proportional to α^2 times the Rydberg constant, but also into relativistic contributions to level energies in general. These scale like

$$\Delta E_{rel} \propto \frac{(Z\alpha)^2}{n_*} \frac{1}{j + 1/2} \tag{8}$$

where Z is the nuclear charge, n_* the effective quantum number and j the electronic angular momentum. It can be seen that relativistic effects are more important in heavy atoms and that a non-relativistic value of a transition energy may be either increased or decreased by the relativistic effects, depending on whether the electronic momentum is bigger in the lower or in the upper level. The absolute values of the relativistic contributions for a number of relevant atomic states have been obtained from relativistic Hartree-Fock calculations [37]. A presumed change of α would therefore produce a predictable characteristic pattern of spectral shifts if a suitable multitude of atomic lines is studied. In spectra from interstellar gas clouds, these shifts would, however, be superimposed on the redshift of the cloud and additional Doppler shifts due to internal dynamics.

A multitude of data has been obtained from quasar absorption spectra by different groups but the present picture that is obtained is not completely consistent: The observations analyzed by Webb et al. suggest that about 10 billion years ago (for the redshift range 0.5 to 3.5), the value of α was smaller than today by $\Delta\alpha/\alpha = (-0.543 \pm 0.116) \cdot 10^{-5}$, representing 4.7σ evidence for a varying α [38]. Assuming a linear increase of α with time, this would correspond to a drift rate $d \ln \alpha/dt = (6.40 \pm 1.35) \cdot 10^{-16}\ \mathrm{yr}^{-1}$ [38]. This result received considerable attention as it could be a first quantitative indication of a change of α on the cosmological timescale. Other evaluations of spectra obtained with a different telescope and using other selections of quasar absorption systems reach similar sensitivity for $\Delta\alpha/\alpha$ but are

Table 2
Functional dependence on the fundamental constants for transition frequencies related to different kinds of energy intervals. Ry stands for the Rydberg constant, μ is the nuclear magnetic moment. The nuclear mass is expressed in units of m_p here.

Transition	Energy scaling
Atomic gross structure	Ry
Atomic fine structure	$\alpha^2 Ry$
Atomic hyperfine structure	$\alpha^2(\mu/\mu_B)Ry$
Molecular electronic structure	Ry
Molecular vibrational structure	$\sqrt{m_e/m_p}\,Ry$
Molecular rotational structure	$(m_e/m_p)Ry$
Relativistic corrections	function of α^2

consistent with $\Delta\alpha = 0$ for all look-back times [39,40], however. It should be kept in mind that the uncertainties cited here are the result of averaging over a large ensemble of data points with much higher individual statistical uncertainties. The significance of the results therefore hinges on a complete understanding of all systematic effects that may introduce bias or correlations in the data and that may not be easy to detect. Efforts are ongoing to take more data with more carefully calibrated spectrographs and there is hope that this important controversy may be resolved.

8. SEARCH IN EXPERIMENTS WITH ATOMIC CLOCKS

Laboratory experiments obviously cannot provide information on temporal evolution on the cosmological timescale. However, they have the important advantage that they can be repeated, parameters can be varied and systematic effects can be studied. This will eventually be crucial for obtaining a consistent and credible signature of a weak effect. Laboratory searches for variations of constants rely mainly on precision comparisons of atomic and molecular frequency standards [41]. One uses the fact that level spacings and thus the transition frequencies in different classes of transitions depend differently on α, on the Rydberg constant Ry and on m_p/m_e [28,42]. The different scalings for the most important types of transitions are summarized in table 2. Relativistic corrections have to be taken into account in heavy atoms, which leads to an additional scaling with $(Z\alpha)^2$ where Z is the nuclear charge [43,37].

If, for example, the frequency of an electronic transition of the atomic gross structure is measured repeatedly with respect to a cesium clock that is based on atomic hyperfine structure, and if this frequency ratio is found to be constant, one can infer the constancy of a product of a power of α and of the ^{133}Cs nuclear magnetic moment. Such a result should not be used to derive a limit for the variation of α alone by postulating the constancy of the other factor. In the framework of unified theories one has to expect that in case of a variation of α the coupling constant of the strong interaction would show even larger variations that would lead to changes in nuclear masses and moments [44,45].

To obtain a model-independent statement on $d\alpha/dt$ one may look at different gross structure transitions frequencies and use a simple parametrization that includes only a minimum of assumptions [28]. The electronic transition frequency is expressed as

$$f = \text{const} \cdot Ry \cdot F(\alpha) \qquad (9)$$

where $Ry = m_e e^4/(8\epsilon_0 h^3) \simeq 3.2898 \cdot 10^{15}$ Hz is the Rydberg frequency, appearing as the common scaling factor. $F(\alpha)$ is a dimensionless function of α that takes the relativistic contributions to the level energies into account. The numerical constant in front depends only on integer or half-integer quantum numbers characterizing the atomic structure and is independent of time. The relative temporal derivative of the frequency f can be written as:

$$\frac{d \ln f}{dt} = \frac{d \ln Ry}{dt} + A \cdot \frac{d \ln \alpha}{dt} \quad (10)$$

with

$$A \equiv \frac{d \ln F}{d \ln \alpha}. \quad (11)$$

The first term $d \ln Ry/dt$ represents a variation that would be common to all measurements of electronic transition frequencies: a change of the numerical value of the Rydberg frequency. The second term in equation 10 is specific for the atomic transition under study. The sensitivity factor A for small changes of α has been calculated for the relevant transitions from relativistic Hartree-Fock calculations [37,46].

The most stringent precision data from absolute frequency measurements of optical atomic transitions are listed in table 3. Those for transitions in Yb^+, Hg^+ have been obtained with single-ion clocks like described in section 5 over the years 2000 to 2008 at PTB and NIST, respectively. The value for Sr is from optical lattice clocks in laboratories in Boulder, Paris and Tokyo and covers the years 2005 to 2008. From the sensitivity factors A it can be seen that the Yb^+ and Hg^+ transition frequencies of the $S \to D$ electrical quadrupole lines would react strongly and with opposite sign to a change of α, whereas H and Sr provide "anchor" lines with small relativistic influence.

In a recent experiment it has been possible to investigate an α-variation without involving a cesium clock: A direct optical frequency ratio measurement of two transitions in Al^+ and Hg^+ was performed at NIST [47]. The Al^+ clock is based on the ${}^1S_0 \to {}^3P_0$ transition frequency that can be realized with very small systematic uncertainty and that provides an anchor line with $A \approx 0$. A frequency comb generator (see section 5) was used to measure the frequency ratio of two lasers stabilized to the ions' transitions. Because of the low systematic uncertainty, the presently most stringent constraint on a change of α was obtained over a time span of only one year [47]:

$$\frac{d \ln \alpha}{dt} = \left(-1.6 \pm 2.3\right) \times 10^{-17}\ \mathrm{yr}^{-1}. \quad (12)$$

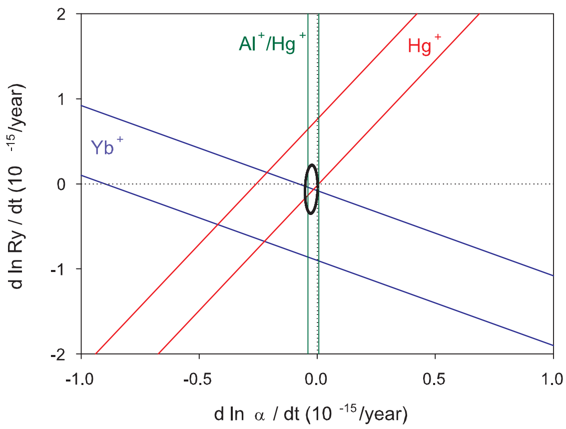

Figure 4. Constraints on $d \ln \alpha/dt$ and $d \ln Ry/dt$ from experiments with single-ion optical clocks. Shown are the individual contributions and the standard uncertainty ellipse of the combined data available at the end of 2008

This constraint obtained from the Al^+/Hg^+ frequency ratio is more stringent than the evaluation of the results of absolute frequency measurements summarized in table 3. Using all the available data in a common least-squares adjustment, a constraint on the variation of the Rydberg constant can be obtained from a combination of equations (10) and (12)

$$\frac{d \ln Ry}{dt} = \left(-0.6 \pm 2.9\right) \times 10^{-16}\ \mathrm{yr}^{-1}\,. \quad (13)$$

Since the Rydberg constant is dimensional, the above mentioned complication arises that this statement refers to a quantity in the SI unit hertz and therefore carries some influence from cesium hyperfine structure, i.e. from the strong interaction. It expresses the limit on a possible common drift of all optical frequencies with respect to the cesium clock.

Table 3
Experimental limits on temporal variations of optical atomic transition frequencies. $d\ln f/dt$ is the observed rate of change in fractional units, typically observed over 3-8 years. A is the sensitivity factor (see equation 11).

Atom, transition	$d\ln f/dt$	Reference	A
^{1}H, $1S \to 2S$	$(-3.2 \pm 6.3) \times 10^{-15}$ yr^{-1}	[48]	0.0
^{171}Yb$^+$, $^2S_{1/2} \to {}^2D_{3/2}$	$(-0.49 \pm 0.41) \times 10^{-15}$ yr^{-1}	[49,50]	1.0
^{199}Hg$^+$, $^2S_{1/2} \to {}^2D_{5/2}$	$(0.37 \pm 0.39) \times 10^{-15}$ yr^{-1}	[51]	-2.9
^{87}Sr, $^1S_0 \to {}^3P_0$	$(-1.0 \pm 1.8) \times 10^{-15}$ yr^{-1}	[52]	0.06

The current combined constraints on variations of α and Ry as obtained from the single-ion optical clocks is plotted in figure 4.

The results from optical clocks indicate that to within an uncertainty of about 3 parts in 10^{17} per year, the fine structure constant *is constant* in the present epoch. This result can be qualified as model-independent because it is only the simple expression equation (10) and the *ab initio* atomic structure calculations of the A-values that have been used in the interpretation of the experimental data. The laboratory experiments have now reached a higher sensitivity for $d\ln\alpha/dt$ than the analysis of quasar absorption spectra, if a linear time evolution of α is assumed. It has to be kept in mind, however, that both approaches look for variations of α in very different cosmological times and regions of space.

From other comparisons between atomic and molecular frequency standards it is possible to derive a limit on the variation of m_p/m_e: A very precise result has been obtained on the ratio of ground state hyperfine frequencies in ^{87}Rb and ^{133}Cs [53,54]: $d\ln(f_{\mathrm{Cs}}/f_{\mathrm{Rb}})/dt = (0.05 \pm 0.53) \cdot 10^{-15}$ yr^{-1}. This measurement was done by comparing atomic fountains (see section 5) working with laser-cooled cesium and rubidium atoms. Since this frequency ratio involves the nuclear magnetic moments, the limit on its change can be combined with the result on the constancy of α to derive a limit on a variation of m_p/m_e using models for the nuclear structure that relate the magnetic moment of the nucleus to the g-factor and mass of the proton [55]. A model-independent result on m_p/m_e can be sought from molecular spectroscopy: the limit

$$\frac{d\ln(m_p/m_e)}{dt} = (-3.8 \pm 5.6) \cdot 10^{-14}\ \mathrm{yr}^{-1} \qquad (14)$$

has been obtained from the measurement of a vibration-rotation transition frequency in SF_6 [56]. Precision spectroscopy of molecules does not yet reach the same level of precision as atomic spectroscopy, because molecules are not directly amenable for laser cooling.

The laboratory search for variations of constants will become even more sensitive, as the precision of frequency standards continues to improve. In addition, a larger variety of systems is now being investigated so that it will be possible to perform tests of the consistency and to disentangle different contributions if a first observation of a variation in the laboratory should be reported. A very promising system in terms of sensitivity is a nuclear optical clock based on a transition at about 7.6 eV in ^{229}Th that is studied in our laboratory [57]. Since the appearance of such a low energy interval in a nucleus can be seen to arise from an accidental cancelation of much larger energy terms (similar to the reasoning applied in the case of the ^{149}Sm resonance in the Oklo reactor), the precise measurement of this transition energy will provide an extremely sensitive probe to look for changes in the electromagnetic and strong coupling [58,59].

9. CONCLUSION

In conclusion, let us summarize the main statements from these lectures:

Work is under way to establish a more precise and consistent system of units based on fixed values of fundamental constants: c, h, e, k_B and N_A. The present focus of the experimental efforts and discussions is on redefinitions of the kilogram, the kelvin and the electrical units. The metrology of time and frequency relies on atomic clocks using suitably chosen transition frequencies that can be measured with high precision. Trapping and cooling of ions and atoms allows to build optical clocks with a relative uncertainty approaching 10^{-18}. This accuracy may be used to test some of the foundations of physics like in the search for variations of fundamental constants. In this field, astrophysical observations looking for changes of the fine structure constant are analyzed at an uncertainty $\Delta\alpha/\alpha \approx 10^{-5}$ over 10^{10} years. The conclusions on constancy or variation of α at this scale are still subject to debate. Laboratory experiments with atomic clocks now reach a sensitivity in $|d \ln \alpha/dt|$ below 10^{-16} yr^{-1} over a few years and are consistent with a constant α. A sensitivity below 10^{-20} yr^{-1} seems accessible in selected systems in the near future.

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