Majorana Transitions in an Atomic Fountain Clock

Robert Wynands, Roland Schröder, and Stefan Weyers

***Abstract*—Recently, we observed quasi-periodic frequency oscillations in CSF1, which is the first cesium fountain clock at Physikalisch-Technische Bundesanstalt, Braunschweig, Germany. These oscillations could be traced back to Majorana transitions after the second Ramsey interaction. A change in field configuration eliminated the Majorana transitions and with it the frequency oscillations.**

***Index Terms*—Atomic fountain clock, frequency bias, magnetometry, Majorana transition.**

I. Introduction

Recently, quasi-periodic frequency oscillations were observed in CSF1, a cesium fountain clock at Physikalisch-Technische Bundesanstalt [1], [2]. When polarization oscillations were introduced for the laser light in the detection zone (with periods of hours), and when the state selection cavity was not used, the effect could be greatly enhanced—with frequency oscillation amplitudes of the order of 10^{-13} (Fig. 1).

An immediate suspect were Majorana transitions. They can occur when the magnetic field seen by a particle possessing a magnetic moment changes its orientation rapidly compared to the Larmor precession frequency. In a multistep process, this can lead to a frequency error in a fountain clock [3]. Indeed, it was found that at one point along the atomic trajectory, such a rapid field change was encountered by the atoms.

II. Experimental Evidence

In a fountain clock, a laser-cooled atom cloud is launched vertically and falls back. At the beginning and the end of the ballistic flight (duration of $\approx$1 s), the cloud interacts with a microwave field. Detuning of the microwave frequency leads to Ramsey interference fringes on the transition probability with subhertz linewidth. An additional microwave field helps to prepare the atoms initially in a single hyperfine state. A fountain clock can reproduce the length of the International System of Units second to within better than one part in 10^{15}. For more details on the principle and state-of-the-art of fountain clocks, see the recent review [4].

The data in Fig. 1 were taken without the state selection cavity in use and for an intentional small misalignment of the linear input polarizations with respect to the easy axis of the optical fibers bringing the laser light to the detection

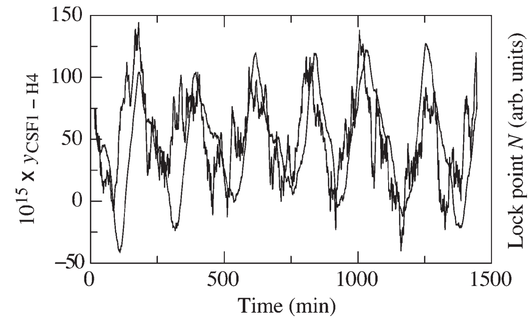

Fig. 1. Correlated oscillations of the fractional output frequency y of CSF1 relative to that of hydrogen maser H4 (noisy curve, left axis) and of the lock point $N = N_4/(N_3 + N_4)$ (smooth curve, right axis). N_4 and N_3 are proportional to the number of atoms detected arriving in states $|F = 4\rangle$ and $|F = 3\rangle$, respectively.

zone. This misalignment leads to polarization changes at the output of the fiber whenever the fiber temperature changes. As explained below, this polarization fluctuation with a period of hours modulates the frequency-shifting effect of Majorana transitions and thus makes the frequency shift easier to detect and characterize.

In order to determine whether Majorana transitions could occur for the given field structure, the atoms themselves were used as a probe for the field inside the vacuum system. In CSF1, the state selection cavity does not have a cutoff tube at its lower end due to geometrical restrictions. Part of the field inside the cavity therefore leaks out into the rest of the apparatus. By coupling a sufficiently strong microwave pulse into the state selection cavity at a time when the atoms are at a height z_1, one can determine the magnetic field at z_1 by monitoring the resonance frequency of the field-sensitive transition $|F = 3, m = -1\rangle \rightarrow |F = 4, m = -1\rangle$. This is similar to the technique used, for example, on FO1 at LNE-SYRTE, where a small antenna was used to provide the microwave field pulse for a measurement of the field above the Ramsey cavity [5].

In this way, we were able to measure the magnetic field all the way down to the detection zone (Fig. 2). A cursory glance at the measured curve does not arouse suspicion regarding the presence of a field zero. However, what is measured by following the resonance frequency of the $|3, -1\rangle \rightarrow |4, -1\rangle$ transition is the absolute value $|B|$ of the magnetic flux density, which is averaged over a spatial region corresponding to the size of the atom cloud (a few millimeters). In fact, as corroborated by qualitative calculations of the magnetic field using the Radia add-on[1] for the Mathematica computation environment, there can be a zero of B_z hiding in the minimum, as indicated

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[1]Radia can be downloaded for free from the website of the European Synchrotron Radiation Facility, http://www.esrf.fr/Accelerators/Groups/InsertionDevices/Software/Radia.

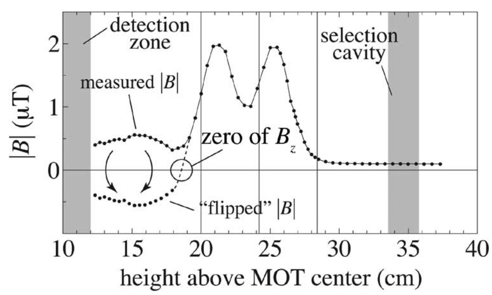

Fig. 2. Measured $|B|$ along the trajectory of the atoms in the original field configuration. The fine vertical lines correspond to the positions of the bottom covers of the three magnetic shield layers. The shaded areas indicate the spatial extent of the detection zone and the state selection cavity, respectively. MOT: Magnetooptic trap.

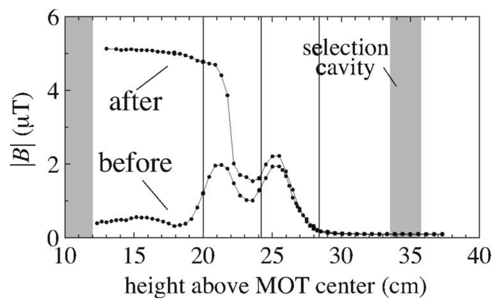

Fig. 3. Measured $|B|$ along the trajectory of the atoms. After adjustment of the coil currents, a field zero no longer occurs.

in the figure. In this region, the horizontal components B_x and B_y are very small due to symmetry, so that Majorana transitions can occur.

Such a zero crossing had been carefully avoided during setup of CSF1. Unavoidable compromises in the construction of the magnetic shields, however, led to a strong sensitivity of the field near the shield covers to mechanical stress in that area, so that over the course of time, a field zero could develop along the atomic trajectory. Unfortunately, this did not show up in the usual "health checks" routinely performed on CSF1.

We have now corrected the field structure by adjusting the current in the coils for the compensation of the geostatic field so that no more Majorana transitions can happen there (Fig. 3). In addition, new steps were added to the list of regular "health check" measures so that no harmful field change can remain undetected in the future. As a result, there are no more correlated oscillations of lock point and output frequency. By switching back and forth between the old and new currents in the field coils, we can reproducibly switch between the presence and absence of frequency oscillations.

In the following, we will give a semi-quantitative explanation of the frequency-shifting effect of a Majorana transition after the second Ramsey interaction in a fountain clock. More details can be found in [3].

III. Majorana Transitions

Majorana transitions can occur when the magnetic field seen by a particle possessing a magnetic moment changes its orientation rapidly compared to the Larmor precession frequency, for instance, near a zero crossing of the magnetic field. The

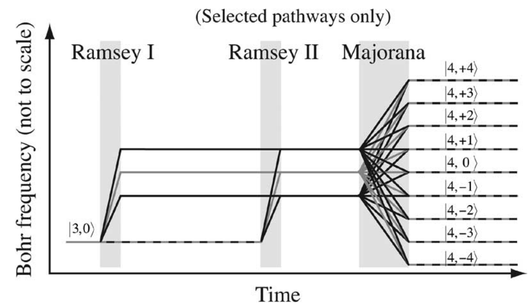

Fig. 4. Visualization of the effect of Majorana transitions, taking place after the second Ramsey interaction, on the final atomic state in a fountain clock with state selection (see text for explanation).

original treatment by Majorana [6] was rephrased and extended by Bloch and Rabi [7]. One of Majorana's results was that the dynamics of the spin orientation can be replaced by a sudden rotation in space by Euler angles determined by the magnitude of the magnetic moment and the field geometry only.

In modern notation, one finds that for an atom initially in a pure state $\Psi_i = |F, m\rangle$, the final state Ψ_f after the Majorana transition is a coherent superposition given by an active rotation of Ψ_i

$$\Psi_f = D^{(F)}(\phi,\alpha,\theta)\Psi_i = \sum_{m'} D^{(F)}_{m',m}|F,m'\rangle \quad (1)$$

$$= \sum_{m'} d^{(F)}_{m',m}(\alpha)\exp(-im\phi - im'\theta)|F,m'\rangle. \quad (2)$$

Here, $d^{(F)}_{m',m}(\alpha)$ are the matrix elements of the rotation operator $d^{(F)}$ for angular momentum $\hbar F$, ϕ and θ are the Euler rotation angles around the initial and the final z axes, respectively, and α is the Majorana angle (rotation angle around the intermediate y-axis).

As in a fountain, where the atoms are quasi-monokinetic (typically $\Delta v/v \approx 0.3\%$ when the atoms are launched), we shall restrict the following discussion to a single velocity, in contrast to the treatment of [8] for a thermal beam clock. One important consequence of the small velocity spread in a fountain is that the frequency-shifting effects are much less washed out compared to the situation in thermal beam clocks. In fact, in those clocks, the most significant features were found for the case of a so-called double zero, where the z component of the magnetic field showed two zeros within a short distance, so that the effects did not wash out [8].

We will consider here the situation of a fountain clock with a state selection cavity (Fig. 4). Following the state selection process, all atoms are in state $|F=3, m=0\rangle =: |3,0\rangle$. This means that any coherences induced by Majorana transitions before the atoms reach the state selection cavity are quenched—no frequency bias is introduced. In the case of CSF1, both cavities are located already in the region of the very homogeneous C-field, so Majorana transitions cannot occur until after the atoms have fallen back through the state selection cavity.

In the course of the Ramsey interaction, the atoms undergo transitions to $|4,0\rangle$, induced by the oscillating magnetic field $\vec{B}_{\mu W}$ inside the Ramsey cavity. This field is nominally parallel to the direction of the C-field (i.e., in the vertical direction) so that selection rules in the ideal case would allow only the clock transition $|3,0\rangle \rightarrow |4,0\rangle$ to occur (π transition, $\Delta m = 0$, dark grey lines in Fig. 4).

However, in a real TE_{011} cavity, the direction of $\vec{B}_{\mu W}$ is truly vertical only along the symmetry axis and on the central horizontal plane. At all other points, there is some component in the horizontal direction. This σ-polarized part of the oscillating magnetic field can induce σ transitions ($\Delta m = \pm 1$, black lines in Fig. 4), leading to a small admixture of the $|4,\pm 1\rangle$ states.

After the second Ramsey interaction, the atomic state is therefore a superposition of three main components

$$|\psi_R\rangle = \beta_{+1}R_{+1}|4,1\rangle + \beta_0 R_0|4,0\rangle + \beta_{-1}R_{-1}|4,-1\rangle. \quad (3)$$

Here, for the relative transition amplitudes β, one has $|\beta_0| \gg |\beta_{\pm 1}|$, and from symmetry, $|\beta_{+1}| = |\beta_{-1}|$. $|R_j|^2$ ($j = -1, 0, 1$) stands for a Ramsey fringe pattern centered on a microwave frequency $f_0 + (j/2)(f_Z - f_0)$, where f_0 is the resonance frequency of the "clock transition" $|3,0\rangle \rightarrow |4,0\rangle$, and f_Z is the resonance frequency of the field-sensitive $|3,1\rangle \rightarrow |4,1\rangle$ transition.

A Majorana transition corresponds to a quantum-mechanical rotation of the state. This means that, in general, each state $|4,m\rangle$ will be transformed into a coherent superposition of all states $|4,m'\rangle$ according to the elements $D^{(4)}_{m',m}$ of the quantum mechanical rotation matrix for spin 4 (Fig. 4)

$$\begin{aligned}|\psi_M\rangle = &\beta_{+1}R_{+1}\left[D_{+1,+1}|4,1\rangle + D_{0,+1}|4,0\rangle + D_{-1,+1}|4,-1\rangle\right.\\ &\left.+ \text{analogous terms with all other } |4,m'\rangle\right] \quad (4)\\ &+ \beta_0 R_0\left[D_{+1,0}|4,1\rangle + D_{0,0}|4,0\rangle + D_{-1,0}|4,-1\rangle\right.\\ &\left.+ \text{analogous terms with all other } |4,m'\rangle\right] \quad (5)\\ &+ \beta_{-1}R_{-1}\left[D_{+1,-1}|4,1\rangle\right.\\ &+ D_{0,-1}|4,0\rangle + D_{-1,-1}|4,-1\rangle\\ &\left.+ \text{analogous terms with all other } |4,m'\rangle\right]. \quad (6)\end{aligned}$$

Here and in the following, we write $D_{m',m}$ instead of $D^{(4)}_{m',m}$. In the detection zone, a projection of $|\psi_M\rangle$ onto the eigenstates $|4,m'\rangle$ is performed, resulting in a detected signal proportional to

$$\begin{aligned}&\cdots + w_{+2}|\beta_{+1}R_{+1}D_{+2,+1} + \beta_0 R_0 D_{+2,0} + \beta_{-1}R_{-1}D_{+2,-1}|^2 \quad (7)\\ &+ w_{+1}|\beta_{+1}R_{+1}D_{+1,+1} + \beta_0 R_0 D_{+1,0} + \beta_{-1}R_{-1}D_{+1,-1}|^2 \quad (8)\\ &+ w_0|\beta_{+1}R_{+1}D_{0,+1} + \beta_0 R_0 D_{0,0} + \beta_{-1}R_{-1}D_{0,-1}|^2 \quad (9)\\ &+ w_{-1}|\beta_{+1}R_{+1}D_{-1,+1} + \beta_0 R_0 D_{-1,0} + \beta_{-1}R_{-1}D_{-1,-1}|^2 \quad (10)\\ &+ w_{-2}|\beta_{+1}R_{+1}D_{-2,+1} + \beta_0 R_0 D_{-2,0}\\ &\quad + \beta_{-1}R_{-1}D_{-2,-1}|^2 + \cdots. \quad (11)\end{aligned}$$

Here, $w_{m'}$ indicates a weighting factor proportional to the number of fluorescence photons that are captured from an atom reaching the detection zone in state $|F,m'\rangle$. Each of the terms $\beta_0 R_0 D_{m',0}$ by itself would give rise to an unshifted Ramsey fringe pattern.

In general, the overall "Ramsey" fringe pattern will have a more complicated shape. However, a frequency error will only result when this change in shape is asymmetric with respect to the undisturbed center frequency f_0. Here, we find from the underlying symmetry of the rotation matrix and the transition probability amplitudes β_j that the asymmetries due to certain terms cancel each other.

For instance, the sum $\beta_{+1}R_{+1}D_{0,+1} + \beta_{-1}R_{-1}D_{0,-1}$ in (9) is symmetric, so it does not contribute to a frequency error. The asymmetries contained in the corresponding sum $\beta_{+1}R_{+1}D_{+1,+1} + \beta_{-1}R_{-1}D_{+1,-1}$ in (8) do not cancel each other, but closer inspection shows that they cancel the asymmetries of the sum $\beta_{+1}R_{+1}D_{-1,+1} + \beta_{-1}R_{-1}D_{-1,-1}$ in (10) in the overall interference pattern. The same pattern holds for the asymmetries in (7) and (11), and so on.

This second type of cancellation, however, happens only if $w_{-m'} = w_{m'}$. This is in general not the case due to various optical pumping processes in the detection zone. Typically, the laser for the detection of $|F=4\rangle$ atoms is circularly polarized. An atom arriving, for example, in state $|4,-2\rangle$ needs more absorption–emission cycles that an atom initially in state $|4,+2\rangle$ to reach the strong cycling transition starting from $|4,+4\rangle$, and also has a higher probability of getting hyperfine pumped into the $|F=3\rangle$ ground state. Therefore, it scatters less photons overall, and $w_{-2} < w_{+2}$.

The actual values of $w_{m'}$ depend on the laser polarizations in the detection zone. This is why a slight misalignment of the laser polarization at the input end of the fiber can influence the occurrence and magnitude of Majorana-induced frequency shifts in such a dramatic way.

IV. Conclusion

With our new technique of using the atom cloud as a probe of the magnetic flux density inside the beam tube even in the area below the Ramsey cavity down to the detection zone, one can detect the presence of field zeros in the vicinity of magnetic shield covers. In a three-step process, such zeros can lead to a frequency error induced by Majorana transitions. This can be understood from a physical model making use of the symmetry of the processes.

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