Effects of microwave leakage in caesium clocks: theoretical and experimental results

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We investigated the effects of microwave leakage fields in an atomic fountain clock. In a series of experiments a test microwave signal was beamed into the vacuum system with controlled amplitude, phase, and timing. The observed frequency shifts can be explained by a theoretical model based on an iterative solution of the Bloch equations up to second order. This model applies to both caesium fountain clocks and conventional thermal beam clocks and provides new and essential insights into the problem of microwave leakage fields.

I. INTRODUCTION

In primary caesium atomic clocks caesium atoms are exposed to phase coherent microwave fields in two well-defined cavity regions (Ramsey configuration) in order to effect transitions between the two hyperfine ground state levels. Unless proper design measures were taken, additional microwave fields can escape from the microwave components of a clock (microwave synthesis setup, cables and the cavity regions) and leak into the vacuum regions outside the cavity. There the presence of such additional microwave fields results in extra rotations of the atoms' magnetic moment and thus can lead to a frequency shift that depends on amplitude and phase of those leakage fields.

Such frequency shifts were already observed in the thermal beam clock CSX at PTB and corrective steps were taken [1]. Later on, several experiments were performed in thermal-beam clocks, by increasing the microwave power inside the Ramsey cavities and looking for corresponding frequency shifts [2–4]. Using a microwave signal that was intentionally beamed into the vacuum system, several groups found an increase of the frequency shift in thermal beam clocks with the amplitude of the microwave test signal [5–7].

For special, idealized cases theoretical considerations were made. For instance, Boussert *et al.* [8] found a rather complicated power dependence for a running-wave leakage field between the two Ramsey zones in a thermal-beam clock, with the first-order term proportional to microwave amplitude and a second-order term proportional to microwave power. Other work [9] found a linear dependence on the amplitude of an additional standing-wave microwave field between the two Ramsey zones in a thermal-beam clock and a more complicated, phase-dependent shift when the Ramsey pulse area (Rabi frequency integrated over time) is changed synchronously with the leakage amplitude.

In atomic fountain clocks it has become a standard approach to check for the presence of microwave leakage fields by running the fountain clock with pulse areas of odd multiples of $\pi/2$. Such modes of operation cannot be realized in thermal beam clocks due to the broad range of velocities present in these clocks, which results in a destructive interference of the resonance features for different velocities. Jefferts *et al.* have treated the leakage problem in fountain clocks theoretically by solving the Schrödinger equation in a first-order approximation [10, 11]. They point out that frequency shifts are generally proportional to the microwave leakage amplitude.

Recently, frequency shifts observed in a primary caesium fountain standard were ascribed to additional microwave fields outside the Ramsey cavity [12, 13]. However, the quadratic dependence of the frequency shift on the microwave leakage field amplitude assumed in [12, 13] is in contradiction to the results of the earlier works.

We have chosen to study experimentally the shift of the output frequency of PTB's fountain clock CSF1 [14, 15] as a function of amplitude and phase of a test "leakage" field that we beam into the apparatus. This test field was time controlled by a PIN modulator, so that we were able to study the effect of leakage fields either after the Ramsey interactions or between the Ramsey interactions, and here for the two cases of a strongly asymmetric or a rather symmetric time structure of the leakage field with respect to the apogee of the ballistic flight of the atoms.

The experimental findings can be explained by our theoretical approach, which consists in the iterative solution of the Bloch equations up to the second order, distinguishing between the cases of leakage fields present between the Ramsey interactions and after the Ramsey interactions. Naturally, our first-order solutions come out identical to that given in Ref. [11]. For leakage fields present between the two Ramsey interactions, the first-order term vanishes in the cases of the optimum power condition, i.e., in the cases of pulse areas which are precisely odd multiples of $\pi/2$. In these cases the leading frequency shift contribution is by the second-order term, which exhibits a quadratic frequency shift dependence on the leakage field amplitude. A corresponding term for beam clocks was already calculated by Boussert *et al.* [8].

We could demonstrate in our fountain clock that in practice this quadratic frequency shift dependence

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will be undiscernible. It requires an unrealistically precise and stable adjustment of the optimum power condition and a strongly asymmetric leakage field for the ascending and descending atoms. Nevertheless, we could measure this quadratic frequency shift, but only by running the experiments under very special conditions not found in normal fountain operation.

However, we discovered another important effect for the case of leakage fields acting between the Ramsey interactions. The usual expectation is that all terms (first and higher order) vanish if the leakage field experienced by the atoms is symmetric with respect to the first and the second half of the free-evolution period between the Ramsey interactions. We could demonstrate that this is only true as long as the amplitudes of the microwave field in the Ramsey cavity seen by the atoms during the first and the second cavity passage, respectively, are equal.

As explained below, this is usually not the case in a real-world fountain clock. As a consequence, even in the "symmetric" leakage case a frequency shift results. Furthermore, this potentially large first-order frequency shift can depend quadratically on the leakage field amplitude.

II. THEORY

In our fountain clock CSF1 the state selection cavity is located immediately below the cut-off tube of the Ramsey cavity. Therefore leakage fields cannot interact with the atoms during the time interval between the state selection and the first Ramsey passage. We will therefore concentrate in the following on the cases of leakage fields either between or after the Ramsey cavity passages. By the way, leakage fields before the first Ramsey passage can be treated in an analog manner as leakage fields after the Ramsey cavity passages [8, 11].

We have solved the Bloch equations iteratively in the presence of time-dependent leakage fields with small amplitudes. We consider in the rotating-wave approximation an oscillating magnetic leakage field $\vec{B}(t) = \vec{B}_0 \cos(\omega t - \phi)$, where ϕ is the phase relative to the Ramsey cavity field. The generalized Rabi vector $\vec{\Omega}_{\text{eff}}(t)$ contains the in-phase (relative to the Ramsey cavity field) and quadrature components of the Rabi frequency of the microwave field $\vec{B}(t)$ present during the respective stage of the interaction. They are given by the real part $\Omega_r(t)$ and the imaginary part $\Omega_i(t)$ of $\vec{\Omega}_{\text{eff}}(t)$. For a two-level system the Bloch vector $\vec{b}(t)$ precesses around $\vec{\Omega}_{\text{eff}}(t) = (\Omega_r(t), -\Omega_i(t), -\delta)$, where $\delta = \omega - \omega_0$ is the detuning of the angular frequency ω of the electromagnetic field from the atomic transition frequency ω_0. The Bloch equations are correspondingly given by

$$\dot{\vec{b}}(t) = \vec{\Omega}_{\text{eff}}(t) \times \vec{b}(t). \tag{1}$$

For the following discussions, without leakage fields $\vec{\Omega}_{\text{eff}}$ can be considered as time independent during the interactions with the Ramsey cavity field (each of duration τ), the free-evolution period between the two Ramsey interactions (duration T) and the free-evolution period between the second Ramsey interaction and the detection (duration T_d). In the first case it is given by

$$\vec{\Omega}_{\text{eff}} = (\Omega_R, 0, -\delta) \approx (\Omega_R, 0, 0), \tag{2}$$

where $\Omega_R \gg \delta$ is the Rabi frequency corresponding to the interaction in the Ramsey cavity and where at first we do not distinguish between possible different Rabi frequencies for the ascending and for the descending atoms. For the free-evolution periods $\vec{\Omega}_{\text{eff}}$ is given by

$$\vec{\Omega}_{\text{eff}} = (0, 0, -\delta). \tag{3}$$

The precession angle α of the Bloch vector is thus obtained from $\alpha = |\vec{\Omega}_{\text{eff}}|\, t_i$ with the respective interaction time t_i equal either to τ, T, or T_d.

Before the first Ramsey interaction the caesium atoms are prepared by a state selection process [16] in a hyperfine substate $|F = 3, m_F = 0\rangle$, here represented by the Bloch vector $\vec{b} = (0, 0, -1)$. Using Eq. (1) and Eq. (2), one finds that after the first Ramsey interaction the atom is in a coherent superposition of the states $|F = 3, m_F = 0\rangle$ and $|F = 4, m_F = 0\rangle$, the latter represented by the Bloch vector $\vec{b} = (0, 0, +1)$. This is because in normal operation the power in the Ramsey cavity is adjusted such that the precession angle due to a single cavity passage is given by $\alpha = \pi/2$ ($\pi/2$-pulse). However, for monokinetic atoms experiencing all the same microwave amplitude any odd multiple $\pi/2$-pulses work, as well. The probability of finding the atom in the states $|F = 3, m_F = 0\rangle$ or $|F = 4, m_F = 0\rangle$ is given by the simple relations $P(|F = 3, m_F = 0\rangle) = 1/2(1 - b_z)$ and $P(|F = 4, m_F = 0\rangle) = 1/2(1 + b_z)$, respectively, where b_z is the z-component of the Bloch vector $\vec{b}(t)$.

During clock operation usually the central Ramsey fringe is alternately probed at its full-width-at-half-maximum (FWHM) points [16], which corresponds to alternately detuning the Ramsey cavity field by $\pm\delta_{\text{FWHM}} = \pm\pi/(2T)$. That means that during the free-evolution period the Bloch vector evolves again according to Eq. (1), but now in combination with Eq. (3) and $\delta = \pm\delta_{\text{FWHM}}$ for the two sides of the central fringe. After the second Ramsey interaction, analogous to the first one, and another subsequent free evolution, the probabilities of finding the atoms in either state are detected. Therefore the detection process corresponds to a measurement of the final z-component of the Bloch vector $b_z(\pm\delta)$ for either detuning. The servo loop of the clock ensures that the final $\Delta b_z = b_z(+\delta) - b_z(-\delta)$ is kept at zero by adjusting the frequency of the probing microwave field.

Any process provoking $\Delta b_z \neq 0$ under open-loop conditions results in a frequency shift proportional to Δb_z when the loop is closed. In fact, from the open-loop Δb_z the resulting frequency shift in the closed-loop operation can be directly calculated by using the slope of the central Ramsey fringe. For a typical fringe width of 1 Hz, one can calculate the relative frequency shift as $\Delta\nu/\nu_0 \approx 2 \cdot 10^{-11} \Delta b_z$, with the caesium hyperfine transition frequency $\nu_0 = 9\,192\,631\,770$ Hz.

In order to take into account the effects of leakage fields present during the free-evolution periods between or after the Ramsey interactions, we used for a first-order solution the unperturbed solutions from above together with the generalized Rabi vector $\vec{\Omega}_{\text{eff}}(t) = (\Omega_r^L(t), -\Omega_i^L(t), 0)$, now defined by in-phase and quadrature components $\Omega_r^L(t)$ and $\Omega_i^L(t)$ corresponding to the leakage field, as input for the Bloch equations (Eq. (1)). For our calculations we have assumed small leakage field amplitudes, i.e. $\Omega_r^L(t), \Omega_i^L(t) \ll \delta$. In a second step, the first-order solutions obtained in this way can be used as $\vec{b}(t)$ in Eq. (1) to obtain second-order solutions.

As a result we obtain for leakage fields between the Ramsey interactions, including first and second-order terms:

$$\begin{aligned}
\Delta b_z \;=\;& \sin(2\,\Omega_R\,\tau)\cos\frac{\delta T}{2}\int_0^T \Omega_i^L(t)\;\sin\delta(t-\frac{T}{2})\;dt \\
&+\;2\,\cos^2(\Omega_R\,\tau)\int_0^T\left[\Omega_i^L(t)\int_0^t \Omega_r^L(t')\;\sin\delta(t-t')\;dt' - \Omega_r^L(t)\int_0^t \Omega_i^L(t')\;\sin\delta(t-t')\;dt'\right]dt \qquad (4)\\
&+\;2\,\sin^2(\Omega_R\,\tau)\int_0^T\left[\Omega_i^L(t)\;\sin\delta(T-t)\int_0^t \Omega_r^L(t')\;\cos\delta t'\;dt' - \Omega_r^L(t)\;\cos\delta(T-t)\int_0^t \Omega_i^L(t')\;\sin\delta t'\;dt'\right]dt,
\end{aligned}$$

where the first term, representing the first-order solution, is in agreement with the solution found in Ref. [11]. This term vanishes due to the $\sin(2\,\Omega_R\,\tau)$ factor at optimum Ramsey pulse area settings $\Omega_R\,\tau = (2N+1)\,\pi/2$ with $N = 0, 1, 2, \ldots$, whereas maximum frequency shifts are obtained at Ramsey pulse areas $\Omega_R\,\tau = (2N+1)\,\pi/4$ with alternating sign for increasing N. The first-order term depends only on the quadrature component $\Omega_i^L(t)$ corresponding to the leakage field. This means that any resulting frequency shift is proportional to the leakage field amplitude. It can also easily be seen that as the sine function within the integral of the first-order term is an odd function with respect to $t = T/2$, for an even quadrature component $\Omega_i^L(t)$ the first-order term vanishes.

At optimum Ramsey pulse area settings $\Omega_R\,\tau = (2N+1)\,\pi/2$ with $N = 0, 1, 2, \ldots$ the only remaining term is the third term, obtained from the second-order solution. As it contains only products of the in-phase component $\Omega_r^L(t)$ and the quadrature component $\Omega_i^L(t)$, the frequency shift becomes proportional to the square of the amplitude of the leakage field. Furthermore, it does not change sign with increasing N. But it is important to note that for real clocks it is impossible to adjust the microwave power in the Ramsey cavity to much better than about 1%. Due to this fact, under normal circumstances the first-order term is dominating, unless the leakage field amplitude becomes rather large.

If leakage fields are present during one of the free-evolution periods, their most realistic and general characterization is that of a field which is variable in amplitude and phase along the atomic trajectories, depending in detail on the particular properties of the respective vacuum chamber. We will restrict the following discussions to the simple case of standing waves, consisting of two counterpropagating plane waves with equal amplitudes and polarizations. We have chosen this case because we have evidence of a strong standing wave character of our experimental test field inside CSF1. Our calculated results for traveling waves are qualitatively similar to those for standing waves and related details will be presented elsewhere [17]. For the polarization of the standing wave fields we assume in the following without loss of generality that the magnetic field vector $\vec{B}(t)$ of the leakage fields is always parallel to the constant magnetic field direction along the atomic trajectories.

For a standing wave field the in-phase and quadrature components of the generalized Rabi vector $\vec{\Omega}_{\text{eff}}(t)$ seen by the atoms along their path $x(t)$ are given by

$$\begin{aligned}
\Omega_r^L(t) \;&=\; \Omega^L(t)\cos\phi = \Omega_0^L\cos(kx(t)+\eta)\cos\phi \\
\Omega_i^L(t) \;&=\; \Omega^L(t)\sin\phi = \Omega_0^L\cos(kx(t)+\eta)\sin\phi, \qquad (5)
\end{aligned}$$

where $k = 2\pi/\lambda$ is the wavenumber for wavelength λ, and η describes the spatial position of the standing wave field envelope with respect to the Ramsey cavity. We note that in the case of a standing wave the atoms experience a leakage field with a time varying amplitude $\Omega^L(t) = \Omega_0^L\cos(kx(t)+\eta)$ but a constant phase ϕ.

By inserting Eqs. (5) into Eq. (4) it can be seen immediately that for $\phi = N\pi$ with $N = 0, 1, 2, \ldots$ there is no frequency shift as in this case $\Omega_i^L(t) = 0$. From numerical integration of Eq. (4) we obtain the final Δb_z for a standing wave leakage field present between the two Ramsey interactions. Let us consider the case of an exposure of the atoms to the leakage field starting immediately after the first Ramsey cavity interaction. It is instructive to replace the upper limit T of the integrals of Eq. (4) by a variable t_l, which indicates the time when the exposure of the atoms to the leakage field stops. The resulting $\Delta b_z(t_l)$ is plotted in Fig. 1 for the case of a standing wave leakage field in an atomic fountain.

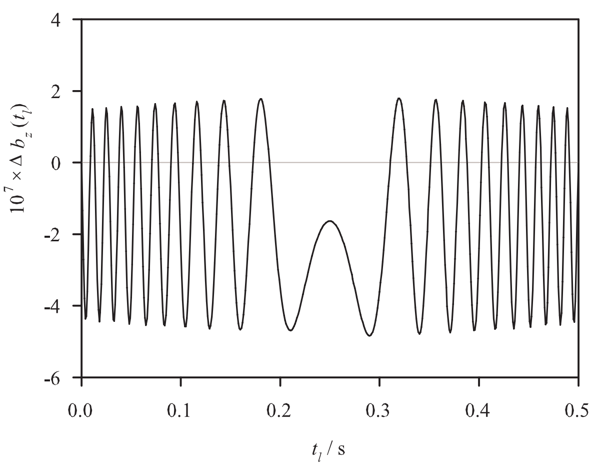

FIG. 1: Final Δb_z as a function of the time t_l (see text). $t_l = 0$ corresponds to the begin of the leakage field interaction just at the end of the first cavity passage and $t_l = 0.5$ s to the end of the leakage field interaction just at the begin of the second cavity passage. The initial velocity of the atoms is $v(t_l = 0) = 2.5$ m/s and the acceleration of gravity is assumed to be $g = 10$ m/s^2, so that $T = 0.5$ s and $\delta = \pi$ (operation at FWHM points). Other parameters have been arbitrarily chosen and do not have any particular significance: $\lambda = 0.0326$ m, $\eta = -\pi/6$, $\phi = \pi/4$, and $\Omega_0^L = 0.01$ rad/s. Depicted is the resulting curve for a realistic situation, where the optimum power is missed by 1%: $\Omega_R\,\tau = 0.99\,\pi/2$.

For $t_l = 0.5$ s we obtain $\Delta b_z = 0$, which is due to the fact that for the calculation of the graph in Fig. 1 we used the properties of the ballistic flight path of atoms in a fountain: in the ideal case the atoms are exposed to the same leakage field during their ascent and their descent. This happens in a symmetric way with respect to the apogee at time $T/2$. On the other hand, from the graph the resulting $\Delta b_z(t_l)$ can be read off when the leakage stops at any other time $0 \le t_l \le 0.5$ s. In this case the leakage field is of course no longer symmetric and consequently in general $\Delta b_z(t_l) \neq 0$ is obtained. But from the zero-crossings of $\Delta b_z(t_l)$ for certain values of t_l we recognize that the asymmetry of a standing wave leakage field is a necessary but not a sufficient condition for frequency shifts to appear.

This result is different from what is obtained if a leakage field with constant amplitude and phase is assumed like in Ref. [11]: in this case we calculate that $|\Delta b_z(t_l)|$ increases monotonically for $0 \le t_l \le 0.25$ s and decreases in a corresponding way for $0.25\text{ s} \le t_l \le 0.5$ s. Moreover, due to the integration of a constant leakage field (Eq. (4)) the maximum value of $|\Delta b_z|$ reached at the apogee is generally much larger compared to the maximum $|\Delta b_z|$ for any t_l in the case of a standing wave or a traveling wave [17]. If we use in the calculation a constant field with parameters analogous to the situation of Fig. 1 ($\Omega_0^L = 0.01$ rad/s, $\phi = \pi/4$), the maximum $|\Delta b_z|$ is about a factor of 40 larger than the maximum $|\Delta b_z|$ obtained from Fig. 1. This means that in general the frequency shift is overestimated if it is calculated based on an un-physical "constant" leakage field, instead of a realistic standing or traveling wave leakage field.

Now we would like to point out another important effect, which at least for fountain clocks may usually be the dominating source of a frequency shift due to leakage fields present between the Ramsey interactions. This effect results from the fact that in general atoms traverse the Ramsey cavity at different distances from the vertical cavity symmetry axis during their first and second Ramsey passages, so that the atoms are exposed to different microwave amplitudes on their way up and on their way down. This is because in typical Ramsey cavities for fountain clocks the microwave amplitude across the cavity apertures varies by several percent. Depicting the Rabi frequencies for the first and the second cavity passage by $\Omega_R(\uparrow)$ and $\Omega_R(\downarrow)$, respectively, we obtain as a result that the angles $\alpha_1 = \Omega_R(\uparrow)\,\tau$ and $\alpha_2 = \Omega_R(\downarrow)\,\tau$ for the ascending and descending atoms can be slightly different. Instead of using Eq. (4) we find for the final Δb_z:

$$\Delta b_z \;=\; 2\Big[\sin(\alpha_1+\alpha_2)\cos\frac{\delta T}{2}\int_0^T \Omega_i^{L,o}(t)\;\sin\delta(t-\frac{T}{2})\;dt \;+\; \sin(\alpha_1-\alpha_2)\sin\frac{\delta T}{2}\int_0^T \Omega_i^{L,e}(t)\;\cos\delta(t-\frac{T}{2})\;dt\Big] + \text{second-order terms}, \qquad (6)$$

where for the first-order term we have split the quadrature component due to the leakage field $\Omega_i^L(t)$ into an odd function of t (with respect to $t = T/2$) $\Omega_i^{L,o}(t)$, and into an even function of t (with respect to $t = T/2$) $\Omega_i^{L,e}(t)$. The second-order terms are identical to those of Eq. (4), with the exception that the factor $\cos^2(\Omega_R \tau)$ has to be replaced by $\cos(\Omega_R(\uparrow)\,\tau)\cos(\Omega_R(\downarrow)\,\tau) = \cos\alpha_1 \cos\alpha_2$ and the factor $\sin^2(\Omega_R \tau)$ has to be replaced analogously.

The first term of Eq. (6) describes the effect of an asymmetric leakage field as discussed before, including now $\alpha_1 \neq \alpha_2$. It can still be made negligible by adjusting the microwave power such that $\alpha_1 + \alpha_2 = (2N+1)\,\pi$ with $N = 0, 1, 2, \ldots$. The second term survives even at optimum power. It describes the frequency shifting effect of the condition $\alpha_1 \neq \alpha_2$ in the presence of a symmetric leakage field between the Ramsey interactions. In contrast to the behavior of this first-order term the second-order terms still vanish for perfectly symmetric leakage fields, even when $\alpha_1 \neq \alpha_2$.

In Fig. 2 the original result of Fig. 1 obtained from Eq. (4) for a standing wave leakage field (Eq. (5)) is compared to the result obtained from Eq. (6), where we have included the effect of $\alpha_1 \neq \alpha_2$. It becomes obvious that in the latter case the frequency shift does not vanish, even when the leakage field is symmetric ($t_l = 0.5$ s). Furthermore, even a very small difference between α_1 and α_2 of 0.5% results in much larger frequency shifts compared to the case $\alpha_1 = \alpha_2$ with asymmetric leakage field ($t_l < 0.5$ s).

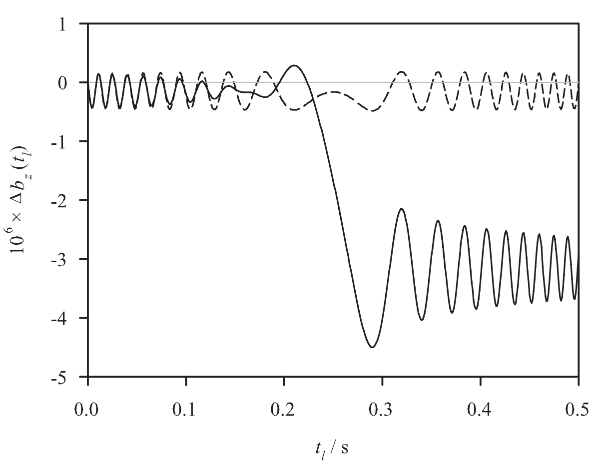

FIG. 2: Final Δb_z as a function of the time t_l. Broken line: Same curve as depicted in Fig. 1 ($\alpha_1 = \alpha_2 = 0.99\,\pi/2$). Full line: Δb_z calculated for $\alpha_1 = 0.99\,\pi/2$ and $\alpha_2 = 0.985\,\pi/2$), but otherwise identical parameters.

When the frequency shift is dominated by the effect due to $\alpha_1 \neq \alpha_2$, the resulting Δb_z for odd multiple $\pi/2$-pulse operation can be written in the explicit form:

$$\begin{aligned}\Delta b_z &= 2\,\sin((2N+1)(\tilde{\alpha}_1 - \tilde{\alpha}_2))\sin\frac{\delta T}{2} \\ &\qquad \times \int_0^T \Omega_i^{L,e}(t)\;\cos\delta(t - \frac{T}{2})\;dt \\ &\approx 2\,(2N+1)(\tilde{\alpha}_1 - \tilde{\alpha}_2)\sin\frac{\delta T}{2} \\ &\qquad \times \int_0^T \Omega_i^{L,e}(t)\;\cos\delta(t - \frac{T}{2})\;dt.\end{aligned} \tag{7}$$

with $N = 0, 1, 2, \ldots$ and $\tilde{\alpha}_1 \approx \tilde{\alpha}_2 \approx \pi/2$. This equation gives the remarkable result that by using odd multiple $\pi/2$-pulse operation modes we obtain a quadratic dependence on the microwave amplitude setting due to the product $(2N+1)\,\Omega_i^{L,e}(t)$, as long as the leakage field amplitude is proportional to the microwave field amplitude in the Ramsey cavity.

Finally, we also give here the calculated first-order solution for arbitrary types of leakage fields and Ramsey pulse area settings close to the optimum ($\alpha_1 \approx \alpha_2 \approx (2N+1)\,\pi/2$ with $N = 0, 1, 2, \ldots$), if leakage fields are present after the second Ramsey interaction ($t = 0$) until the atoms reach the detection region ($t = t_d$):

$$\Delta b_z = 2\,\sin\alpha_1 \sin\delta T \int_0^{t_d} \Omega_i^L(t)\;\cos\delta t\;dt, \tag{8}$$

where we have omitted terms which due to Ramsey pulse area settings close to the optimum only give small contributions compared to the contributions from Eq. (8). Frequency shifts are proportional to the amplitude of the leakage field and alternate in sign for increasing N.

From Eq. (8) we obtain the same result as Levi *et al.* obtained for the special case of a leakage field of short duration after the second Ramsey interaction with amplitude and phase constant in time [11].

III. EXPERIMENTAL SETUP

The PTB caesium fountain CSF1 is described in detail elsewhere [14, 15]. For the experimental microwave leakage investigations we beam a test microwave field either through one of the optical access ports near the detection region or through the upper window for the vertical laser beams (Fig. 3). The electrical part of the experimental setup is relatively simple. The microwave signal produced by the 9-GHz synthesis chain for the Ramsey cavity is split into two paths. One of them supplies the interrogation microwave field for the Ramsey cavity in the usual way. The other path, which provides the test microwave field, consists of an X-band waveguide containing an adjustable attenuator, a variable phase shifter and a

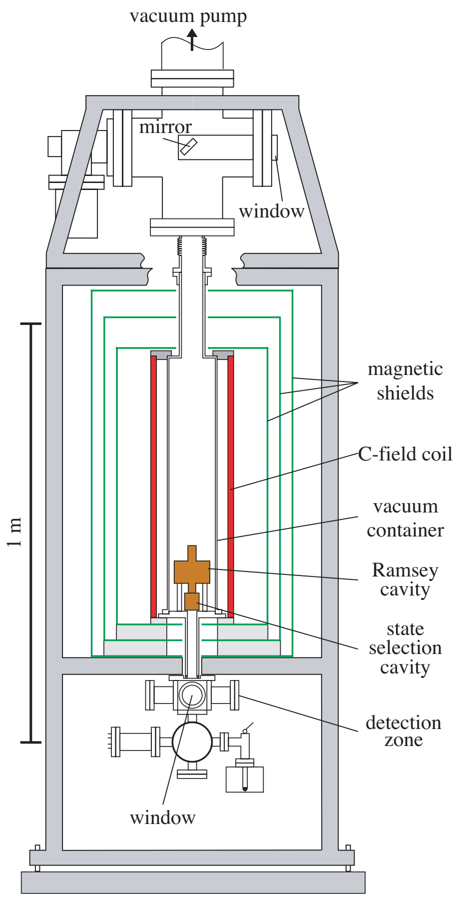

FIG. 3: Geometry of the vacuum subsystem of PTB's CSF1 caesium fountain clock. The horn antenna for the application of a test microwave field was positioned as close as possible to one of the two windows marked in the sketch.

PIN modulator, used as rf switch (extinction 117 dB). The X-band waveguide leads to a cable-coupled microwave horn, which is firmly positioned as close as possible to an unused window at the vacuum cross containing the detection zone, or at the top where one of the vertical laser beams enters the vacuum system (Fig. 3). This setup allows us to set the amplitude and phase of the test microwave field independently of the pulse area in the Ramsey cavity and to switch the test field on and off at well-defined times during the fountain operating cycle, i.e., when the atom cloud is at a specific position along its trajectory.

We note that any values of the phase of our test microwave field given in this publication refer to the reading of the dial of the phase shifter. There is a fixed, but unknown propagation delay between this device and the interior of the fountain. In particular, it would be an improbable coincidence if our "phase 0°" coincided with the "in-phase" situation described in Section II.

In the standard mode of operation of CSF1 the 9-GHz synthesis chain is locked to the center of the Ramsey fringe by controlling the frequency of a 5-MHz quartz oscillator. Its relative frequency is compared to the average relative frequency $y(\mathrm{H})$ of two hydrogen masers using a phase comparator. Depending on the size of the frequency shift an averaging time of 1-5 minutes is chosen in order to clearly resolve the shift within the frequency noise of the fountain at such short integration times. In the plots shown below a difference frequency of zero corresponds to the fountain output frequency in the absence of the test microwave field, while the statistical uncertainty is of the order of the data symbol size.

IV. EXPERIMENTAL RESULTS

In the fountain CSF1 atoms are launched with an initial velocity of 4.04 m/s. They pass through the Ramsey cavity after 130 ms for the first time and after 695 ms for the second time. The upper detection region is reached 795 ms after the launch. In the experiments described in the following, for a fixed amplitude setting the phase of the simulated leakage field was varied over a full cycle. It was checked in an auxiliary experiment that changing the setting of the phase shifter does not cause any noticeable amplitude change of the microwave signal passing through.

In fountain clocks leakage fields present between the first and the second Ramsey passage would be expected to exibit a highly symmetric time structure with respect to $t = T/2$. In our fountain such a leakage field was simulated by beaming a test field through the upper port starting when the atoms pass through the Ramsey cavity for the first time, and stopping when they pass through the Ramsey cavity for the second time. In Fig. 4 the resulting frequency shift is shown for a fixed test field power and for different Ramsey pulse areas.

In each case one observes an oscillating phase dependence that can very well be fitted by a sine function. These experimental results are explained by Eq. (7). From this equation we see that only the symmetric part of the test leakage field, described by $\Omega_i^{L,e}(t)$, contributes, so that the phase dependence is given by $\sin\phi$ (see Eq. (5)). Furthermore, in Fig. 4 one sees that for a fixed phase the frequency shift increases linearly with the pulse area, as expected from Eq. (7). Additional measurements of the frequency shift for a fixed phase and a fixed Ramsey pulse area (not depicted here) revealed that the frequency shifts for leakage fields between the Ramsey interactions are proportional to the leakage field amplitude over several orders of magnitude [17]. The combination of these experimental findings confirms a quadratic dependence of the frequency shift as predicted by Eq. (7), in the case that the leakage field amplitude is proportional to the amplitude in the Ramsey cavity. This is probably the explanation of the observed quadratic frequency shift dependence reported in Refs. [12, 13].

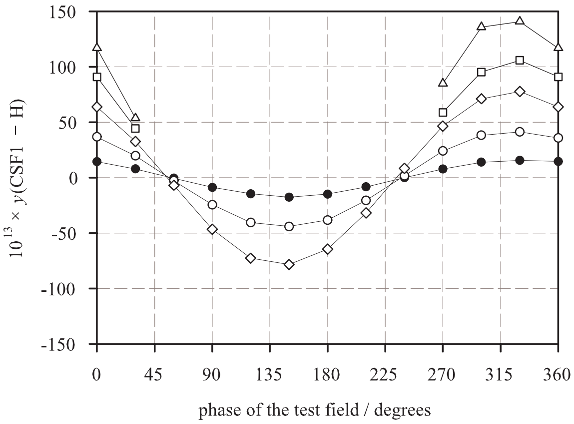

FIG. 4: Phase dependence of the relative frequency shift y(CSF1 − H) due to a microwave test field of fixed amplitude, which is beamed into the apparatus from the upper port during the free-evolution period between the first and the second Ramsey cavity passage of the atoms (full circles: $1\,\pi/2$-pulse, open circles: $3\,\pi/2$-pulse, open diamonds: $5\,\pi/2$-pulse, open squares: $7\,\pi/2$-pulse, open triangles: $9\,\pi/2$-pulse). The solid lines connect the data points to guide the eye.

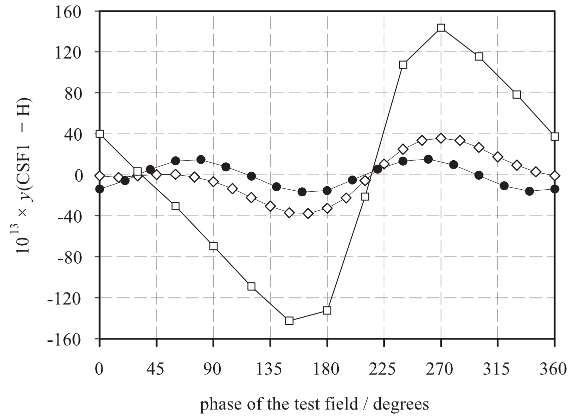

FIG. 5: Phase dependence of the relative frequency shift y(CSF1 − H) due to pulsed microwave radiation, which is beamed into the apparatus from the upper port 220 ms after the launch of the atoms on their way up for a duration of 5 ms (full circles: optimized Ramsey pulse area ($\Omega_R\,\tau \approx \pi/2$), open diamonds: optimized Ramsey pulse area + 0.8 dB, open squares: optimized Ramsey pulse area + 3.5 dB). The solid lines connect the data points to guide the eye.

We should note here that the reason why the approximate solution Eq. (7) is sufficient to describe the measured frequency shifts of Fig. 4 is that other contributions from the more complete solution, Eq. (6), are negligible: the first term of Eq. (6) is small due to the factor $\sin(\alpha_1 + \alpha_2) \approx \sin(2N+1)\,\pi$ ($N = 0, 1, 2, \ldots$) and the second-order terms of Eq. (6) are small as they depend quadratically on the leakage field amplitude and the test leakage field is highly symmetric (second- and higher-order terms vanish for perfectly symmetric leakage fields).

In order to make visible the contributions by the second-order terms we induced a test leakage field exhibiting a highly asymmetric time structure with respect to $t = T/2$ by beaming a 5 ms test field into CSF1 through the upper port at a time 220 ms after the launch of the atoms. In Fig. 5 the resulting frequency shift is shown for a fixed test field power but varying Ramsey pulse area.

Due to the rather asymmetric character of the applied test field in this case, we recognize that now the second-order terms of Eq. (6) contribute to the frequency shift and may even become dominant when the test field amplitude is large enough. This is most clearly the case in the optimized Ramsey pulse area situation (full circles), where the contribution of the first term of Eq. (6) even vanishes due to the factor $\sin(\alpha_1+\alpha_2) \approx \sin\pi$. In this situation contributions to the frequency shift mainly come from the product of the in-phase and quadrature components, $\Omega_r^L(t)$ and $\Omega_i^L(t)$, so that the resulting frequency shift depends on $\sin\phi\cos\phi \propto \sin(2\phi)$. The other curves of Fig. 5 show that away from the optimized Ramsey pulse area situation the first term of Eq. (6) becomes increasingly important and finally dominates, so that the resulting frequency shift depends mainly on $\sin\phi$ (open squares). Qualitatively similar results were calculated by Dorenwendt and Bauch [9].

Finally, in Fig. 6 the frequency shifts are plotted for experiments where a test field was applied between the second Ramsey interaction and the detection, 744 ms after the launch and for a duration of 5 ms.

The observed frequency shifts exhibit a sinusoidal dependence on the phase with alternating sign for an increasing odd number $2N + 1$ ($N = 0, 1, 2, \ldots$) of $\pi/2$-pulse areas as predicted by Eq. (8). The reason for the increased amplitude of the curves for $7\pi/2$- and $9\pi/2$-pulses is currently not clear.

Also in this case we performed measurements of the leakage amplitude dependence for a fixed phase and a fixed Ramsey pulse area (not depicted here), which revealed frequency shifts proportional to the test field amplitude as predicted by Eq. (8) [17].

V. CONCLUSION

By solving iteratively the Bloch equations we obtained general expressions for the frequency shifts in caesium clocks due to the presence of microwave leakage fields between or after the Ramsey interactions. An important finding, overlooked in the past, is the strong sensitivity of the clock frequency to leakage fields when the effective pulse areas for the first and second Ramsey pulse are not precisely equal. This may even be the dominating source of leakage-induced frequency shifts. It was also pointed out that it is nec-

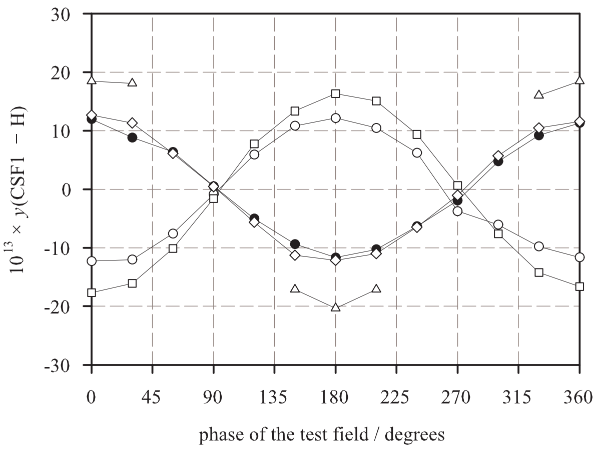

FIG. 6: Phase dependence of the relative frequency shift $y(\mathrm{CSF1} - \mathrm{H})$ due to a microwave test field of fixed amplitude, which is beamed into the apparatus from the lower port during 5 ms after the second Ramsey cavity passage and before the detection of the atoms (full circles: $1\,\pi/2$-pulse, open circles: $3\,\pi/2$-pulse, open diamonds: $5\,\pi/2$-pulse, open squares: $7\,\pi/2$-pulse, open triangles: $9\,\pi/2$-pulse). The solid lines connect the data points to guide the eye.

essary to take into account realistic leakage fields like standing wave fields or traveling wave fields in order to describe the effects due to leakage fields qualitatively and quantitatively correctly. Finally, conclusions from these theoretical findings are in full agreement with our experimental results.

All this has an important consequence for the operation of a primary frequency standard like CSF1. In case one finds significant frequency shifts due to microwave leakage upon variation of the Ramsey pulse area it is unrealistic to expect that a proper correction could be applied because this would require a model with enough sophistication to take account of the specific characteristics of the leakage fields along the trajectory of the atom cloud. Based on the results presented here one therefore has no choice but to work to remove any pulse-area dependence in the first place or to include the observed frequency variation into the uncertainty budget.

Acknowledgments

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