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New experimental limit on the validity of local position invariance

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Local position invariance (LPI) is part of the more general Einstein equivalence principle (EEP) which in turn is a foundation of Einstein's theory of general relativity. The EEP predicts a dependence of clock rates on the local gravitational potential U. LPI predicts that the gravitational shift is independent of the atomic species involved as a reference in the clock. It can thus be tested by comparing two different kinds of atomic frequency standard in the same time-varying gravitational potential $U(t)$. In our experiment we made use of the time dependence of $U(t)$ due to Earth's annual elliptical orbital motion. $U(t)/c^2$ varies between $\pm 3.3\times 10^{-10}$ (c is the speed of light). Comparing a cesium atomic fountain frequency standard with a hydrogen maser for about one year allowed us to set an upper limit on a possible frequency variation of 2.1×10^{-5} of this amount. Compared to previous similar experiments the limit of the notional violation of LPI was reduced by a factor of more than 30.

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I. INTRODUCTION

Time and frequency metrology is one of the rare fields in physics where general and special relativity manifest themselves immediately and need to be taken into account in the everyday practice of comparing clock rates and time scales over long distances. Therefore the methods of time metrology are also well suited for experimental tests of relativistic theories or for the search for variations of the fundamental constants. Such tests have recently gained renewed interest [1–4]. This is stimulated by the current availability of frequency standards with improved characteristics compared to the situation in previous years. The recent development of comparatively easy means for measurements of optical frequencies [5] will also facilitate such studies. Several space projects have been proposed [6] or already approved [7] during which such tests will be conducted in a space environment. The analysis described in this Rapid Communication resumes earlier ground-based work [8,9], searching for a time variation of frequency differences between nonidentical atomic clocks subjected to the same time variations of the local gravity potential.

II. THEORETICAL BACKGROUND

One part of Einstein's equivalence principle (EEP) is known as local position invariance (LPI), stating that "in local, freely falling frames, the outcome of any nongravitational test experiment is independent of where and when in the universe it is performed" [10]. The gravitational redshift of the clock frequency,

$$y_U=\frac{\nu_U-\nu_0}{\nu_0}=\frac{\Delta U}{c^2}, \tag{1}$$

is a consequence of the EEP [10]. The frequency difference $\nu_U-\nu_0$ occurs because of the difference ΔU in Newtonian gravitational potential between the clock's actual location (in space or time) and a reference value for which the clock frequency is ν_0. If LPI was not satisfied, just to mention one possible consequence, Eq. (1) would have to be modified to

$$y_U=(1+\beta_k)\frac{\Delta U}{c^2}. \tag{2}$$

The parameter β_k would be either a function of position or a function of the atomic species k used, or of both. Historically, clock transportation experiments were made for the purpose of demonstration of relativistic effects, Hafele's and Keating's probably being the best known [11]. More refined experiments were made in order to test the condition $\beta_k=0$. During the Gravity Probe A mission, the frequency of a hydrogen maser (H) that was operated in a well determined time-varying gravity potential was recorded with reference to a stationary ground-based maser, and a limit of $|\beta_{\mathrm{H}}|<7\times 10^{-5}$ was deduced [12]. The second type of experiment made in this context comprises the comparison of stationary frequency standards based on two different atomic species (a and b) subjected to the same variation of the gravitational potential. In such experiments one determines the relative frequency difference

$$y_a-y_b=(\beta_a-\beta_b)\frac{U(t)}{c^2} \tag{3}$$

which is zero if LPI is valid. In the pioneering experiments, the diurnal variation of the local gravity potential due to Earth rotation [8] and the annual variation due to the eccentricity of the earth orbit around the Sun [9] were considered. We report here on an experiment of the latter kind. Earth's orbital motion entails a temporal variation of the solar gravitational potential on the geoid, described by

$$\frac{U(t)}{c^2}=-\frac{2GM_S}{ac^2}e\cos\phi(t), \tag{4}$$

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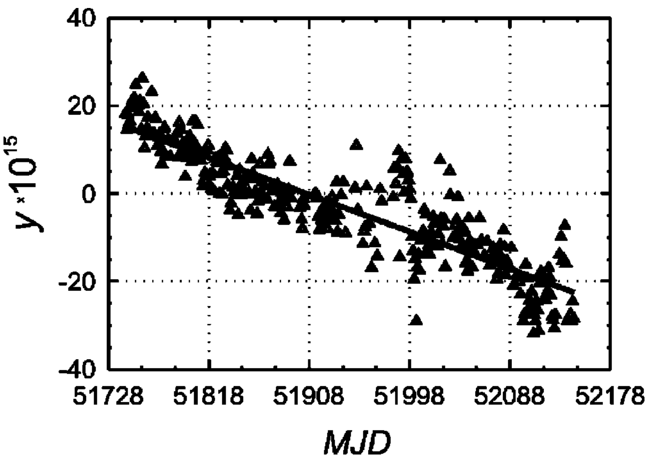

FIG. 1. Results of comparisons of a hydrogen maser (HM) with respect to CSF1, PTB's cesium atomic fountain frequency standard, expressed as relative frequency difference $y := y_H - y_{Cs}$ as a function of time. Here and in all other figures MJD designates the modified Julian date. MJD 51728 corresponds to 2000-07-03. The solid line represents a least-squares fit to the data points and is explained further in the text.

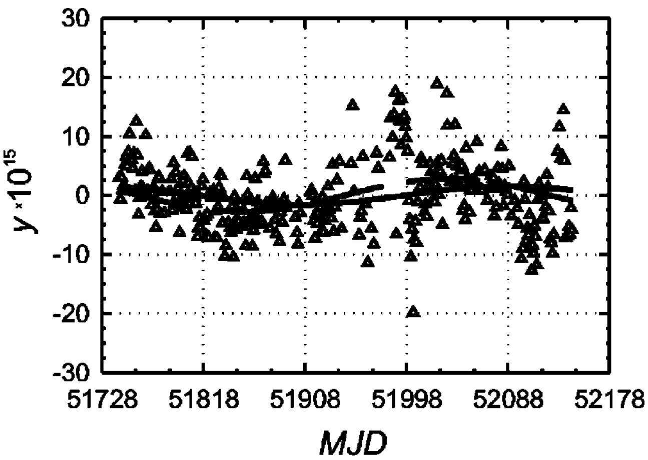

FIG. 2. Detrended data from Fig. 1. The two sinusoidal curves represent least-squares fits with an annual period using equal statistical weight for each data point. The dashed line represents a simultaneous fit of amplitude and phase whereas the solid line represents a fit with Φ fixed so that a maximum (or minimum) resulted at perihelion (MJD 51913).

where the product of the gravitational constant G and the solar mass M_S amounts to $1.327\times10^{20}\,\mathrm{m}^3/\mathrm{s}^2$, $a=1.496\times10^{11}$ m is the semimajor axis of the Earth orbit, and $e=0.0167$ is the eccentricity of the Earth orbit. $\phi(t)$ is the true anomaly, zero at perihelion occurring on January 4th 2001 [modified Julian date (MJD)=51913]. The peak to peak variation in $U(t)/c^2$ thus amounts to 0.66×10^{-9}. Of course, this variation is undetectable using clocks that experience the same $U(t)$ as long as LPI is satisfied. The difference $(\beta_a-\beta_b)$ would become nonzero if the (nongravitational) fundamental constants which determine the energy of hyperfine states, e.g., the fine structure constant α, were a function of the external gravitational potential [8,9]. In fact, the hyperfine splitting in the ground state of hydrogen and cesium, the two atomic species of interest in the context of this study, are known to have a different dependence on α [13]. Recording the temporal variation of the hyperfine splitting frequencies of cesium and hydrogen simultaneously with the variations of the gravity potential allows one to deduce a limit on $|\beta_a-\beta_b|$.

III. DESCRIPTION OF THE EXPERIMENT AND RESULTS

Physikalisch-Technische Bundesanstalt (PTB) started operation of its cesium atomic fountain frequency standard CSF1 in 1999. It is a primary frequency standard using laser cooled cesium atoms [14]. Essentially continuous records of the frequency of an active hydrogen maser with reference to the SI hertz, as realized with CSF1, are available since summer 2000. The hydrogen maser is operated including a cavity tuning procedure whereby the resonance frequency of the maser's microwave cavity is tuned to the hydrogen resonance frequency. The tuning system employs a stable reference signal delivered by a second maser [15]. Its design allows compensation for mechanical aging or settling of the cavity to a large degree. The remaining frequency drift, observed with respect to CSF1 and also with respect to other stable references, is not calculable from first principles. It is a common conviction that it reflects the aging of the Teflon coating covering the inner surface of the storage bulb [16]. The raw comparison data are displayed in Fig. 1. Each data point (total number 321) represents an average value over between 16 and 24 hours of measurement. The relative statistical measurement uncertainty for each point varies between 1×10^{-15} and 3×10^{-15}. The excess noise during the middle period reflects an imperfect tuning of the maser cavity caused by a defect in the second maser delivering the stable frequency reference. From a linear least-squares fit, the drift of the maser frequency is deduced as -0.094×10^{-15}/day and is determined with a standard uncertainty of 0.003×10^{-15}/ day (1σ). Maser frequency drifts of similar small values and with both signs have also been reported elsewhere [17].

Data reporting the comparison results of PTB's primary clock CS2 and a hydrogen maser [15] were used to analyze a possible variation of the fine structure constant α in time [13], explicitly using the maser frequency drift determined in [15]. Because of the above mentioned causes of the maser's frequency drift we restrict ourselves to a determination of the magnitude of the annual variation of the frequency difference and remove the constant drift from the data in Fig. 1. The detrended data are depicted in Fig. 2, including two sinusoidal curves $y_G \sin(2\pi t/365+\Phi)$, with t expressed in days. They represent least-squares fits with an annual period using equal statistical weight for each data point. The dashed line represents a simultaneous fit of amplitude and phase which yields $y_G=2.76(44)\times10^{-15}$ and Φ corresponding to a maximum at MJD=51861(9). Here and in the following the values in parentheses represent 1σ standard uncertainties. For the fit represented by the solid line, Φ was fixed so that a maximum (or minimum) resulted at perihelion. Under this constraint $y_G=1.32(45)\times10^{-15}$ was found. When the data

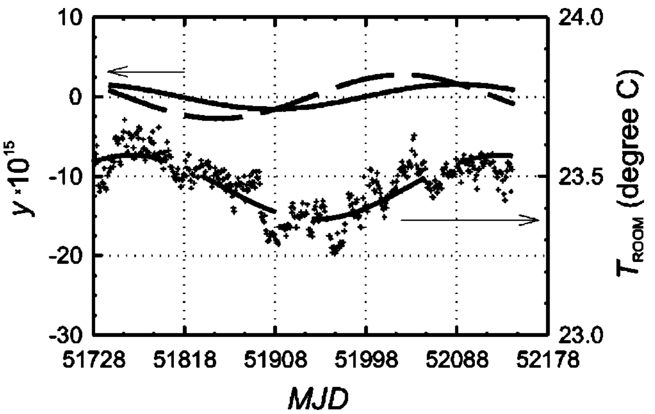

FIG. 3. Recorded air temperature in PTB's clock room (right scale) and leastsquares fit of a sinusoid with an annual period shown as dash-dotted curve. For comparison, the two fitted curves from Fig. 2 have been included (left scale).

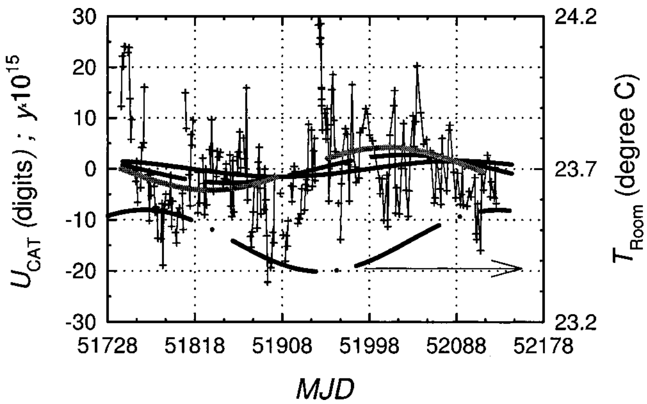

FIG. 4. Cavity control voltage U_{CAT} of the maser's cavity tuning system (left scale), expressed in digits of the digital-to-analog. converter. A linear trend of -1.9 digit/day was subtracted and a least-squares fit of a sinusoid with an annual period is shown as the dashed gray curve. As one digit approximately corresponds to a step of 1×10^{-15} in relative maser frequency it is possible to plot the sinusoid fit curves from Fig. 2 in the same scale. For comparison, the temperature fit from Fig. 3 is added (right scale).

set is split into two halves, taking always one point out of two, two independent fits to the two data sets give about the same y_G values with uncertainties increased by about $\sqrt{2}$, as expected.

When a linear and a sinusoidal term are fitted in one step to the original data of Fig. 1, the amplitude of the sinusoid comes out as $4.25(54)\times10^{-15}$. The process of detrending apparently partially absorbs the Fourier component with a frequency 1/year from the original data. For an estimate of $|\beta_a-\beta_b|$ we will later use this (larger) amplitude value, but in the following discussion we use the fits to the detrended data as they allow a better graphical presentation.

IV. DISCUSSION

In a similar previous study [9], making use of the comparison of a magnesium frequency standard at 601 GHz and a commercial cesium clock, the equivalent fitted amplitude was about 10^{-13} in magnitude. The improvement due to more stable frequency standards and a larger number of data points is obvious. However, it appears not straightforward to safely state an upper bound for $|\beta_a-\beta_b|$. The observation of the annual frequency variation with a maximum shifted in time with respect to perihelion calls for an examination of other possible frequency shifting effects in CSF1 as well as in the hydrogen maser.

The CSF1 uncertainty was evaluated for the first time in early 2000 [14]. Since February 2001 (MJD>51950) CSF1 has been operated including a selection of atoms in one hyperfine substate prior to the excitation of the clock transition, and the CSF1 uncertainty was reduced to 1.0×10^{-15} for a certain standard operation condition [18]. During this study, CSF1 was operated at conditions which deviated therefrom and we estimate the average CSF1 uncertainty as 2.0×10^{-15}. This uncertainty estimate reflects knowledge of effects that might systematically shift the otherwise unperturbed frequency. It also implies that the CSF1 frequency should not exhibit fluctuations under all conditions of operation which exceed plus or minus twice the stated uncertainty for more than 5% of the operating time. This latter statement, of course, neglects the short-term frequency instability, characterized by $\sigma_y\leqslant3\times10^{-13}/\sqrt{\tau(\mathrm{s})}$ which is a property of the standard and requires sufficiently long measurement times. For averaging times τ exceeding half a day the observed frequency instability (see Figs. 1 and 2) is to a large extent determined by the frequency instability of the hydrogen maser. Ideally, any long-term fluctuations of CSF1 should be significantly smaller than the stated uncertainty, but due to the lack of a frequency reference similar in quality to CSF1 this is difficult to verify.

The manufacturer of the hydrogen maser specified a potential frequency dependence on ambient temperature of below $\pm10^{-14}$/K. This specification was later said to be conservative. In case of proper function of the cavity autotuning system and of the two-stage temperature control shields around the maser cavity the relative frequency dependence should be about 2×10^{-15}/K [19]. Achievement of such a performance is a challenge, as without cavity tuning, the cavity temperature would have to be stabilized to better than about 20 μK to achieve a frequency instability below 1×10^{-14}, as calculated from Table 6.7.3 in [16]. The maser and CSF1 were operated in the same temperature controlled room whose air temperature is constantly monitored. In Fig. 3 the recorded temperature values and a sinusoidal least-squares fit with an annual period are depicted (right scale), together with the fitted curves to the frequency data (from Fig. 2). Temperature clearly shows an annual term with peak-to-peak variations of 0.2 K and a minimum around the end of February, which is close to the observation of the lowest temperatures in winter 2001 in Braunschweig.

Records were kept of the control voltage fed to the cavity tuning varactor. In the maser this voltage is generated from a 16-bit digital to analog converter whose state is displayed. The control voltage exhibits a dominant linear trend with a small annual term superimposed. The annual term is almost out of phase by $\pi/2$ with respect to the temperature variations as if it were caused by the time derivative of the temperature variations. Its amplitude and phase could well explain the observed annual frequency dependence out of phase with the variation of the gravitational potential as one step of the least significant bit approximately corresponds to a step of 1×10^{-15} in relative maser frequency [19]. For clarification, in Fig. 4 all four fitted sinusoids are overlaid.

One may argue that such a frequency variation due to technical reasons could mask or mimic the effect under study. We thus conservatively estimate the potential magnitude $|\beta_a-\beta_b|$ (67% confidence) as

$$|\beta_a-\beta_b|\leqslant\frac{(4.3+0.5+2)\times10^{-15}}{0.33\times10^{-9}}=2.1\times10^{-5}, \qquad (5)$$

where the first term represents the maximum observed annual variation, the second term represents the 1σ standard measurement uncertainty, and the third term is the average CSF1 1σ combined uncertainty during the measurement period. The new limit on $|\beta_a-\beta_b|$ is about a factor of 30 tighter than the one reported in [9]. If one considered strictly only the sinusoidal variation in phase with the variation of the gravity potential, the limit on $|\beta_a-\beta_b|$ would be even tighter by another factor of 2.

CSF1 can now be operated quasicontinuously, and it is obvious from the discussion above that comparison with one or two well-behaved hydrogen masers over periods of a few years will allow one to confirm the validity of LPI much tighter. At PTB we may in the near future start regular comparisons between an optical frequency standard based on a single trapped ytterbium ion [20] and CSF1. It was estimated that the ytterbium frequency standard currently realizes the unperturbed transition frequency of the $6s\,^2S_{1/2}(F=0)\rightarrow5d\,^2D_{3/2}(F=2)$ transition in $^{171}Yb^+$ at $\lambda=435$ nm with a relative uncertainty below 10^{-14} [21]. Significant improvements can be expected in the oncoming years. Such optical frequency measurements could also be used for testing the validity of LPI much more tightly.

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