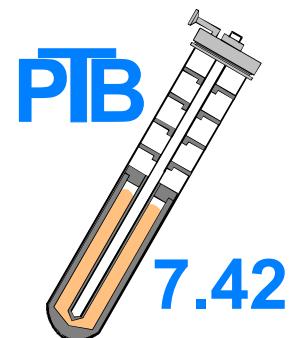


Temperature Measurements According to the International Temperature Scale of 1990 and Associated Uncertainties

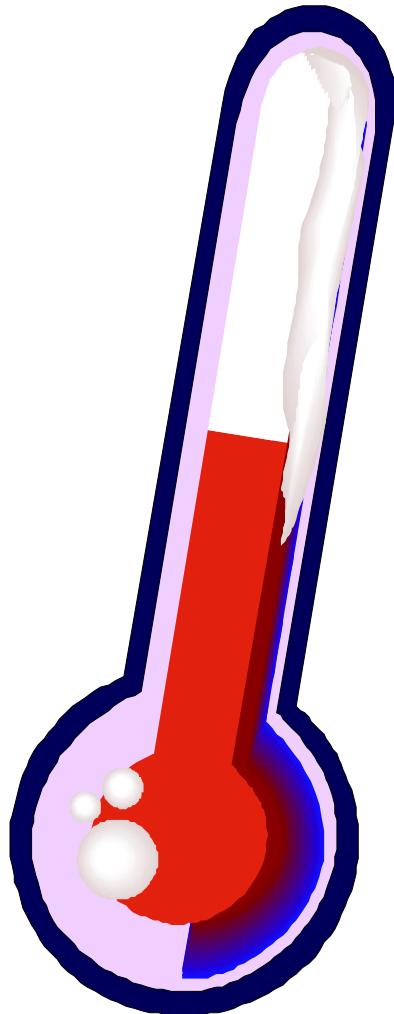
Steffen Rudtsch



Content

- ➡ Thermodynamic temperatures and temperature scales
 - ➡ Definition of the unit kelvin
 - ➡ ITS-90 and fixed points
 - ➡ Resistance thermometry
 - ➡ Error sources and uncertainty budget

Definition of a Temperature Scale



Problem: Temperature is an intensive thermodynamic property (value does not depend on the amount of the substance).

How can we define a universal temperature scale whose properties does not depend on specific substances or thermometers?

W. Thomson (later Lord Kelvin, 1848)

- Absolute zero and a second fixed point
- Efficiency of reversible Carnot-cycle

$$T_1 = T_2(1-\eta)$$

Measurement of Thermodynamic Temperatures

Gas thermometer: equation of state of an ideal gas

$$p V = N k T \text{ bzw. } \frac{T}{T_{\text{tpw}}} = \frac{p V}{p_{\text{tpw}} V_{\text{tpw}}} \quad k = 1,38065 \cdot 10^{-23} \text{ J/K}$$

(Boltzmann constant)

$V=\text{const.}, p \rightarrow 0$

N : Number of atoms

Dielectric constant gas thermometer (DCGT)

$$\varepsilon = \varepsilon_0 + \alpha_0 \frac{N}{V} \quad \alpha_0 - \text{atomic polarisability}$$

$$p = k T \frac{\varepsilon - \varepsilon_0}{\alpha_0} \quad \frac{T}{T_{\text{tpw}}} = \frac{p}{p_{\text{tpw}}} \cdot \frac{\varepsilon_{\text{tpw}} - \varepsilon_0}{\varepsilon - \varepsilon_0}$$

Measurement of the change in capacitance of a gas-filled capacitor after evacuation

Basis: Fundamental Physical Laws

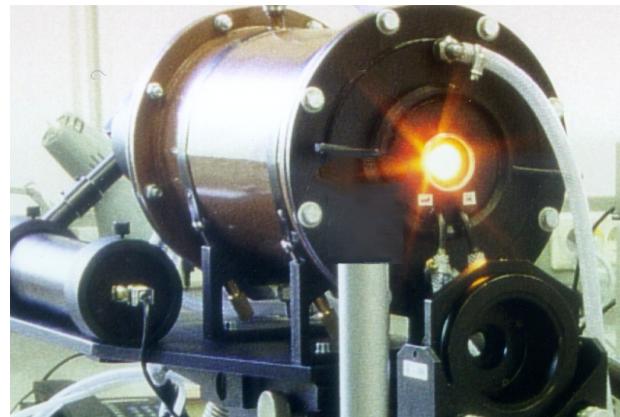
Gas Properties:

- Equation of state of an ideal gas
- Dielectric constant
- Speed of sound
- Spectral-line Doppler broadening



Electron Gas:

- Thermal noise



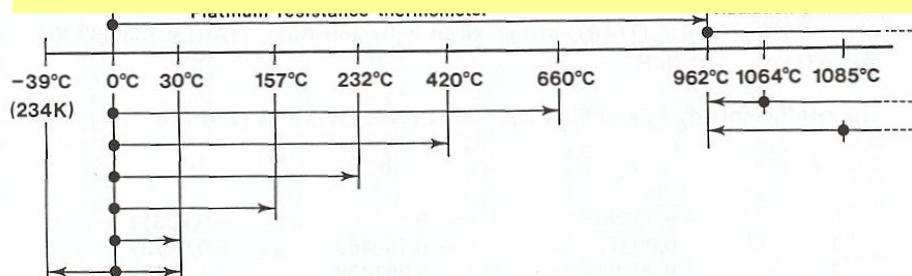
Photons:

- Total radiation of a blackbody
- Spectral emission of a blackbody

International Temperature Scale of 1990

Figure 1.1. Schematic representation of the ranges, sub-ranges and interpolation instruments of ITS-90. The temperatures shown are approximate only.
Section 1.2.6

- ◆ Definition of the unit kelvin
- ◆ Fixed points of the ITS-90
- ◆ Interpolation instruments
- ◆ Interpolation equations



TABLES, FIGURES AND REFERENCES

TABLE 1.1

The defining fixed points of the ITS-90.

Number	Temperature		Substance ^{1,7}	State ^{1,8}	$W_r(T_{90})$
	T_{90}/K	$t_{90}/^\circ\text{C}$			
1	3 to 5	-270,15 to -268,15	He	vp	
2	13,8033	-259,3467	e-H ₂	tp	0,001 190 07
3	≈ 17	≈ -256,15	e-H ₂ (or He)	vp (or gp)	(0,002 296 46) ^{1,9}
4	≈ 20,3	≈ -252,85	e-H ₂ (or He)	vp (or gp)	(0,004 235 36) ^{1,9}
5	24,5561	-248,5939	Ne	tp	0,008 449 74
6	54,3584	-218,7916	O ₂	tp	0,091 718 04
7	83,8058	-189,3442	Ar	tp	0,215 859 75
8	234,3156	-38,8344	Hg	tp	0,844 142 11
9	273,16	0,01	H ₂ O	tp	1,000 000 00
10	302,9146	29,7646	Ga	mp	1,118 138 89
11	429,7485	156,5985	In	fp	1,609 801 85
12	505,078	231,928	Sn	fp	1,892 797 68
13	692,677	419,527	Zn	fp	2,568 917 30
14	933,473	660,323	Al	fp	3,376 008 60
15	1234,93	961,78	Ag	fp	4,286 420 53
16	1337,33	1064,18	Au	fp	
17	1357,77	1084,62	Cu	fp	

1.7 All substances except helium (both ³He and ⁴He are used) are of natural isotopic composition, e-H₂ is hydrogen at the equilibrium concentration of the ortho- and para-molecular forms.

1.8 For complete definitions and advice on the realization of these various states, see Section 2. The symbols have the following meaning: vp: vapour pressure point; tp: triple point (temperature at which the solid, liquid and vapour phases are in equilibrium); gp: gas thermometer point; mp, fp: melting point, freezing point (temperature, at a pressure of 101 325 Pa, at which the solid and liquid phases are in equilibrium).

1.9 The values corresponding to fixed points numbers 3 and 4 are calculated for $T_{90} = 17,035 \text{ K}$ and $T_{90} = 20,27 \text{ K}$ respectively (see Section 2.3.4).

Definition of the Kelvin and Triple Point of Water



The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

13. CGPM (1967): Metrologia, **4** (1968), 147

$$T_{\text{tpw}} = 273,16 \text{ K} \text{ (Definition)}$$

$$p_{\text{tpw}} = 611,66 \text{ Pa}$$

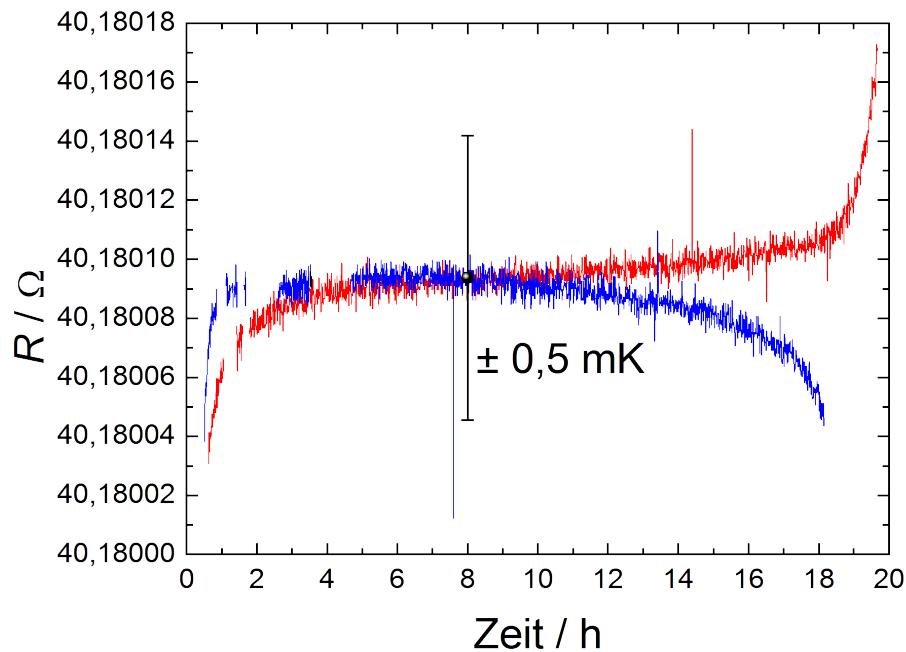
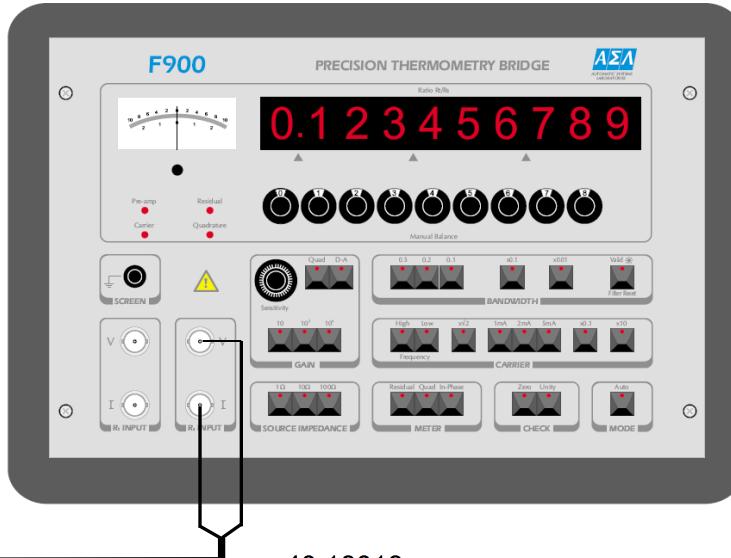
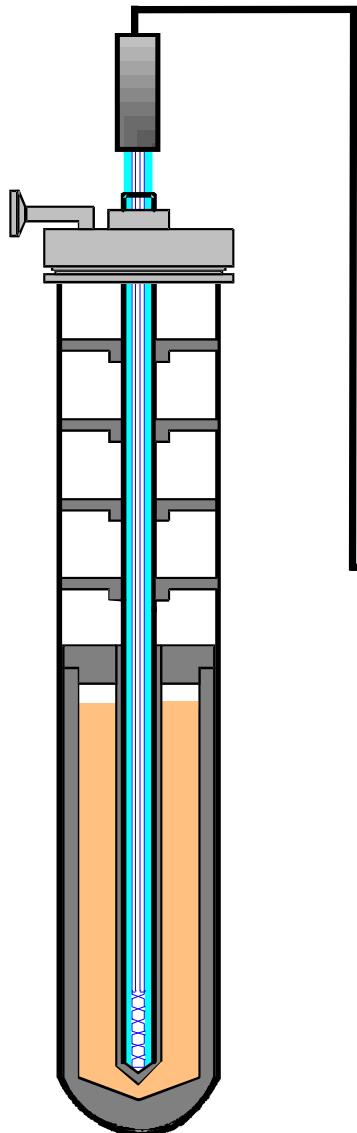
Defined within ITS-90	Research
Hydrostatic pressure (correction) Preparation ice mantle Mesurement procedure Isotopic composition (since 2005 V-SMOW)	Purity (traceable to the SI) Cell materials (contamination) Dissolved gases Pressure caused by buoyancy Purity (traceable to the SI)

Fixed Points of the ITS-90



		T_{90}	$\Delta T_{\text{th.dyn}} / \text{mK}$	$\Delta T_{\text{ITS-90}} / \text{mK}$
←	Cu (fp)	1084,62 °C	60	15
←	Au (fp)	1064,62 °C	50	10
←	Ag (fp)	961,78 °C	40	3
←	Al (fp)	660,323 °C	25	2
←	Zn (fp)	419,527 °C	13	1
←	Sn (fp)	231,928 °C	5	1
←	In (fp)	156,5985 °C	3	1
←	Ga (mp)	29,7646 °C	1	0,25
←	H ₂ O (tp)	273,16 K	0	0,1
←	Hg (tp)	234,3156 K	1,5	0,25
←	Ar (tp)	83,8058 K	1,5	0,3
←	O ₂ (tp)	54,3584 K	1	0,4
←	Ne (tp)	24,5561 K	0,5	0,4
←	e-H ₂ (vp)	≈ 20,3 K	0,5	0,4
←	e-H ₂ (vp)	≈ 17,0 K	0,5	0,4
←	e-H ₂ (tp)	13,8033 K	0,5	0,4
←	⁴ He (vp)	4,2221 K	0,3	0,1

Fixed-Point Measurement



ITS-90: Interpolation Instruments and Equations

0,65 K to 5,0 K:

Vapour pressure ${}^3\text{He}$ und ${}^4\text{He}$

$$T_{90} / \text{K} = A_0 + \sum_{i=1}^9 A_i \left[\frac{\ln(p) / \text{Pa} - B}{C} \right]^i$$

3,0 K to 24,5561 K:

Gasthermometer (He)

$$T_{90} = \frac{a + bp + cp^2}{1 + B_x(T_{90})N/V}$$

13,8033 K bis 961,78 °C:

Standard Platinum Resistance Thermometer

$$W_r(T_{90}) = C_0 + \sum_{i=1}^9 C_i \left\{ \frac{T_{90} / \text{K} - 754,15}{481} \right\}^i$$

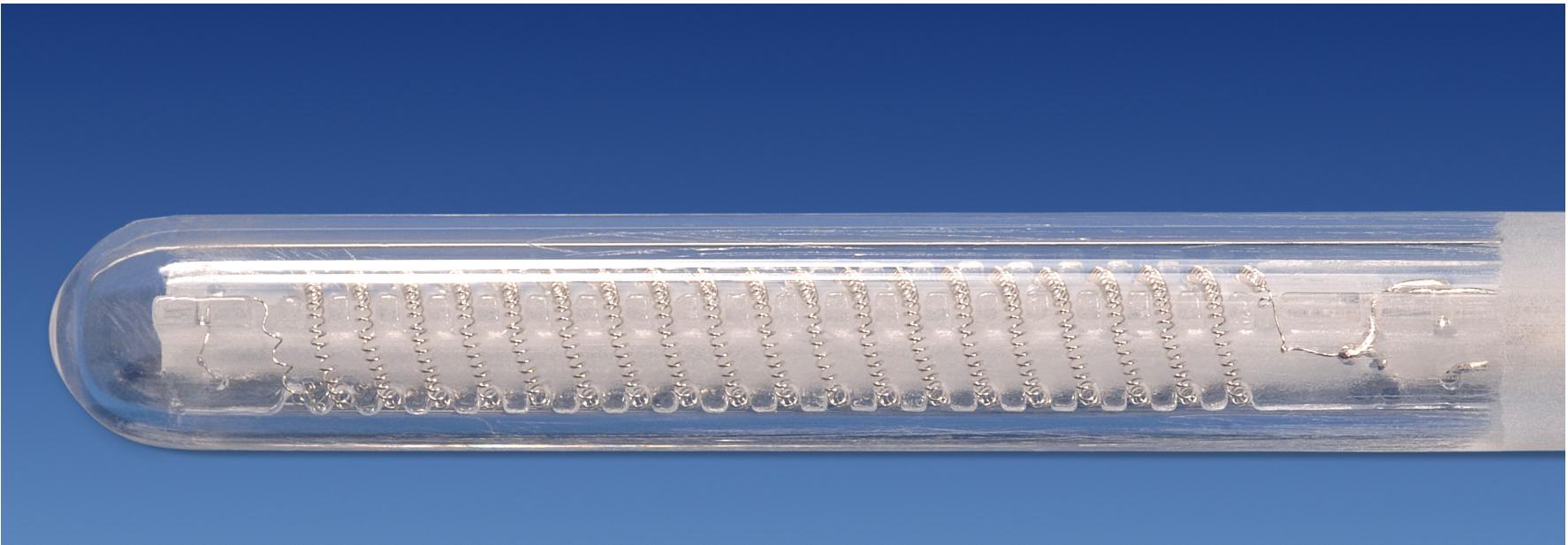
above 961,78 °C:

Radiation Thermometer

$$\frac{L_\lambda(T_{90})}{L_\lambda(T_{90}(X))} = \frac{\exp(c_2[\lambda T_{90}(X)]^{-1}) - 1}{\exp(c_2[\lambda T_{90}]^{-1}) - 1}$$

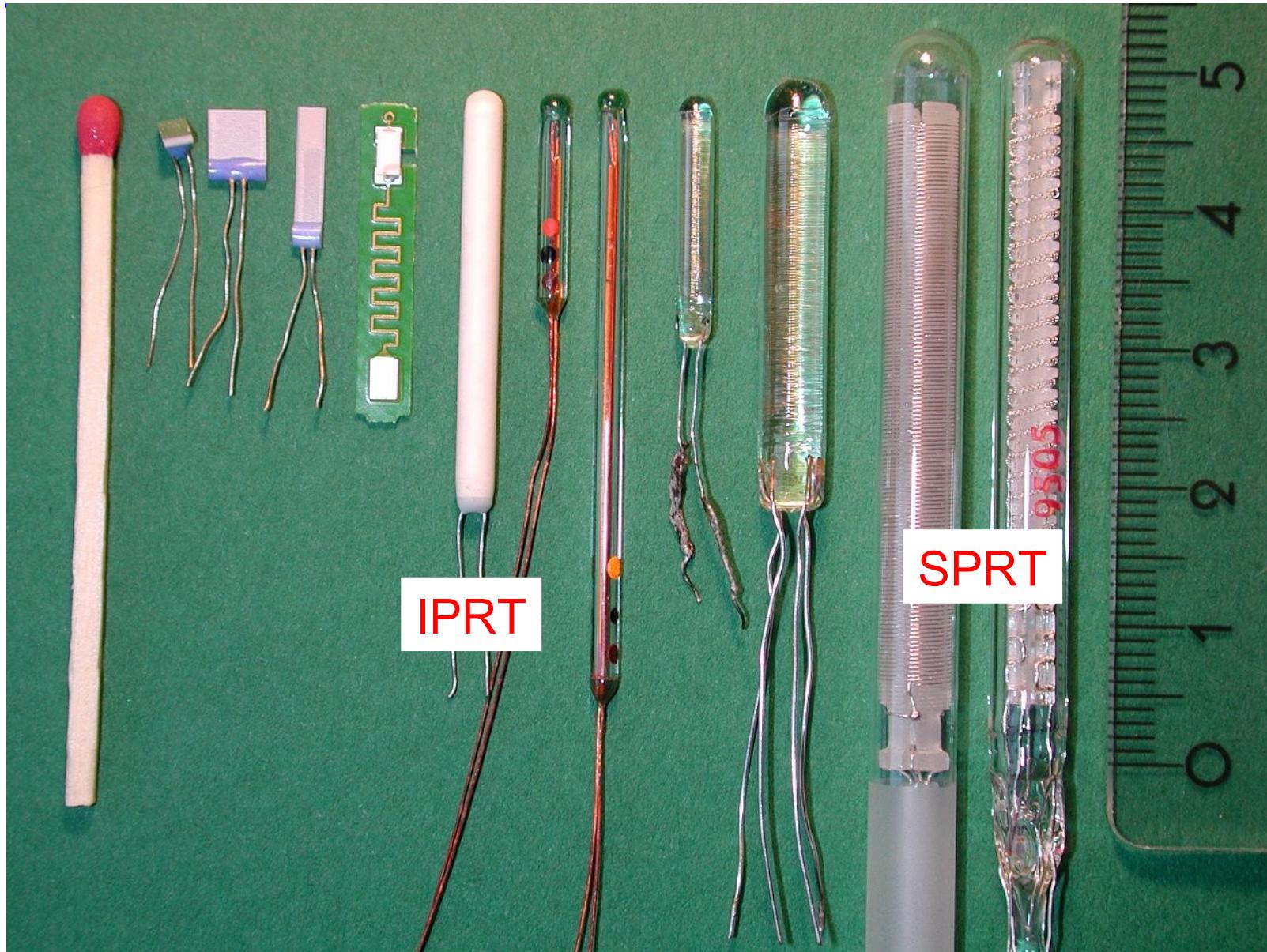
$$X: \text{Ag, Au, Cu}; \quad c_2 = 0,014388 \text{ m}\cdot\text{K}$$

Standard Platinum Resistance Thermometer



- Platinum wire of highest purity
- Induction free winding,
- Free of mechanical stress
- Specific gas filling in order to control oxidation
- Special heat treatment....

Resistance Thermometers



Callendar-van Dusen: $R(t) = R(0^\circ\text{C}) \cdot [1 + At + Bt^2 + C(t - 100)t^3]$

ITS-90 (SPRT calibration)

Determination of
resistance ratio W is required.
- extrapolated to $I = 0$ mA

$$W = \frac{R_{\text{SPRT}}(\text{Fixed Point})}{R_{\text{SPRT}}(\text{TPW})} = \frac{R}{R_0}$$

Reference Function
 0°C to 961.78°C :

$$W_r(T_{90}) = C_0 + \sum_{i=1}^9 C_i \left\{ \frac{T_{90}/\text{K} - 754,15}{481} \right\}^i$$

$$T_{90}/\text{K} = 273,15 + D_0 + \sum_{i=1}^9 D_i \left\{ \frac{W_r(T_{90}) - 2,64}{1,64} \right\}^i$$

Purity of the Platinum: $W(29,7646 \text{ } ^\circ\text{C}) \geq 1,1807$
 $W(-38,8344 \text{ } ^\circ\text{C}) \leq 0,844235$

Calibration Certificate for SPRTs:

Deviation Function (deviation from reference function)

$$W(T_{90}) - W_r(T_{90}) = a(W - 1) + b(W - 1)^2 + \sum_{i=1}^5 c_i [\ln(W)]^{2+i}$$

SPRT-Calibration: Uncertainty Budget

Quantity	$u(T_{\text{Ga}})$ mK	$u(T_{\text{In}})$ mK	$u(T_{\text{Sn}})$ mK	$u(T_{\text{Zn}})$ mK	$u(T_{\text{Al}})$ mK	$u(T_{\text{Ag}})$ mK
Impurities	0,06	0,25	0,31	0,54	0,40	0,65
Hydrost. Pressure	0,01	0,02	0,02	0,02	0,02	0,08
Gas Pressure	0,01	0,10	0,08	0,12	0,30	0,30
Standard Resistor	0,01	0,01	0,01	0,01	0,01	0,01
Bridge	0,02	0,11	0,12	0,16	0,20	0,25
TPW	0,08	0,09	0,11	0,15	0,20	0,28
Self-heating	0,05	0,15	0,20	0,20	0,20	0,20
Immersion	0,01	0,20	0,10	0,10	0,10	0,10
Fixed Point Value	0,03	0,06	0,06	0,06	0,20	0,20
Type B (combined)	0,12	0,40	0,43	0,64	0,65	0,87
Type A	0,05	0,20	0,15	0,15	0,30	0,30
Standard Uncertainty	0,13	0,45	0,45	0,66	0,71	0,92

Resistance Measurement

Resistance Thermometer
 typically between 25Ω and $10 \text{ k}\Omega$

1. Digital Multimeter (8 ½ digits)

$$U(T) \approx 5 \text{ mK}$$

- Problems:
- Stability reference resistor
 - Self-heating
 - Offset thermovoltages

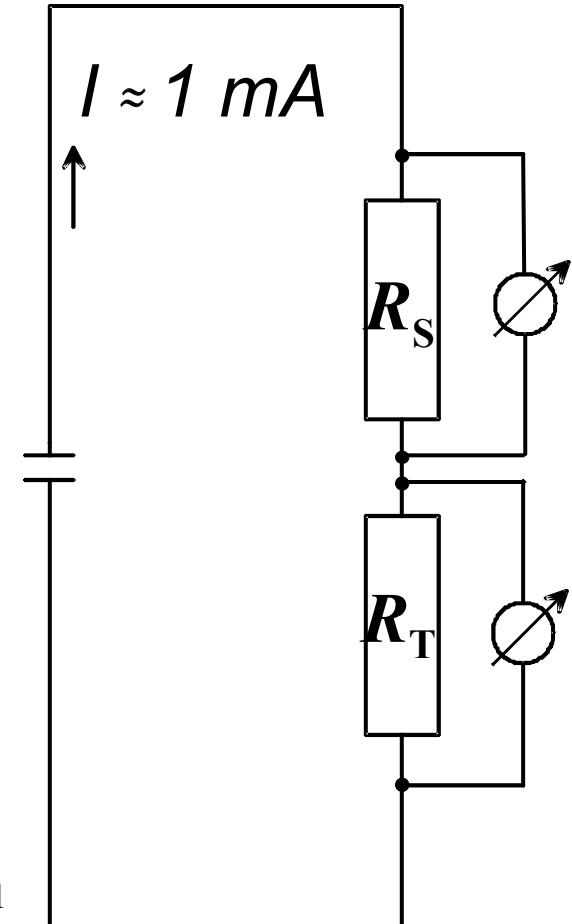
2. Potentiometric Method

$$U(T) < 1 \text{ mK}$$

High-precision standard resistor R_s

including its temperature control
 different currents are used

$$\left(\frac{d R_s}{d t} \right)_{\text{rel}} < 2 \cdot 10^{-7} \text{ a}^{-1}$$



Highest level with
commercial equipment

$$U \left(\frac{R_T}{R_S} \right) < 2 \cdot 10^{-8} \quad U(T) < 5 \mu\text{K}$$

AC-Bridges

Inductive voltage divider (ratio transformer)
typically 25 Hz or 30 Hz

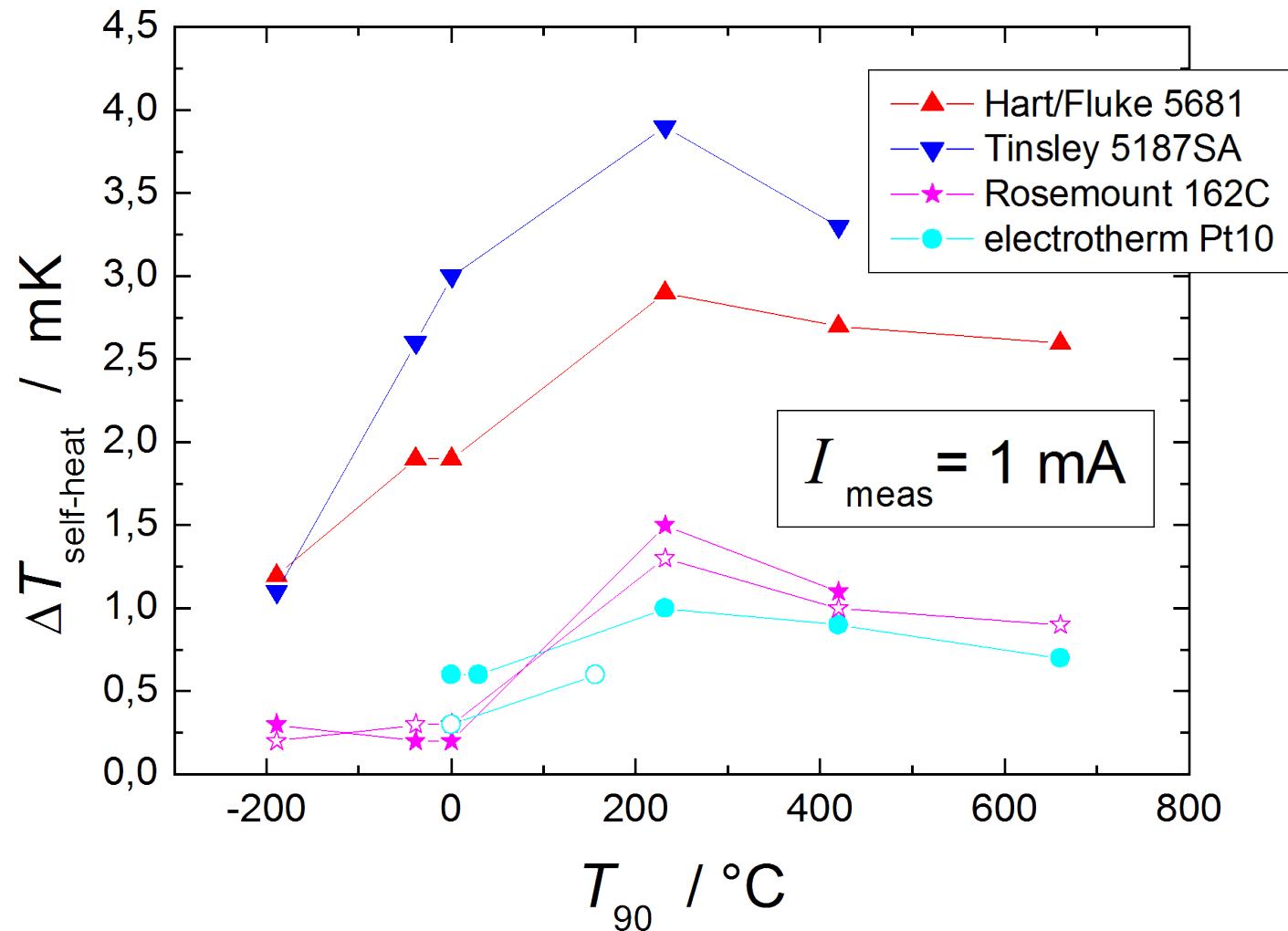
Manufacturer: Automatic Systems Laboratories (ASL)

“DC”-Bridges

Current comparator (current reversal necessary)
Manufacturer: Measurement International, Guildline

Measurement Errors and Corrections

1. Self Heating of the Resistance Thermometer



Typical Measurement Errors

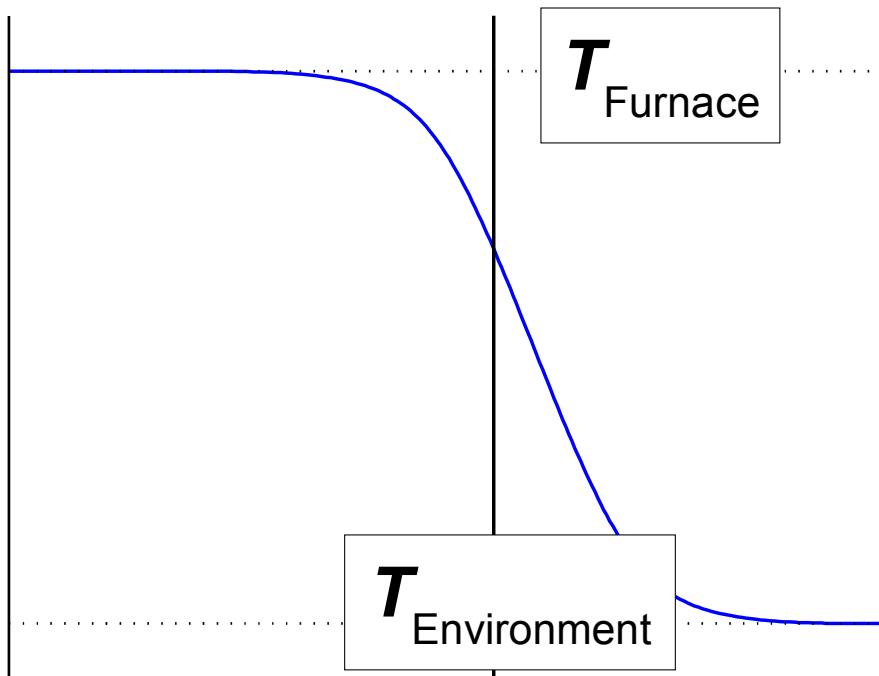
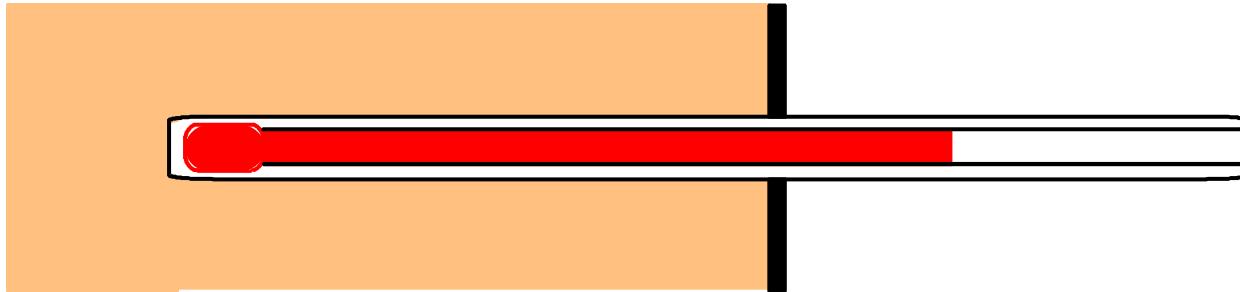
1. Stability of the Thermometer
 - resistance might be changed by mechanical shock or oxidation (SPRT, IPRT)
 - Drift even after careful heat treatment (Thermistors)
6. Hysteresis
 - up to $\Delta T = 500 \text{ mK}$ at IPRTs (not at SPRTs)
 - depend on thermal “history” of the IPRT
 - caused by thermal expansion of the ceramic substrate / powder

Thermal Measurement Errors

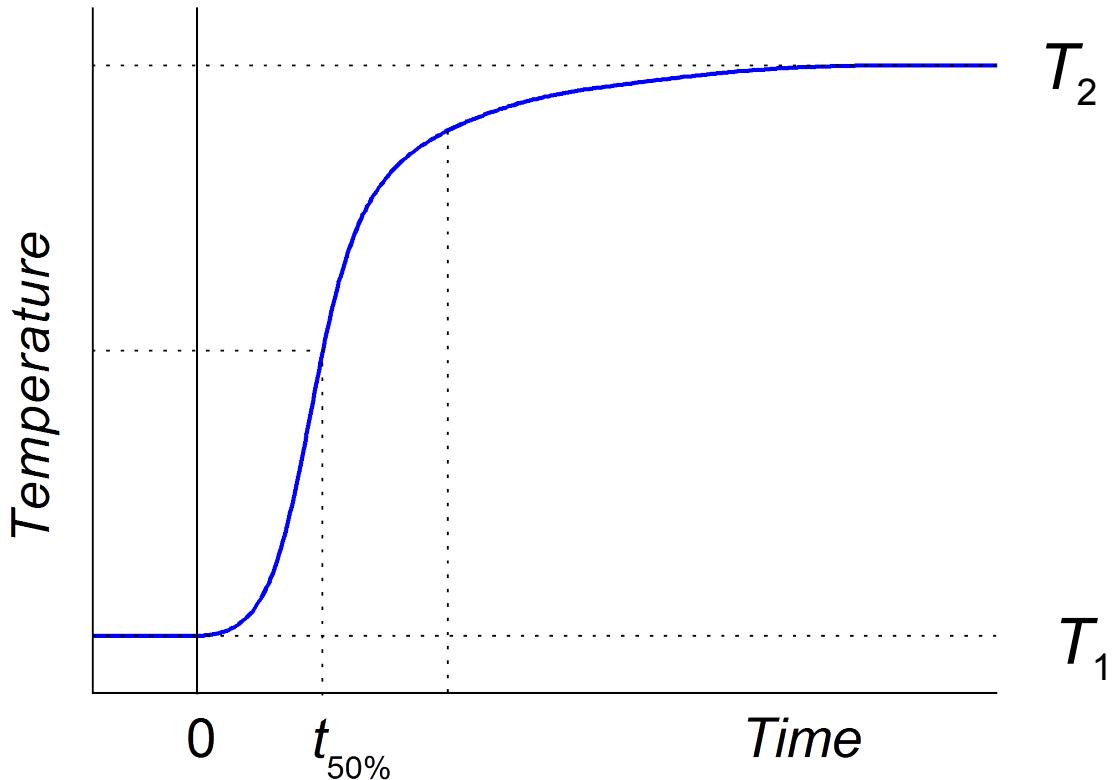
A contact thermometer measures only its own temperature.

4. Change of the temperature of the investigated object
 - heat conduction
 - heat storage/capacity
5. Heat dissipation and thermal coupling
i.e. thermometer and object have different temperatures
6. Dynamic behavior and thermal lag
7. Radiation errors

Heat Dissipation and Thermal Coupling



Dynamic Behavior and Thermal Lag



The dynamic behavior depend on the construction of the thermometer and the thermal coupling between object and thermometer.

Example of an Uncertainty Budget

Calibration of a resistance thermometer in a liquid bath (more details in DKD-R 5-1)

$$t_x = t_N + \delta t_{\text{Kal}} + \delta t_{\text{Drift}} + c_R \delta R_R + \delta t_{\text{Br}} + \delta t_{\text{WaN}} + \delta t_{\text{WAP}} + \delta t_{\text{EWN}} + \delta t_{\text{Hom}} + \delta t_{\text{Stab}}$$

t_x : ITS-90 temperature of the resistance thermometer to be calibrated

t_N : Mean of the temperature of the SPRT (reference)

δt_{Kal} : Correction based on the SPRT calibration

δt_{Drift} : Correction based on the drift of the SPRT

$c_R \delta R_R$: Correction based on the calibration of the standard resistor

c_R : Sensitivity of the bridge

δt_{Br} : Correction based on the bridge calibration

δt_{WaN} : Correction based on heat conduction of the SPRT

δt_{EWN} : Correction based on self-heating of the SPRT

δt_{WAP} : Correction based on heat conduction of the resistance thermometer

δt_{Hom} : Correction based on the in-homogeneity of the bath

δt_{Stab} : Correction based on the stability of the bath

Example of an Uncertainty Budget

Calibration of a resistance thermometer in a liquid bath
 (more details in DKD-R 5-1)

Quantity		Estimate	Standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution
t_N	Temperature SPRT	180.234 °C	1.2 mK	Normal	1	1.2 mK
δt_{Kal}	Calibration SPRT	0 K	7.5 mK	Normal	1	7.5 mK
δt_{Drift}	Drift SPRT _{Ref}	0 K	3.5 mK	Rectang.	1	3.5 mK
δR_R	Standard Resistor	0 Ω	0.07 mΩ	Normal	10 K/Ω	0.7 mK
δt_{Br}	Bridge	0 K	1.5 mK	Normal	1	1.5 mK
δt_{WaN}	Heat cond. SPRT	0 K	1.2 mK	Rectang.	1	1.2 mK
δt_{Hom}	Homogeneity bath	0 K	4.6 mK	Rectang.	1	4.6 mK
δt_{Stab}	Stability bath	0 K	3.5 mK	Rectang.	1	3.5 mK
t_x	Temperature	180,234 °C	10 mK			

Summary

- Definition of the unit kelvin
- International Temperature Scale of 1990
- Sensors and instruments
- Sources of uncertainty
- Uncertainty budget