

Uncertainty of VNA S-parameter measurement due to non-ideal TMSO or LMSO calibration standards

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Abstract. For the widespread 12-term TMSO and LMSO calibration of 4-sampler vector network analyzers (VNA), the sensitivity coefficients of the S-parameters of two-ports are developed as functions of the deviations of the reflection coefficients of the one-port calibration standards and of an imperfect through or line connection. Expressions representing the deviations of the S-parameters with respect to the error terms and for the deviations of the error terms with respect to the non-ideal calibration standards are also given. It is shown that the deviations of the S-parameters become quite large particularly for high-reflective two-port test objects. If applying a broadband load (instead of using the time consuming “ideal” sliding load routine) and the VNA-internal firmware-operated calibration and evaluation routines where the reflection coefficient is set to zero, deviations may appear of some 0,1 dB for the attenuation and some degrees of the transmission phase angle.

1 Introduction

When assuming the 12-term error model with a 4-sampler vector network analyzer (VNA) and applying the widespread TMSO (or SOLT) calibration method, the one-port standards of different reflection – usually a low-reflective termination M, a short-circuit S and an open circuit O – may be non-ideal, i.e. their reflection coefficients M_i , K_i , and L_i of M, S and O connected to the VNA test port numbered $i = 1$ or 2 show deviations δM_i , δK_i , and δL_i from the ideal values (or values assumed true). The through or line connection necessary for the calibration may also be non-ideal. The assumption of ideal reflection values with the firmware-operated performance and evaluation of calibration and measurement of a commercial VNA then leads to deviations of the S-parameters S_{jk} of the test object. To establish an uncertainty budget for the S-parameters of a test object measured after calibration, according to well known guidelines (GUM, 1995; Guidelines, 2000) it is necessary to calculate the individual uncertainty contributions – and the sensitivity coefficients – associated with the different input

estimates. It is also necessary to investigate what impact the deviations of the reflection coefficients of the calibration standards have on the uncertainty of the S_{jk} of test objects which include not only matched attenuators (often used as attenuation transfer standards) but also high-reflective two-ports of low and high attenuation (Stumper, 2002). In the present paper, the expressions for the sensitivity coefficients are extended to the LMSO calibration method, i.e. where a line representing the characteristic line impedance of known length between the test port reference planes, is used instead of a through connection, e.g. with on-wafer measurements. The ideal through-matrix $\mathbf{T}_{ideal} = \begin{pmatrix} 0 & S_{12}^T \\ S_{21}^T & 0 \end{pmatrix}$ changes

to $\mathbf{T} = \begin{pmatrix} \delta S_{11}^T & S_{12}^T + \delta S_{12}^T \\ S_{21}^T + \delta S_{21}^T & \delta S_{22}^T \end{pmatrix}$ for an imperfect line, where the S_{12}^T , S_{21}^T are the transmission coefficients of the ideal line and the δS_{11}^T , δS_{22}^T and δS_{12}^T , δS_{21}^T are the reflection and transmission deviations, respectively. The general set of expressions obtained for the sensitivity coefficients is explicitly displayed, as well as the sets of expressions for the deviations of the S-parameters with respect to the error terms and for the deviations of the error terms with respect to the non-ideal calibration standards. In the calculations, the effect on the raw values e.g. of noise, non-linearity, and cross-talk is not considered (i.e. only 10 error terms are taken into account and the two additional cross-talk (or leakage) error terms are ignored in the following calculations).

2 Measurement: dependence of the S-parameter deviations on error term deviations

Ignoring the two cross-talk error terms, we consider the 10 complex error terms E_D , E'_D (directivity), E_S , E'_S (source match), E_R , E'_R (reflection tracking), E_T , E'_T (transmission tracking), and E_L , E'_L (load match). The undashed quantities denote an active test port 1 and the dashed ones an active test port 2. The S_{jk} of a two-port test object are obtained from a set of four linear equations (Thumm, 1997; Gronau, 2001) given here in matrix form:

$$\begin{pmatrix} (E_S m_{11} + E_R - E_S E_D) \cdot E_T & E_R E_L m_{21} & 0 & 0 \\ 0 & 0 & (E_S m_{11} + E_R - E_S E_D) \cdot E_T & E_R E_L m_{21} \\ E'_R E'_L m_{12} & (E'_S m_{22} + E'_R - E'_S E'_D) \cdot E'_T & 0 & 0 \\ 0 & 0 & E'_R E'_L m_{12} & (E'_S m_{22} + E'_R - E'_S E'_D) \cdot E'_T \end{pmatrix} \times \begin{pmatrix} S_{11} \\ S_{12} \\ S_{21} \\ S_{22} \end{pmatrix} = \begin{pmatrix} (m_{11} - E_D) \cdot E_T \\ E_R m_{21} \\ E'_R m_{12} \\ (m_{22} - E'_D) \cdot E'_T \end{pmatrix}. \quad (1)$$

For commercial VNAs, the tracking terms are $|E_R| \approx 1$, $|E'_R| \approx 1$, $|E_T| \approx 1$, $|E'_T| \approx 1$, while the magnitudes of the directivity and the source and load match are about 0,1 or smaller. The $m_{11} = m_1/m_2$, $m_{21} = m_4/m_2$, $m_{12} = m'_1/m'_3$, and $m_{22} = m'_4/m'_3$ are normalized raw values, where m_2 , m_1 , and m_4 are the raw (sampler) values of the incident, re-

flected, and transmitted signal when test port 1 is active (cf. Fig. 1), and m'_3 , m'_4 , and m'_1 the raw (sampler) values of the incident, reflected, and transmitted signal, respectively, when test port 2 is active. After rewriting the equation system and solving it with respect to the raw values, we obtain:

$$m_{11} = \{[S_{11} \cdot (E_R - E_D E_S) + E_D] \cdot (1 - S_{22} E_L) + S_{21} S_{12} E_L \cdot (E_R - E_D E_S)\} / \{(1 - S_{11} E_S) \cdot (1 - S_{22} E_L) - S_{21} S_{12} E_S E_L\}, \quad (2)$$

$$m_{21} = S_{21} E_T / \{(1 - S_{11} E_S) \cdot (1 - S_{22} E_L) - S_{21} S_{12} E_S E_L\}, \quad (3)$$

$$m_{12} = S_{12} E'_T / \{(1 - S_{22} E'_S) \cdot (1 - S_{11} E'_L) - S_{12} S_{21} E'_S E'_L\}, \quad (4)$$

$$m_{22} = \{[S_{22} \cdot (E'_R - E'_D E'_S) + E'_D] \cdot (1 - S_{11} E'_L) + S_{12} S_{21} E'_L \cdot (E'_R - E'_D E'_S)\} / \{(1 - S_{22} E'_S) \cdot (1 - S_{11} E'_L) - S_{12} S_{21} E'_S E'_L\}. \quad (5)$$

We assume that the deviations δS_{jk} are not affected by variations of the m_{jk} . Two total differentials, which are set to zero, can then be established considering the S-parameters S_{11} , S_{12} , S_{21} , S_{22} , and the undashed error terms E_D , E_S , E_R , E_T , E_L (or the dashed) as variables. After execution of the

differentiations we finally obtain a set of four linear equations which are uniquely solved for the four S-parameter deviations δS_{11} , δS_{12} , δS_{21} , and δS_{22} as functions of the 10 deviations δE_D , δE_S , δE_R , δE_T , δE_L , $\delta E'_D$, $\delta E'_S$, $\delta E'_R$, $\delta E'_T$, $\delta E'_L$. For δS_{11} and δS_{12} we obtain:

$$\begin{aligned} N \cdot \delta S_{11} = & -\delta E_S \cdot S_{11} \cdot [S_{11} \cdot (1 - S_{22} E_L) + S_{21} S_{12} E_L] \cdot (1 - S_{11} E'_L) + \delta E'_S \cdot S_{21} S_{12} \cdot [S_{22} \cdot (1 - S_{11} E'_L) + S_{12} S_{21} E'_L] \cdot E_L \\ & - \delta E_D \cdot [(1 - S_{11} E_S) \cdot (1 - S_{22} E_L) - S_{21} S_{12} E_S E_L] \cdot (1 - S_{11} E_S) \cdot (1 - S_{11} E'_L) / E_R \\ & - \delta E'_D \cdot S_{21} S_{12} \cdot [(1 - S_{22} E'_S) \cdot (1 - S_{11} E'_L) - S_{12} S_{21} E'_S E'_L] \cdot E'_S E_L / E'_R - \delta E_R \cdot [S_{11} \cdot (1 - S_{22} E_L) \\ & + S_{21} S_{12} E_L] \cdot (1 - S_{11} E_S) \cdot (1 - S_{11} E'_L) / E_R - \delta E'_R \cdot S_{21} S_{12} \cdot [S_{22} \cdot (1 - S_{11} E'_L) + S_{12} S_{21} E'_L] \cdot E'_S E_L / E'_R \\ & + \delta E_T \cdot S_{21} S_{12} \cdot (1 - S_{11} E'_L) \cdot E_L / E_T + \delta E'_T \cdot S_{12} S_{21} \cdot (1 - S_{11} E'_L) \cdot E_L / E'_T - \delta E_L \cdot S_{21} S_{12} \cdot \\ & \cdot (1 - S_{11} E'_L) + \delta E'_L \cdot S_{11} S_{12} S_{21} E_L \end{aligned} \quad (6)$$

$$\begin{aligned} N \cdot \delta S_{12} = & \delta E_S \cdot S_{11} S_{12} \cdot [S_{11} \cdot (1 - S_{22} E_L) + S_{21} S_{12} E_L] \cdot E'_L - \delta E'_S \cdot S_{12} \cdot [S_{22} \cdot (1 - S_{11} E'_L) + S_{12} S_{21} E'_L] \cdot \\ & \cdot (1 - S_{22} E_L) + \delta E_D \cdot S_{12} \cdot [(1 - S_{11} E_S) \cdot (1 - S_{22} E_L) - S_{21} S_{12} E_S E_L] \cdot (1 - S_{11} E_S) \cdot E'_L / E_R + \delta E'_D \cdot S_{12} \cdot \\ & \cdot [(1 - S_{22} E'_S) \cdot (1 - S_{11} E'_L) - S_{12} S_{21} E'_S E'_L] \cdot (1 - S_{22} E_L) \cdot E'_S / E'_R + \delta E_R \cdot S_{12} \cdot [S_{11} \cdot (1 - S_{22} E_L) + S_{21} S_{12} E_L] \cdot \\ & \cdot (1 - S_{11} E_S) \cdot E'_L / E_R + \delta E'_R \cdot S_{12} \cdot [S_{22} \cdot (1 - S_{11} E'_L) + S_{12} S_{21} E'_L] \cdot (1 - S_{22} E_L) \cdot E'_S / E'_R - \delta E_T \cdot S_{21} \cdot \\ & \cdot S_{12}^2 E_L E'_L / E_T - \delta E'_T \cdot S_{12} \cdot (1 - S_{22} E_L) \cdot (1 - S_{11} E'_L) / E'_T + \delta E_L \cdot S_{21} S_{12}^2 E'_L - \delta E'_L \cdot S_{11} S_{12} \cdot (1 - S_{22} E_L) \end{aligned} \quad (7)$$

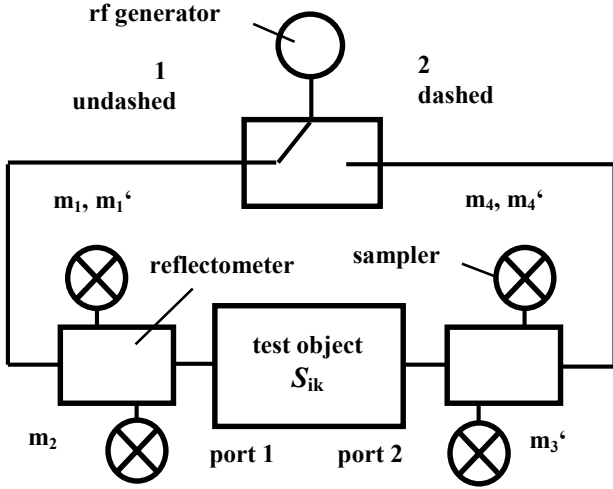


Fig. 1. Block diagram of a 4-sampler VNA, with test port 1 active and the raw (sampler) values of the incident, reflected, and transmitted signal m_2 , m_1 , and m_4 measured, m_3 omitted.

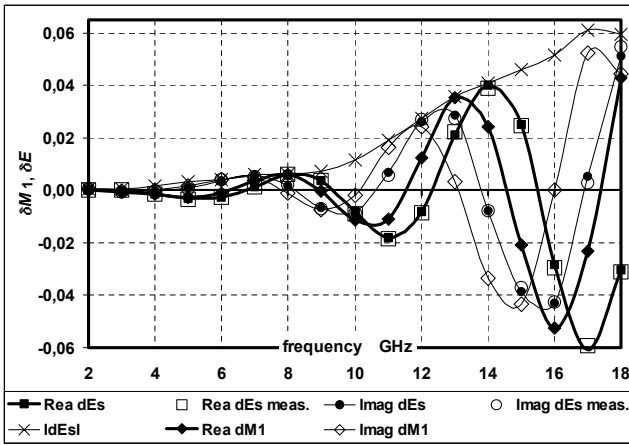


Fig. 2. Calculated (lines) and measured (signs) error term deviations δE_S due to a non-ideal broadband load (of reflection $\text{Rea } \delta M_1$ and $\text{Imag } \delta M_1$) at port 1 for the TMSO calibration.

where $N = (1 - S_{22}E_L) \cdot (1 - S_{11}E'_L) - S_{21}S_{12}E_LE'_L$.

As the magnitudes of the directivity and the source and load match are small, δS_{11} is primarily dependent on δE_S , δE_D , δE_R , and δE_L , and δS_{12} primarily depends on the deviations $\delta E'_S$, $\delta E'_T$, and $\delta E'_L$ of the “dashed” error terms whereas the influence of the deviations of the other error terms is smaller. Due to the symmetry of the 12-term error model, the corresponding expressions for δS_{22} and δS_{21} are obtained by replacing index 1 by 2 and vice versa and undashed error terms by dashed ones in Eqs. (6) and (7).

3 Calibration: dependence of the error term deviations on reflection and through deviations

The second step is to find out, for a full TMSO or LMSO calibration, the dependence of the error term deviations $\delta E_D, \dots, \delta E'_L$ on the reflection deviations $\delta M_1, \delta M_2, \delta K_1, \delta K_2, \delta L_1, \delta L_2$, and on the four deviations $\delta S_{11}^T, \delta S_{12}^T, \delta S_{21}^T, \delta S_{22}^T$ of the S-parameters of the through-connection or, alternatively, of the line inserted instead of the “through”. The error terms assigned to test port 1 and test port 2 are ob-

tained from a set of ten linear equations, which can be solved with respect to the raw values $m_{jk}^M, m_{jk}^K, m_{jk}^L$, and m_{jk}^T (T for “through” or, alternatively, “line”). These depend on the MSO standards of reflection coefficients $M_1, M_2, K_1, K_2, L_1, L_2$ and the S-parameters $S_{11}^T, S_{12}^T, S_{21}^T$, and S_{22}^T of the through-connection. For active test port 1 the raw values are:

$$m_{11}^M = [M_1 \cdot (E_R - E_S E_D) + E_D] / (1 - M_1 E_S), \quad (8)$$

$$m_{11}^K = [K_1 \cdot (E_R - E_S E_D) + E_D] / (1 - K_1 E_S), \quad (9)$$

$$m_{11}^L = [L_1 \cdot (E_R - E_S E_D) + E_D] / (1 - L_1 E_S), \quad (10)$$

$$m_{11}^T = \left([S_{11}^T \cdot (E_R - E_D E_S) + E_D] \cdot (1 - S_{22}^T E_L) + S_{21}^T S_{12}^T E_L \cdot (E_R - E_D E_S) \right) / \left((1 - S_{11}^T E_S) \cdot (1 - S_{22}^T E_L) - S_{21}^T S_{12}^T E_S E_L \right), \quad (11)$$

$$m_{21}^T = S_{21}^T E_T / \left((1 - S_{11}^T E_S) \cdot (1 - S_{22}^T E_L) - S_{21}^T S_{12}^T E_S E_L \right). \quad (12)$$

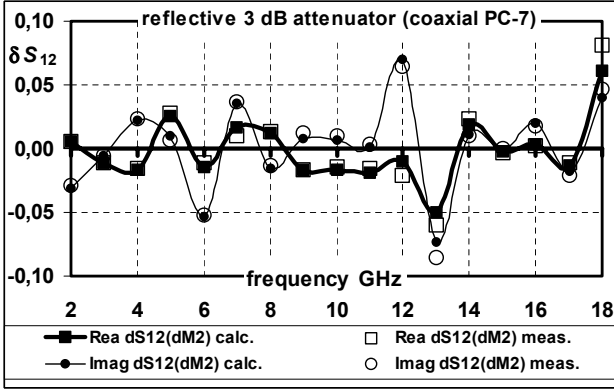


Fig. 3. Comparison of calculated and measured deviations δS_{12} for a high-reflective 3 dB attenuator, using a mismatch instead of a sliding load at port 2 for the TMSO calibration.

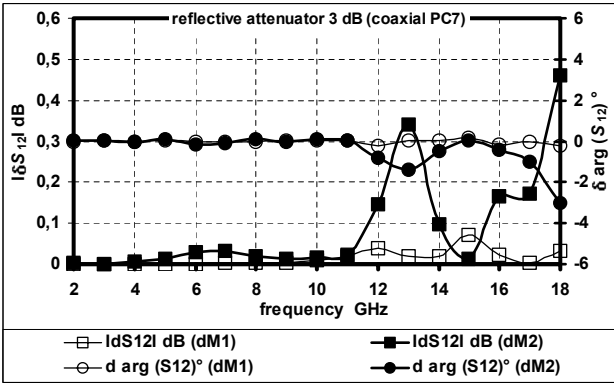


Fig. 4. Calculated deviations $|\delta S_{12}|$ in dB and phase deviations $\delta \arg(S_{12})$ for a high-reflective 3 dB attenuator using a low-reflective broadband load instead of a sliding load at port 1 (δM_1) or port 2 (δM_2) for the TMSO calibration (its reflection is plotted in Fig. 2).

For active test port 2 we obtain a second set by replacing index 1 by 2, and vice versa and undashed error terms by dashed ones. We assume that the deviations δE , $\delta E'$ are not affected by variations of the raw values and establish 10 total differentials, namely five for active test port 1, considering M_1 , K_1 , L_1 , S_{11}^T , S_{12}^T , S_{21}^T , S_{22}^T and the undashed error terms E_D , E_S , E_R , E_T , E_L as variables and a second set of five for active test port 2 by replacing index 1 by 2 and vice versa, and undashed error terms by dashed ones. After execution of the differentiations we finally obtain a set of five linear equations assigned to the undashed error terms and a second set of five linear equations assigned to the dashed error terms. These 10 linear equations uniquely deliver the 10

error term deviations $\delta E_D, \dots, \delta E'_L$ as functions of δM_1 , δM_2 , δK_1 , δK_2 , δL_1 , δL_2 and δS_{11}^T , δS_{12}^T , δS_{21}^T , δS_{22}^T . The general expressions obtained are symmetrical with reference to the M_i , K_i , and L_i which can be of arbitrary values, i.e. M, S and O can be not only match, short and open circuit as used with the TMSO or LMSO calibration method but, for alternative calibration methods, can be also e.g. three offset short circuits which provide different reflection phase angles. The influence of the δS_{ik}^T deviations due to a non-ideal through connection is only on transmission tracking and load match i.e. on δE_T and δE_L , not depending on M_i , K_i , and L_i . We obtain for the deviation of the undashed source match:

$$\delta E_S = -\frac{(1 - K_1 E_S) \cdot (1 - L_1 E_S)}{(K_1 - M_1) \cdot (L_1 - M_1)} \cdot \delta M_1 - \frac{(1 - M_1 E_S) \cdot (1 - L_1 E_S)}{(M_1 - K_1) \cdot (L_1 - K_1)} \cdot \delta K_1 - \frac{(1 - K_1 E_S) \cdot (1 - M_1 E_S)}{(K_1 - L_1) \cdot (M_1 - L_1)} \cdot \delta L_1, \quad (13)$$

directivity:

$$\delta E_D = -\frac{E_R K_1 L_1}{(K_1 - M_1) \cdot (L_1 - M_1)} \cdot \delta M_1 - \frac{E_R M_1 L_1}{(M_1 - K_1) \cdot (L_1 - K_1)} \cdot \delta K_1 - \frac{E_R K_1 M_1}{(K_1 - L_1) \cdot (M_1 - L_1)} \cdot \delta L_1, \quad (14)$$

reflection tracking:

$$\delta E_R = -\frac{E_R [2K_1 L_1 E_S - (K_1 + L_1)]}{(K_1 - M_1) \cdot (L_1 - M_1)} \cdot \delta M_1 - \frac{E_R [2M_1 L_1 E_S - (M_1 - L_1)]}{(M_1 - K_1) \cdot (L_1 - K_1)} \cdot \delta K_1 - \frac{E_R [2K_1 M_1 E_S - (K_1 + M_1)]}{(K_1 - L_1) \cdot (M_1 - L_1)} \cdot \delta L_1 \quad (15)$$

transmission tracking:

$$\begin{aligned} \delta E_T = & -\frac{E_T[K_1 L_1 E_S - E_L]}{(K_1 - M_1) \cdot (L_1 - M_1)} \delta M_1 - \frac{E_T[M_1 L_1 E_S - E_L]}{(M_1 - K_1) \cdot (L_1 - K_1)} \cdot \delta K_1 - \frac{E_T[K_1 M_1 E_S - E_L]}{(K_1 - L_1) \cdot (M_1 - L_1)} \cdot \delta L_1 \\ & - \frac{E_T}{S_{21}^T} \cdot \delta S_{21}^T - E_T E_L \cdot \delta S_{22}^T, \end{aligned} \quad (16)$$

load match:

$$\begin{aligned} \delta E_L = & \frac{K_1 L_1 - E_L \cdot (K_1 + L_1) + E_L^2}{(K_1 - M_1) \cdot (L_1 - M_1)} \cdot \delta M_1 + \frac{M_1 L_1 - E_L \cdot (M_1 + L_1) + E_L^2}{(M_1 - K_1) \cdot (L_1 - K_1)} \cdot \delta K_1 + \\ & + \frac{K_1 M_1 - E_L \cdot (K_1 + M_1) + E_L^2}{(K_1 - L_1) \cdot (M_1 - L_1)} \cdot \delta L_1 - \frac{1}{S_{21}^T S_{12}^T} \cdot \delta S_{11}^T - \frac{E_L}{S_{12}^T} \cdot \delta S_{12}^T - \frac{E_L}{S_{21}^T} \cdot \delta S_{21}^T - E_L^2 \cdot \delta S_{22}^T \end{aligned} \quad (17)$$

The expressions simplify for the TMSO method ($M_1 = 0$). Particularly, $\delta E_D = -\delta M_1 \cdot E_R$ whereas the influence of δK and δL on directivity deviations is zero. To obtain the expressions for the dashed error term deviations $\delta E'_S, \dots, \delta E'_L$, index 1 is replaced by 2 and vice versa, and E_S, \dots, E_L by E'_S, \dots, E'_L . The influence of a given deviation δM_1 of a non-ideal low-reflective broadband load at port 1 of the VNA on the δE_L , δE_S , and δE_D has been tested. The values of E_S , E_R , and E_L were obtained by either an “ideal” TMSO calibration of the VNA including a sliding load technique (i.e. $M_1 = 0$), a plane short circuit for S and a standard open circuit for O taken from the calibration kit or a “non-ideal” calibration with the low-reflective load at port 1, while all other standards were considered as ideal. The calculated error term deviations δE_L , δE_S , and δE_D agreed well with the differences of the values of E_S , E_R , and E_L , respectively, which were obtained experimentally after the “non-ideal” or the “ideal” calibration. As $|E_L|$ is small, $\delta E_L \approx \delta M_1$ is valid. The other deviations are also of nearly the same magnitude as δM_1 . An example for δE_S is shown in Fig. 2. The results may be used for comparison with results of the well-known “ripple method” (Guidelines, 2000) where δE_S and δE_D are evaluated from the modulus of reflections measured

vs. frequency of a precision air line impedance standard terminated either by a short circuit or by a low-reflective load, respectively. The applicability of the ripple method could be tested especially for thin coaxial line systems (2,92 mm and 2,4 mm) where reflections of an imperfect inner conductor contact at the test port become significant and therefore an increase to the $|\delta E_S|$ and $|\delta E_D|$ values may occur due to these reflections.

4 Sensitivity coefficients: dependence of the S-parameters on reflection and through deviations

By inserting Eqs. (13) to (17) – and the corresponding expressions for the dashed error term deviations – into Eqs. (6) and (7), we finally obtain 40 sensitivity coefficients of the S-parameters S_{ik} of arbitrary two-port test objects with respect to the non-ideal calibration standards, represented by the deviations $\delta M_{1,2}$, $\delta K_{1,2}$, $\delta L_{1,2}$, δS_{11}^T , δS_{12}^T , δS_{21}^T , and δS_{22}^T . These expressions are functions of the actual values of S_{ik} , $M_{1,2}$, $K_{1,2}$, $L_{1,2}$, and the parameters S_{21}^T , S_{12}^T of the through (line) connection. We obtain for δS_{11} , δS_{12} and the deviations δM_1 , δM_2 :

$$\delta S_{11}(\delta M) = \frac{(1 - S_{11} E'_L) \cdot Z_{1M}}{(K_1 - M_1) \cdot (L_1 - M_1) \cdot N} \cdot \delta M_1 + \frac{S_{21} S_{12} E_L Z_{2M}}{(K_2 - M_2) \cdot (M_2 - L_2) \cdot N} \cdot \delta M_2, \quad (18)$$

$$\delta S_{12}(\delta M) = \frac{S_{12} E'_L Z_{1M}}{(K_1 - M_1) \cdot (M_1 - L_1) \cdot N} \cdot \delta M_1 + \frac{S_{12} \cdot (1 - S_{22} E_L) \cdot Z_{2M}}{(K_2 - M_2) \cdot (L_2 - M_2) \cdot N} \cdot \delta M_2, \quad \text{where} \quad (19)$$

$$Z_{1M} = (S_{11} - K_1) \cdot (S_{11} - L_1) \cdot (1 - S_{22} E_L) - S_{21} S_{12} \cdot \left(\frac{K_1 L_1}{S_{21}^T S_{12}^T} - S_{11} E_L \right), \quad (20)$$

$$Z_{2M} = S_{22} \cdot (1 - S_{11} E'_L) + (S_{12} S_{21} - S_{12}^T S_{21}^T) \cdot E'_L + S_{11} \cdot \left[(K_2 + L_2) \cdot E'_L - \frac{K_2 L_2}{S_{12}^T S_{21}^T} \right], \quad (21)$$

for the deviations δK_1 , δK_2 :

$$\delta S_{11}(\delta K) = \frac{(1 - S_{11} E'_L) \cdot Z_{1K}}{(M_1 - K_1) \cdot (L_1 - K_1) \cdot N} \cdot \delta K_1 + \frac{S_{21} S_{12} E_L Z_{2K}}{(M_2 - K_2) \cdot (K_2 - L_2) \cdot N} \cdot \delta K_2, \quad (22)$$

$$\delta S_{12}(\delta K) = \frac{S_{12}E'_L Z_{1K}}{(M_1 - K_1) \cdot (K_1 - L_1) \cdot N} \cdot \delta K_1 + \frac{S_{12} \cdot (1 - S_{22}E_L) \cdot Z_{2K}}{(M_2 - K_2) \cdot (L_2 - K_2) \cdot N} \cdot \delta K_2, \quad \text{where} \quad (23)$$

$$Z_{1K} = (S_{11} - M_1) \cdot (S_{11} - L_1) \cdot (1 - S_{22}E_L) - S_{21}S_{12} \cdot \left(\frac{M_1 L_1}{S_{21}^T S_{12}^T} - S_{11}E_L \right), \quad (24)$$

$$Z_{2K} = S_{22} \cdot (1 - S_{11}E'_L) + (S_{12}S_{21} - S_{12}^T S_{21}^T) \cdot E'_L + S_{11} \cdot \left[(M_2 + L_2) \cdot E'_L - \frac{M_2 L_2}{S_{12}^T S_{21}^T} \right], \quad (25)$$

for the deviations $\delta L_1, \delta L_2$:

$$\delta S_{11}(\delta L) = \frac{(1 - S_{11}E'_L) \cdot Z_{1L}}{(K_1 - L_1) \cdot (M_1 - L_1) \cdot N} \cdot \delta L_1 + \frac{S_{21}S_{12}E_L Z_{2L}}{(K_2 - L_2) \cdot (L_2 - M_2) \cdot N} \cdot \delta L_2, \quad (26)$$

$$\delta S_{12}(\delta L) = \frac{S_{12}E'_L Z_{1L}}{(K_1 - L_1) \cdot (L_1 - M_1) \cdot N} \cdot \delta L_1 + \frac{S_{12} \cdot (1 - S_{22}E_L) \cdot Z_{2L}}{(K_2 - L_2) \cdot (M_2 - L_2) \cdot N} \cdot \delta L_2, \quad \text{where} \quad (27)$$

$$Z_{1L} = (S_{11} - K_1) \cdot (S_{11} - M_1) \cdot (1 - S_{22}E_L) - S_{21}S_{12} \cdot \left(\frac{K_1 M_1}{S_{21}^T S_{12}^T} - S_{11}E_L \right), \quad (28)$$

$$Z_{2L} = S_{22} \cdot (1 - S_{11}E'_L) + (S_{12}S_{21} - S_{12}^T S_{21}^T) \cdot E'_L + S_{11} \cdot \left[(K_2 + M_2) \cdot E'_L - \frac{K_2 M_2}{S_{12}^T S_{21}^T} \right]. \quad (29)$$

The sensitivity coefficients are symmetrical with reference to the M_i, K_i , and L_i which can – as well as the S_{jk} – be of arbitrary values. For the through (line) deviations $\delta S_{11}^T, \delta S_{12}^T, \delta S_{21}^T, \delta S_{22}^T$:

$$\begin{aligned} \delta S_{11}(\delta S_{ik}^T) &= \frac{S_{12}S_{21} \cdot \left(\frac{1 - S_{11}E'_L}{S_{21}^T S_{12}^T} - E_L E'_L \right)}{N} \cdot \delta S_{11}^T - \frac{S_{11}S_{12}S_{21}E_L E'_L}{S_{12}^T \cdot N} \cdot \delta S_{12}^T - \frac{S_{11}S_{21}S_{12}E_L E'_L}{S_{21}^T \cdot N} \cdot \delta S_{21}^T \\ &\quad - \frac{S_{11}S_{12}S_{21}E_L}{S_{12}^T S_{21}^T \cdot N} \cdot \delta S_{22}^T, \end{aligned} \quad (30)$$

$$\begin{aligned} \delta S_{12}(\delta S_{ik}^T) &= \frac{S_{12}E'_L \cdot \left(1 - S_{22}E_L - \frac{S_{21}S_{12}}{S_{21}^T S_{12}^T} \right)}{N} \cdot \delta S_{11}^T + \frac{S_{12} \cdot (1 - S_{22}E_L - S_{21}S_{12}E_L E'_L)}{S_{12}^T \cdot N} \cdot \delta S_{12}^T + \\ &\quad + \frac{S_{12}S_{11}E'_L \cdot (1 - S_{22}E_L)}{S_{21}^T \cdot N} \cdot \delta S_{21}^T + \frac{S_{12}S_{11} \cdot (1 - S_{22}E_L)}{S_{12}^T S_{21}^T \cdot N} \cdot \delta S_{22}^T, \end{aligned} \quad (31)$$

$$\text{where } N = (1 - S_{22}E_L) \cdot (1 - S_{11}E'_L) - S_{21}S_{12}E_L E'_L. \quad (32)$$

To obtain the sensitivity coefficients for δS_{21} and δS_{22} , index 1 is replaced by 2 and E_L by E'_L and vice versa.

5 Experimental and calculations

The sensitivity coefficients have been experimentally verified with a set of coaxial (PC-7) two-port test objects of (nominal) attenuation 0 dB to 60 dB. A high-reflective test object was a step attenuator with three attenuation settings (0 dB, 3 dB, and 60 dB). The S_{11} of these objects varied with frequency (2–18 GHz) between about $-0,6$ and $+0,6$. Low-reflective

devices were of nominal 0 dB, 10 dB, and 60 dB attenuation. A rather large reflection coefficient of a mismatch of VSWR 1,5 as M standard was used for δM_1 (or δM_2), while M_1 and M_2 were set to zero in the calibration routine and in the sensitivity coefficients. The calculated deviations δS_{11} and δS_{12} agreed well with the differences of the values of S_{11} and S_{12} , respectively, which were obtained experimentally after calibrations using either this mismatch or an “ideal” sliding load. Results are shown in Fig. 3 for δM_2 .

To show the influence of small values of $\delta M_i, \delta K_i$, and δL_i at either port 1 or port 2 as they may occur with real calibration items from commercial calibration kits, additional

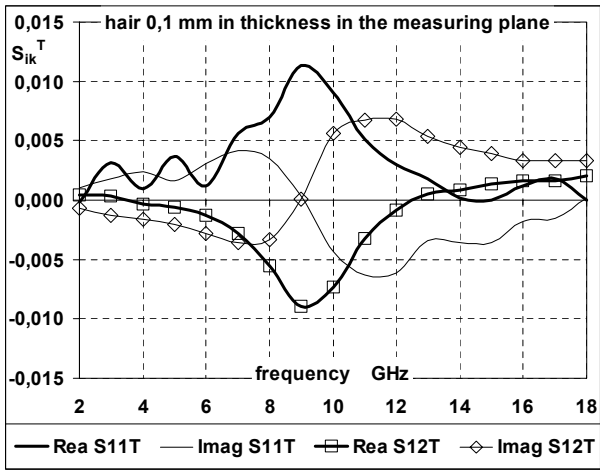


Fig. 5. S-parameters S_{ik}^T of a hair 0,1 mm in thickness in the through connection when stretched across the connection plane of the two VNA flexible PC-7 coaxial test cables, measured with “ideal” TMSO calibration. S_{22}^T is nearly equal to S_{11}^T and S_{21}^T is nearly equal to S_{12}^T .

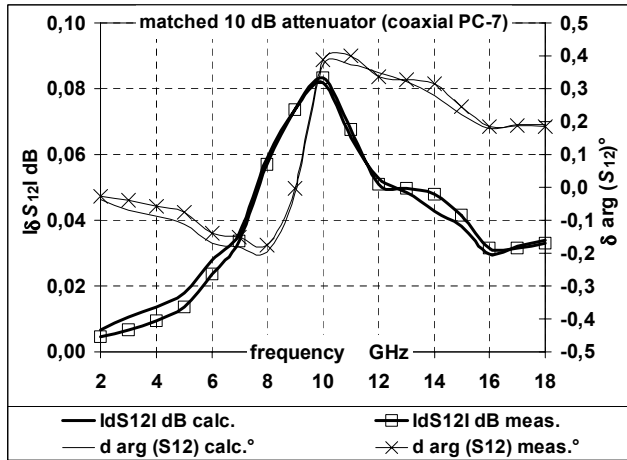


Fig. 6. Calculated and measured deviations $|\delta S_{12}|$ in dB and phase deviations $\delta \arg(S_{12})$ for a matched 10 dB attenuator using a non-ideal through with a hair whose S-parameters are shown in Fig. 5, instead of an ideal through, for the TMSO calibration.

calculations have been carried out. The experimentally obtained S_{jk} values of the test objects and the values for δM_1 or δM_2 of the broadband low-reflective termination (shown in Fig. 2) were used. In Fig. 4, calculated deviations of transmission δS_{12} are shown, for the high-reflective 3 dB attenuator, using the low-reflective load (δM_2) instead of a sliding load at port 2 for the TMSO calibration. This example shows that, if applying a broadband load of maximum reflection of 0,05 (instead of using the time consuming “ideal” sliding load routine) and the VNA-internal firmware-operated calibration and evaluation routines where the reflection coefficient of M is set to zero, deviations may appear of the order of 0,4 dB and 3° respectively. For low-reflective test objects, also significant deviations of about 0,1 dB and 0,5° may occur. The influence of a non-ideal open or short circuit with an offset length deviation of 0,01 mm or a short circuit loss of about $\delta K_1 = \delta K_2 = 0,02$ at the upper coaxial band end (18 GHz) has been found to be significantly smaller than the influence of the non-ideal broadband load (Stumper, 2003).

The influence of a “non-ideal” through was separately investigated. To generate a non-ideal through, after having carried out a TMSO calibration with an ideal through which was then measured ($S_{ik}^{T ideal}$), small disturbing matters of different shape and material were then inserted in the connection plane and the S-parameters $S_{ik}^{T non-ideal}$ measured. With this non-ideal through, a second calibration – now non-ideal – was then carried out. To calculate the deviations δS_{ik} using Eqs. (30) and (31), the differences $S_{ik}^T = S_{ik}^{T non-ideal} - S_{ik}^{T ideal}$ were used as the δS_{ik}^T . For different test objects, the calculated deviations δS_{ik} agreed well with the differences of the values of the S-parameters obtained experimentally using either the error terms of the non-ideal or of the ideal calibration. In laboratory practice, small lints may get in between the end planes of the connectors. As an example, the measured four S-parameters S_{ik}^T (of about maximum 0,01) of a human hair 0,1 mm in thickness in the through connection are shown in Fig. 5. The deviations of modulus and phase of the transmission coefficient of a matched 10 dB attenuator – due to the S-parameters of this hair – are shown in Fig. 6. Even with this thin hair, the devia-

tions are of maximum values of about 0,1 dB and about $0,4^\circ$, respectively.

6 Conclusions

Analytical expressions have been developed for the estimation of uncertainties in S-parameter measurement due to deviations from the ideal values of the reflection coefficients of the standards used for the 12-term LMSO or TMSO calibration and due to an imperfect line or through connection. The expressions are free of raw values and error terms except the load match. The results are experimentally verified and show that even for small deviations in the reflection values of the used standards from the “ideal” state – as assumed in the firmware evaluation – considerable deviations may be expected in the S-parameters in particular if the test objects are reflective. Even for a matched attenuator, the deviation is of the order of 0,1 dB if a thin hair is introduced while making the through connection.

The expressions for the sensitivity coefficients can be helpful particularly for on-wafer measurements or for coaxial lines of small diameter where the usual estimation of uncertainties including the “ripple method” yields unsatisfactory results.

Acknowledgements. The author is indebted to Thorsten Schrader for performing the verification measurements.

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