Analytical and Numerical Investigation of the Sound Power Emission of a Vibrating Baffled Piston Into a Hemi-Anechoic Room

Katharina Völkel, Martin Schmelzer, and Volker Wittstock

Physikalisch-Technische Bundesanstalt (PTB), Bundesallee 100, 38116 Braunschweig, Germany, Email: katharina.voelkel@ptb.de

Introduction

At this point in time there is no option of a direct measurement of sound power levels. Instead it is common to measure sound pressure levels and calculate the corresponding sound power levels. However, the calculated sound power levels critically depend on the assumptions made on the sound field; free field or diffuse field for example. Thus, sound powers determined by different methods differ from each other and their uncertainties are difficult to obtain. For this reason, the traceability of sound power is now being addressed in a project funded by the European Metrology Research Programme. One goal of this project is the development of a primary realisation of the unit Watt based on a vibrating piston whose sound power emission can be calculated from non-acoustic quantities such as velocity and acceleration as described in [5].

Comparisons of measurement data and analytical as well as numerical data for a first set up were analyzed and reported in [4]. In this paper further analytical and numerical studies will be detailed, which aim at providing suggestions for set-up improvements.

Analytical Studies

As described above, the object of interest in this study was a vibrating piston; more specifically the piston was chosen to be a circular disk that is driven by an electrodynamic shaker (see Fig. 1). This assembly was installed inside a cavity in the floor of a hemi-anechoic room.

Figure 1: Teflon piston attached to a shaker. The piston moves inside a cylinder to avoid tilting motions. The creme colored foam material decouples the shaker from the aluminum bearings that connect the bottom and top plate.

To calculate the sound power output of this set-up a lumped parameter model was developed. From it the emitted sound power level was calculated (Fig. 2).

To calculate the Eigenmodes of a rigid piston the wave equation for a plate was analyzed (see [1]). These oscillations then had to be sampled and inserted into Rayleigh’s integral in discretised form to yield the emitted sound power into an ideal free field.

Calculations of sound power levels were performed for a piston with radius \( R = 0.0495 \) m comparing Rayleigh’s integral to the case of a rigid piston. The rigid piston was assumed to move uniformly with the same velocity \( v_0 \) as the center point of the bending disc. Results from these calculations were compared to sound intensity and sound pressure measurement data from PTB’s hemi-anechoic room. It can be seen that analytical results using the assumption of a bending plate provide the better fit (Fig. 3(a)) while there is no significant difference in calculating sound power level from sound intensity or sound pressure measurements (Fig. 3(b)).

Figure 2: Sound power level of a 0.1 kg piston. Red line: tonal excitation at \( U = 1 \) V. Black line: Pink noise with \( U_{tot}=1 \) V.

Figure 3: Comparison of measurement data with calculated values

(a) Pressure Measurements

(b) Intensity Measurements

Summarizing the results of the analytical analysis one can say that a thicker plate at the piston end will lead to higher first eigenfrequencies as long as thickness modes of vibration can be avoided. Also, to have an adequate coverage of the frequency range from 50 Hz to 20 kHz, two pistons will most likely be needed.
Numerical Studies
Comparison of Sound Power Level

Of practical interest in the determination of accurate measurement procedures was the question whether a rigid piston behaves like a monopole in terms of sound power emission - at least for low frequencies. If so, calculation of sound power level from sound pressure measurements should yield the same results as calculations from sound intensity measurements, i.e. Eq. (1) should hold according to [3].

\[ P_F = \frac{1}{\rho c} \int p^2 dS = \bar{P}_I = \int I_n dS \]  

(1)

To test this a 3D numerical model of a rigid piston with radius \( R = 0.05 \) m was developed in COMSOL@. Excitation of the piston was modeled to occur with prescribed acceleration at the center point and piston movement was modeled to be uniform without any tilting or bending.

Data for sound pressure and sound intensity were accumulated for ten points each along radial arcs with distances of \( r_1 = 0.5 \) m, \( r_2 = 1.0 \) m, and \( r_3 = 3.0 \) m from the source. The numerically calculated sound power levels from pressure and intensity data were referenced to analytically calculated sound power levels from the prescribed acceleration data. Results indicate good agreement between all methods of sound power level determination as all methods provide results that - independent of distance \( r_i \) - differ by no more than 0.03 dB from each other in the range up to 200 Hz. Frequencies higher than 200 Hz could not be investigated due to lack of computational power.

Near Field Effect

The near field was defined to be that volume of air around the piston surface where sound pressure and sound velocity are out of phase, i.e. \( \phi_p \neq \phi_v \). For a monopole the phase angle between sound pressure and the radial component of sound velocity depends on the wavenumber \( k \) and radial distance \( R \) as given by eq. (2).

\[ \angle (p_v) = -\arctan \left( \frac{1}{kR} \right) \]  

(2)

So, to test whether a rigid piston behaves like a monopole an analysis similar to the one for sound power level was carried out. In particular, the phase angles of sound pressure and the radial component of sound velocity were calculated for ten points along arcs of angles \( \theta_1 = 30^\circ \), \( \theta_2 = 50^\circ \), and \( \theta_3 = 70^\circ \) from the numerical data obtained for the three radial distances \( r_1 \), \( r_2 \), and 3 named above and compared to calculations via eq. (2). Again, the differences for all points lie within one degree of each other for frequencies up to 200 Hz which indicates that a rigid piston indeed behaves like a monopole for low frequencies.

Directivities

Lastly, the directivity of sound emission of a rigid piston was investigated. This was done by evaluating the directivity factor given in [2]. This analytical value was compared to the numerical data for the ten points along the radial arc \( r_1 \) (Fig. 4). Again, there is very good agreement between the numerical and analytical data. Moreover, sound emission is uniform for low frequencies which supports the hypothesis of monopole like behavior of a rigid piston for those frequencies.

Figure 4: Directivity of Sound Power Emission of Rigid Piston. Dots indicate numerical data, lines analytical calculations.

Conclusion

Investigated here was the sound power emission of a rigid piston. Calculated sound power levels correspond to measurement data. However, measurement data indicate that the piston used underwent bending. Refinement of the set-up thus needs to be considered. Numerical studies analyzing sound power level, near field effect, and directivity of emission confirm that for frequencies up to 200 Hz a rigid piston behaves like a monopole.

This study is part of the EMRP project SIB56 sound power. The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union.

References


