A new definition of the SI for the new century

to celebrate the PTB's first 3×42 –1 years = 1111101 years

Presentation from Ian Mills, President of the CCU

We have known for more than 50 years that the international prototype of the kilogram (IPK) is ‘drifting’ (changing in mass) relative to other Pt-Ir standards. We do not know by how much in an absolute sense, nor do we know whether all the Pt-Ir standards may be drifting together.

The kilogram is not the only base unit whose definition needs attention.

We desire new definitions, referenced to fundamental constants (constants of nature), for all the base units of the SI. It should be possible to realise them by anyone, anywhere, at any time, as precisely as the best measurements require.

Resolution 1 of the 24th CGPM is a proposal for a possible future revision of the SI to achieve these objectives.

Brief summary of the changes in the New SI proposed in Draft Resolution A

- New definitions are proposed for the kg, A, K and mol referenced to the fundamental constants h, e, k and N_A ;
- A new presentation of the SI that emphasises the importance of the fundamental constants that are used as references to define the units rather than the base units and derived units:

  Part I specifies the seven reference constants that define the entire SI, without reference to base units and derived units,

  Part II is the traditional presentation based on defining the traditional base units by linking each to one of the seven reference constants.

The seven base units are also presented in the revised order:
s, m, kg, A, K, mol, and cd, so that no definition of a base unit depends on others that come later in the list.

Four new reference constants

Unit		***Reference constant used to define the unit***		
		in the current SI	***in the New SI***	
second,	s	$\mathrm{D}n\,(^{133}\mathrm{Cs})_{\mathrm{hfs}}$	$\mathrm{D}n\,(^{133}\mathrm{Cs})_{\mathrm{hfs}}$	Cs hyperfine splitting
metre,	m	c	c	speed of light in vacuum
kilogram,	kg	$m(K)$	h	Planck constant
ampere,	A	m_0	e	elementary charge
kelvin,	K	T_{TPW}	k	Boltzmann constant
mole,	mol	$M(^{12}\mathrm{C})$	N_{A}	Avogadro constant
candela,	cd	K_{cd}	K_{cd}	luminous efficacy of a 540 THz source

A revised presentation of the SI

First, define the entire SI by specifying the seven reference constants, without introducing base and derived units

Part I

The International System of Units, the SI, will be the system of units in which:

- the ground state hyperfine splitting frequency of the caesium 133 atom $\Delta n\,(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ is exactly 9 192 631 770 hertz,
- the speed of light in vacuum *c* is exactly 299 792 458 metre per second,
- the Planck constant *h* is exactly 6.626 06**X** … $\times 10^{-34}$ joule second,
- the elementary charge *e* is exactly 1.602 17**X** … $\times 10^{-19}$ coulomb,
- the Boltzmann constant *k* is exactly 1.380 6**X** … $\times 10^{-23}$ joule per kelvin,
- the Avogadro constant N_A is exactly 6.022 14**X** … $\times 10^{23}$ reciprocal mole,
- the luminous efficacy K_{cd} of monochromatic radiation of frequency 540 $\times 10^{12}$ Hz is exactly 683 lumen per watt,

where the symbol **X** denotes additional digits to be determined from the latest CODATA adjustment when the new definitions are adopted.

Then, give the individual definitions of the traditional base units, second, metre, kilogram, ampere, kelvin, mole, candela in terms of the definitional constants that we propose to adopt.

Part II

second

> **The second, symbol s, is the SI unit of time; its magnitude is set by fixing the numerical value of the ground state hyperfine splitting frequency of the caesium 133 atom, at rest and at zero thermodynamic temperature, to be equal to exactly 9 192 631 770 when it is expressed in the SI unit s^{-1}, which is equal to hertz.**

Thus we have the exact relation $\Delta n\,(^{133}\mathrm{Cs})_{\mathrm{hfs}} = 9\,192\,631\,770$ Hz. The effect of this definition is that the second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

metre

The metre, symbol m, is the SI unit of length; its magnitude is set by fixing the numerical value of the speed of light in vacuum to be equal to exactly 299 792 458 when it is expressed in the SI unit m s^{-1}.

Thus we have the exact relation $c = 299\ 792\ 458$ m/s. The effect of this definition is that the metre is the length of path travelled by light in vacuum during the time interval of 1/299 792 458 of a second.

kilogram

The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly 6.626 06X … $\times 10^{-34}$ when it is expressed in the SI unit s^{-1} m^2 kg, which is equal to J s.

Thus we have the exact relation $h = 6.626\ 06X \ldots \times 10^{-34}$ J s. The value of the Planck constant is a constant of nature, which may be expressed as the product of a number and the unit of action joule second, where J s = s^{-1} m^2 kg. The effect of this definition, together with those for the second and the metre, is to express the unit of mass in terms of the unit of frequency through the fundamental equations $E = mc^2$ and $E = hn$.

ampere

The ampere, symbol A, is the SI unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be equal to exactly 1.602 17X … $\times10^{-19}$ when it is expressed in the SI unit A s, which is equal to coulomb, C.

Thus we have the exact relation e = 1.602 17X… $\times10^{-19}$ C. The effect of this definition is that the ampere is the electric current corresponding to the flow of 1/(1.602 17X … $\times10^{-19}$) elementary charges per second.

kelvin

The kelvin, symbol K, is the SI unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant to be equal to exactly 1.380 6X … $\times10^{-23}$ when it is expressed in the SI unit s^{-2} m^{2} kg K^{-1}, which is equal to J K^{-1}.

Thus we have the exact relation k = 1.380 6X… $\times10^{-19}$ J/K. The effect of this definition is that the kelvin is equal to the change of thermodynamic temperature T that results in a change of thermal energy kT by 1.380 6X... ´ 10^{-23} J.

mole

The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle, or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be equal to exactly 6.022 14X… ×10^{23} when it is expressed in the SI unit mol^{-1}.

Thus we have the exact relation N_A = 6.022 14X… ×10^{23} mol^{-1}. The effect of this definition is that the mole is the amount of substance of a system that contains 6.022 14X… ×10^{23} specified elementary entities.

candela

The candela, symbol cd, is the unit of luminous intensity in a given direction; its magnitude is set by fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency 540 ´ 10^{12} Hz to be equal to exactly 683 when it is expressed in the SI unit s^{3} m^{-2} kg^{-1} cd sr, or cd sr W^{-1}, which is equal to lm W^{-1}.

Thus we have the exact relation K_{cd} = 683 lm/W, or lm/W = K_{cd} / 683, where K_{cd} is the luminous efficacy of monochromatic radiation of frequency ν = 540 ´ 10^{12} Hz. The effect of this definition is that the candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 ×10^{12} Hz and that has a radiant intensity in that direction of 1/683 W/sr.

Summary

The New SI as proposed in Resolution 1 will meet the growing needs of science, technology and commerce in the 21st century by providing:

- a definition of the kilogram based on an invariant of nature,
- exactly known values of h and e, and hence of K_J and R_K, thus bringing electromagnetic units within the SI,
- a simpler and more fundamental definition of the ampere referenced to the value of the elementary charge e,
- a definition of the kelvin referenced to the Boltzmann constant k, an invariant of nature that can be realised in many different ways, at a variety of temperatures,
- a definition of the mole that is clearly related to the quantity amount of substance and to counting entities,
- definitions of all seven SI base units based on seven reference constants in a standard explicit-constant format,
- significant improvements in our knowledge of many of the fundamental constants used in science.

Relative standard uncertainties for some fundamental constants in the current SI and the New SI, multiplied by 10^8 (in parts per hundred million) based on CODATA 2010 values of the fundamental constants

constant	***current SI***	***New SI***
$m(\mathrm{K})$	0	4.4
c	0	0
h	4.4	0
e	2.2	0
k	91	0
N_A	4.4	0
R	91	0
F	2.2	0
σ	360	0
m_e	4.4	0.064
m_u	4.4	0.070
$m(^{12}\mathrm{C})$	4.4	0.070
$M(^{12}\mathrm{C})$	0	0.070

constant	***current SI***	***New SI***
a	0.032	0.032
M_u	0	0.070
K_J	2.2	0
R_K	0.032	0
μ_0	0	0.032
ε_0	0	0.032
Z_0	0	0.032
$N_\mathrm{A} h$	0.070	0
J↔kg	0	0
$\mathrm{J}\leftrightarrow\mathrm{m}^{-1}$	4.4	0
J↔Hz	4.4	0
J↔K	91	0
J↔eV	2.2	0

(i) Using a reference constant to define a unit

If Q is a fundamental constant, or a constant of nature, then the *value* of Q is not for us to choose; that is why we call it a “constant of nature”.

But the *value* of Q is the product of a *numerical value* $\{Q\}$, and a *unit* $[Q]$:

$Q = \{Q\}\,[Q]$ (example: $c = \{c\}[c] =$ 299 792 458 m/s)

We may choose the two factors in different ways. If we define the unit $[Q]$ independently, we must determine the numerical value $\{Q\}$ by experiment.

But if we choose to define the numerical value $\{Q\}$ to suit our convenience, then that **fixes** (or **defines**) the unit $[Q]$. We describe this as using the value of Q as a *reference constant* to define the unit $[Q]$.

> (example: If we choose the numerical value $\{c\}$ to be exactly 299 792 458, then in the process we define the unit m/s. This is how the metre was re-defined in 1983, by using c as a reference constant to define the unit m/s.)

Thus by choosing the *numerical value* of a fundamental constant, we may define its *unit*. We choose the numerical value to maintain continuity with the previous value of the unit.

(ii) How do we realise the definition of the kilogram when it is defined by fixing *h* ?

Answer: any experiment that is used to determine the value of *h* in the current SI becomes an experiment to realise the kilogram in the new SI, in which the value of *h* is fixed by the definition. The uncertainty in the mass of the IPK, and in the value of *h*, are shown in the table below, **first** for the current SI in which the mass of the IPK is fixed, and **then** for the new SI in which the value of *h* is fixed.

reference constant fixed in the definition	*uncertainty in the current SI*	*uncertainty in the New SI*
mass of the IPK, $m(K)$ (in the current SI)	0 (exact)	4.4×10^{-8} (expt)
Planck constant, h (in the new SI)	4.4 ´ 10^{-8} (expt)	0 (exact)

Thus the uncertainty in the value of *h* in the curent SI is not lost in the new SI, in which the value of *h* is known exactly, but it becomes the uncertainty in the mass of the IPK on making the change. Similarly for each of the other new definitions.

Is the Avogadro constant N_A really a fundamental constant, analogous to the speed of light c or the Planck constant h ?

It is sometimes argued that N_A is a man-made constant, chosen for our convenience to establish the magnitude of the unit mole, and that it is not a fundamental constant like c, h, e, and k.

But that is a mis-understanding: it is the *numerical value* of N_A, not the *value* of N_A, that defines the mole.

The value of N_A is given by the product of a *numerical value* × *unit*,

$$N_A = \{N_A\}[N_A] = 6.022\ldots\times10^{23}\ \text{mol}^{-1}$$

The ***value*** of N_A is a constant of nature, just like the Planck constant h. It is an invariant. It is the conversion factor between the quantities number of entities N and amount of substance n: $N = N_a n$,

just as h is the conversion factor between frequency n and energy E: $E = hn$.

It is the ***numerical value*** of N_A that determines the ***unit*** mole,

just as the ***numerical value*** of c determines the ***unit*** m/s,

and the ***numerical value*** of h determines the ***unit*** $\text{kg m}^2\ \text{s}^{-1}$.

The kilogram ampere, kelvin and mole …

To define the kilogram ampere, kelvin and mole
is our role
It really wont do
if it is not all tickety-boo

Examples of the **clerihue** verse form:

Samuel Pepys
said it gave him the creeps
to see Nell Gwynn beckoned
by Charles the Second

Cecil B. de Mille
somewhat against his will
was persuaded to leave out Moses
from the wars of the roses

Liszt
used to bang the piano with his fist
– that was the way
he used to play

Further information on the New SI as proposed in Resolution 1 may be found in:

1. Resolution 1 itself, adopted by the 24th CGPM in October 2011,
2. Draft Chapter 2 for the SI Brochure as it might appear when the New SI is adopted, drafted by the CCU, available at http://www.bipm.org/en/si/new_si/
3. Frequently asked questions and answers (FAQs) about the New SI, drafted by the CCU, also available at http://www.bipm.org/en/si/new_si/
4. The paper "Adapting the International System of Units to the 21st century" by Mills IM, Mohr PJ, Quinn TJ, Taylor BN and Williams ER, in *Philosophical Transactions of the Royal Society A,* 2011, **369** (1953), 3903-4142 and in other papers in the same issue of *Phil Trans* presented at the Discussion Meeting on this subject held at the Royal Society in January 2011.

All these papers are available on the BIPM website. There are many further published papers discussing particular aspects of the proposals in this Resolution, some of which are listed in reference 4 above.

The history of 24th CGPM Resolution 1

This Resolution is the result of many years of discussion by many expert committees and individual metrologists. The CCU produced their final draft after lengthy discussions at their meetings in 2009 and 2010.

The CCU is made up of representatives from the following bodies and NMIs:

CIE, International Lighting Commission
ICRU, International Commission for Radiation Units
IEC, International Electrotechnical Commission
ISO, International Organisation for Standardisation
OIML, International Organisation for Legal Metrology
IUPAC, International Union for Pure and Applied Chemistry
IUPAP, International Union for Pure and Applied Physics
CODATA task group on Fundamental Constants
IAU, International Astronomical Union
IFCC, International Federation of Clinical Chemists

CEM, Madrid
NIM, Beijing
NMIJ / AIST Tsukuba
NIST Gaithersburg
NPL Teddington
PTB Braunschweig
Rostekhregulirovaniye, Moscow

- plus two personal members:

T J Quinn CBE FRS, Emeritus Director of the BIPM,
M Himbert, Professeur de la Conservatoire Nationale des Arts et Metier,

- and ex-officio: the Director of the BIPM and the President of the CCU.

We have also had about 12 further invited guests at our recent meetings, including the Executive Secretaries of all the relevant Consultative Committees, to provide us with further expert advice and with the views of a wider community.

The CCU voted unanimously to present the draft for Resolution 1 to the CIPM for their approval in October 2010 (only the representatives from ISO and the IAU were not able to be present for the vote).

This draft was then slightly modified by the CIPM and then by the CGPM to arrive at the final draft, which has now been adopted unanimously by the CGPM.

CGPM-24 (2011) Resolution 1 is a proposal for a possible future revision of the SI. It is still open for discussion, although the present form of the resolution has been extensively discussed before it was presented to the 24th CGPM, and at the 24th CGPM the proposal in Resolution 1 was given unanimous approval.

The proposed revisions to the SI will be adopted by the CGPM as soon as further experimental results on the numerical values in the new definitions are regarded as acceptable.

The remaining slides in this presentation explain briefly the reasons behind the choice of the new definitional constants for the kg, A, K and mole.

What were the difficult choices?

The most common points of difficulty and subjects discussed when formulating the new definitions were as follows.

(i) How do you define a unit by fixing the value of a fundamental constant?

(ii) Would it not be better to use the mass of an atom, such as $m(^{12}\mathrm{C})$, rather than the Planck constant h as a reference to define the kilogram? Similarly why choose e in place of μ_0 to define the ampere, and N_A rather than $M(^{12}\mathrm{C})$ to define the mole?

(iii) How are the proposed changes going to affect our ability to realise and use the new definitions?

(iv) How can you fix exactly the value of a fundamental constant? How do you know what value to fix it to? What if you choose the wrong value? What happens to the present uncertainty in a fundamental constant when you fix it?

(v) Is the Avogadro constant truly a fundamental constant, and the quantity ‘amount of substance’ anything more than a shorthand for counting entities?

(ii) Choosing the reference constant to define the kilogram: *h* or *m*(^{12}C) ?

Some people argue that the reference constant for the kilogram should be a mass, just as the reference constant for the second is a time (the period of the caesium transition). But note:

1. The reference constant for the metre is the speed of light *c*, which is a speed – and is not a length. Fixing *c* actually defines the unit m/s, not the m. The reference constant for the ampere in the present SI is the magnetic constant μ_0, which is not an electric current. Fixing μ_0 actually defines the unit H/m, not the ampere A.
2. Thus reference constants do not ***have*** to be tied to base units. They do not ***have*** to be the same dimension as the unit. The reference constants are best regarded as a set of constants which together define the entire SI.
3. The quantities *h* and *m*(^{12}C) are related by the equation : in which the value of the product on the right hand side is at present known with a relative standard uncertainty of 3.2 ×10^{-10}. Thus we can calculate *h* from *m*(^{12}C), within an uncertainty of 3.2 ×10^{-10} .

$$\frac{h}{m(^{12}\mathrm{C})} = \frac{\alpha^2 c}{2R_\infty}\left(\frac{m_\mathrm{e}}{m(^{12}\mathrm{C})}\right)$$

5. Fixing *h*, combined with fixing *e* to define the ampere, has the clear advantage that we would have exactly known values of K_J and R_K , thus bringing electromagnetic units within the SI. Thus fixing *h* is strongly preferred by the electromagnetic community.
6. Also *h* and *e* are amongst the most fundamental constants of physics.
7. For these reasons it is preferable to fix *h* to define the kilogram in the new SI.

Choosing the reference constant to define the ampere: *e* or μ_0 ?

This choice defines all the electrical units.

1. Both ***e*** **or** $\boldsymbol{\mu}_0$ are fundamental constants.
2. But the elementary charge ***e*** is a simpler concept to most people than the magnetic constant $\boldsymbol{\mu}_0$. The magnetic constant calls for a deeper understanding of electromagnetic theory than is common among bio-scientists, for example.
3. Fixing ***e*** to define the ampere and ***h*** to define the kilogram has the clear advantage that we would have exactly known values for the Josephson constant $K_J = 2e/h$, and the von Klitzing constant $R_K = h/e^2$, thus bringing electromagnetic units within the SI. For this reason fixing ***e*** is strongly preferred to fixing $\boldsymbol{\mu}_0$ by the electromagnetic community.
4. Fixing the values of *c*, *h* and *e* give exact values for three of the most fundamental constants of physics. To have the values of these constants exactly known would be a significant advantage for fundamental physics.
5. For reasons 3 and 4 it is preferred to choose *e* to define the ampere.

Relative standard uncertainties for some fundamental constants in the current SI and the new SI, multiplied by 10^8 (in parts per hundred million)

constant	***current SI***	***new SI***
$m(\mathrm{K})$	0	4.4
h	4.4	0
e	2.2	0
k	91	0
N_A	4.4	0
R	91	0
F	2.2	0
σ	360	0
m_e	4.4	0.064
m_u	4.4	0.070
$m(^{12}\mathrm{C})$	4.4	0.070
$M(^{12}\mathrm{C})$	0	0.070

constant	***current SI***	***new SI***
a	0.032	0.032
K_J	2.2	0
R_K	0.032	0
μ_0	0	0.032
ε_0	0	0.032
Z_0	0	0.032
$N_\mathrm{A} h$	0.070	0
$\mathrm{J}\leftrightarrow\mathrm{kg}$	0	0
$\mathrm{J}\leftrightarrow\mathrm{m}^{-1}$	4.4	0
$\mathrm{J}\leftrightarrow\mathrm{Hz}$	4.4	0
$\mathrm{J}\leftrightarrow\mathrm{K}$	91	0
$\mathrm{J}\leftrightarrow\mathrm{eV}$	2.2	0

Base units and derived units, or reference constants ?

The traditional presentation of the SI is made in terms of ***base units*** and ***derived units***. The choice of the seven base units is somewhat arbitrary; it could be made in many different ways without changing the system.

The particular choice in the current SI, **m, kg, s, A, K, mol,** and **cd,** is based on history rather than logic.

The Resolution 1 recommends a more fundamental presentation of the SI by specifying the values of the ***seven reference constants*** used to define the units, without explicitly connecting the individual constants to individual base units. We describe these as ‘explicit-constant definitions’.

The equivalent alternative presentation in terms of base and derived units is also retained, in order to preserve the historical connection with the traditional language. But the choice of reference constants is more fundamental than the choice of base units.

We also recommend changing the order used to present the base units to **s, m, kg, A, K, mol, cd** so that no definition of a unit involves one of the other units later in the list.

Is the Avogadro constant N_A really a fundamental constant, analogous to the speed of light *c* or the Planck constant *h* ?

It is sometimes argued that N_A is a man-made constant, chosen for our convenience to establish the magnitude of the unit mole, and that it is not a fundamental constant like *c*, *h*, *e*, and *k*.

But that is a mis-understanding: it is the *numerical value* of N_A, not the *value* of N_A, that defines the mole.

The value of N_A is given by the product of a *numerical value* × *unit*,

$$N_A = \{N_A\}\,[N_A] = 6.022\ldots\times 10^{23}\ \mathrm{mol}^{-1}$$

The ***value*** of N_A is a constant of nature, just like the Planck constant *h*. It is an invariant. It is the conversion factor between the quantities number of entities *N* and amount of substance *n*: $N = N_a n$,

just as *h* is the conversion factor between frequency *n* and energy *E*: $E = hn$.

It is the ***numerical value*** of N_A that determines the ***unit*** mole,

just as the ***numerical value*** of *c* determines the ***unit*** m/s,

and the ***numerical value*** of *h* determines the ***unit*** kg m^2 s^{-1}.