Quantum Measurement and Control

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Quantum control: Can we get more for less?

- Achieve universal q control using NO local addressing?
- Global Control: Transport/Computing
- Scheme to perform N-cavity mirror gate in Circuit-QED using global pulses
- Can we perform FT QEC with little local addressing or little/no measurement?

Nonlinear quantum optics

- Optical nonlinearities: single photon switch/blockade, deterministic q gates with single photons?
- Can we use the strong coupling to generate “huge” optical nonlinearities?
- Can we measure the photon anti-bunching: Hanbury-Brown-Twiss?

Coupling microwave photons to atomic ensembles

- Use atomic ensembles for memory etc, How to scale down the ensemble size but still retain large coupling strengths?
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Can we do without local control in Q Devices?

Huge technological challenge to engineer the control circuitry for each and every qubit in a device....

Can we perform q operations **without local control**?

- e.g. Q Transport: A N-N coupled chain, many local SWAPs to shuttle qubits
- e.g. Q Computing: Can we perform without ANY local addressing at all?
- e.g. Q Error Correction: how much local addressing does one really need?

Earliest Design: Lloyds ABCABC...chain

A Potentially Realizable Quantum Computer

Seth Lloyd

Science (1993)

Arrays of weakly coupled quantum systems might compute if subjected to a sequence of electromagnetic pulses of well-defined frequency and length. Such pulsed arrays are true quantum computers: bits can be placed in superpositions of 0 and 1, logical operations take place coherently, and dissipation is required only for error correction. Operated with frequent error correction, such a system functions as a parallel digital computer. Operated in a quantum-mechanically coherent manner, such a device functions as a general-purpose quantum-mechanical micromachinist, capable of both creating any desired quantum state of the array and transforming that state in any desired way.

Fig. 1. Two wires. In (1), data is encoded in the A units in a section, and all the B's and C's, except for one unit, are set to 0. Call the unit in which C = 1 the control unit. In (2), the array has been subjected to a series of pulses that realizes a Fredkin gate on each triple ABC. The only triple affected is the one in which the control unit sits. Here, the bit of data has been moved to the B unit. In (3), the information in the BC units has been moved 3 triples to the right by the information-swapping process given in the text. In (4), the operation of a Fredkin gate on all triples has swapped x₁ with x₄, all other triples are unchanged. In (5), the information in the BC units has been moved back three triples to the left. In (6), the operation of a Fredkin gate on all triples has restored x₄ to the A unit in the triple. The first three configurations show the action of the first wire, moving x₁ adjacent to x₄; the second three configurations show the action of the second wire, moving x₄ back to x₁’s old place. The set of pulses transporting the data is independent of the data being transported.
Can we do without local control in Q Devices?

- e.g. Q Transport: using no local addressing?
  - Perfect Q Transport of bosonic modes within a chain of CPW resonators
  - ✔️

- e.g. Q Computing: Can we perform without ANY local addressing at all?
  - Quick reference to two types of global QIP schemes: verified only in NMR to date
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- e.g. Q Error Correction: without ANY local addressing?
  - No Fault Tolerant QEC scheme using only global control
  - New results on Unitary FT semi-global QEC
  - ❓
Q Transport using no local addressing?

- How to transport quantum information in solid-state: q-wires?
- What is quantum mirror transport?
- Basic qubit mirror scheme for perfect transport
- Mirror transport scheme for continuous variables (CV)
- Possible scheme in Circuit-QED for CV q-mirror transport
Q-Wires

Quantum Wires:

Transport information around a Q.I. Processor:

- Will probably be required in all physical implementations
- Must transport with near perfect fidelity
- Q-Wire should require no detailed control to work
- Would like massive parallelism...?
First Work: S. Bose

S. Bose PRL 91, 207901 (2003), Heisenberg Chain, S=1/2, in Ground state.

- Alice injects unknown $|\psi_A\rangle$
  - Not an eigenstate of chain
  - Evolves in a wave excitation to Bob

- Wave suffers dispersion
  - $\langle \psi_A | \rho_B | \psi_A \rangle \sim 1/L^{2/3}$

- Small chains ok. Other ways?
Q Transport protocols....

Inhomogenous coupling strengths

\[ J_i \sim \sqrt{i(N - i)} \]

\( \Omega \)

(+): Perfect transport in finite time,
(-): Precise values for coupling strengths


Adiabatic transport

\[ |2\rangle \xrightarrow{\Omega_1} |3\rangle \xrightarrow{\Omega_2} |1\rangle \]

Longhi et al, PRB 76, 201101 (2007)
Schroer et al, PRB 76, 075306 (2007)

Dark state transport, (+): Robust control, (-): only 1 qubit at a time

Gapped system transport

\( j \ll J \)

(-): Not perfect transport, (+): could be easy to implement?

M. Plenio and F. L. Semio, NJP 7, 73 (2005)

Wavepacket system transport

(-): Not perfect, (-): implement?

T. J. Osborne and N. Linden, PRA 69, 052315 (2004).
Quantum Mirroring

- Transport of quantum information crucial in all QC designs

- What is Q Mirroring?

  \[ |\phi_1, \phi_2, \ldots, \phi_{N-1}\phi_N \rangle \rightarrow |\phi_N, \phi_{N-1}, \ldots, \phi_2\phi_1 \rangle \]

- Pure or mixed: flipped spatially, can be used to transport **entire registers** of quantum information at once

- Scheme of Christandl et al, performs q-mirroring but requires precise engineering of the coupling strengths and \( \max J_i \sim N \)

- Another way for perfect qubit mirroring?
Quantum Mirroring

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- What is Q Mirroring?

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Qubit Transport made simple?

SWAP Gate

Qubit Transport made simple?

If one qubit is $|0\rangle$, can omit gate in RED.
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Rewrite $|0\rangle$

$$|0\rangle = H|+\rangle$$
Qubit Transport made simple?

Decompose CNOT into Hadamards and CPHASE

$$\text{CNOT}_{12} = (I \otimes H) \text{CZ}_{12} (I \otimes H)$$

|a, b\rangle \rightarrow (-1)^{a \cdot b} |a, b\rangle

\begin{array}{|c|c|}
\hline
|00\rangle & \rightarrow |00\rangle \\
|01\rangle & \rightarrow |01\rangle \\
|10\rangle & \rightarrow |10\rangle \\
|11\rangle & \rightarrow -|11\rangle \\
\hline
\end{array}

$$\text{CZ}_{12}$$

$$|\psi\rangle \rightarrow$$

$$H |\psi\rangle$$

$$|+\rangle$$
Qubit Transport made simple?

- Chain to multiple qubits
Qubit Transport made simple?

Code new input states from beginning with extra Hadamards
Qubit Transport made simple?

Code extra CPHASEs which nominally do nothing
Qubit Transport made simple?

- CPHASE applied homogeneously along the chain
- Hadamards applied homogeneously
Qubit “Automata” Mirror Scheme

Global Gates

\[ S \equiv \prod_{s=1}^{N-1} CZ(s, s+1) \]

\[ H \equiv \prod_{s=1}^{N} H(s) \]

Mirroring Rules

Rules for propagating Pauli Ops

\[
S \sigma_z^{(a)} = \sigma_z^{(a)} S \\
S \sigma_x^{(0)} = \sigma_x^{(0)} \sigma_z^{(1)} S \\
S \sigma_x^{(N)} = \sigma_z^{(N-1)} \sigma_x^{(N)} S \\
S \sigma_x^{(a)} = \sigma_z^{(a-1)} \sigma_x^{(a)} \sigma_z^{(a+1)} S \\
H \sigma_z^{(a)} = \sigma_x^{(a)} H
\]
Mirroring Rules

\[ (HS)^{N+1} \sigma^{(a)}_x = \sigma^{(N-a)}_x (HS)^{N+1} \]

\[ (HS)^{N+1} \sigma^{(a)}_z = \sigma^{(N-a)}_z (HS)^{N+1} \]

\[ \rho_I = \sum_{\{i,j\}} C_{i,j} Z^{i_1} X^{j_1} \otimes \ldots \otimes Z^{i_N} X^{j_N} \]

\[ \rho_F = \sum_{\{i,j\}} C_{i,j} Z^{i_N} X^{j_N} \otimes \ldots \otimes Z^{i_1} X^{j_1} \]

Complete Mirror:

Mirroring

\[ Z^i X^j \text{ are basis for state} \]
F&T Scheme

C13 Labeled Alanine

Universal QIP

Have nearest neighbor coupling, have Hadamard gates homogeneously on all sites: can do Clifford operations
Universal QIP

Have nearest neighbor coupling, have Hadamard gates homogeneously on all sites: can do Clifford operations

For universal QIP: allow single qubit ops on end sites only!


Another scheme: R. Raussendorf, PRA 72, 052301 (2005)
Demonstration of QIP with Global addressing

3 qubit NMR Demo of Deutsch-Jozsa

Generalise to qudits and continuous variables?

QuDits

Continuous Variables

Qudits

Hilbert space $H_d$ spanned by $Z^i X^j$, where $i,j \in \mathbb{Z}^d$ and basis $\{|0\rangle, ..., |d-1\rangle\}$

**Pauli Ops**

$Z_d |n\rangle = \zeta^n |n\rangle$

$X_d |n\rangle = |n \oplus 1\rangle$

$\zeta = e^{i2\pi/d}$

**Gate Ops**

**Fourier Gate**

$F |n\rangle = \sum_{s=0}^{s=d-1} e^{i2\pi ns/d} |s\rangle$

$F^2 |n\rangle = |-n\rangle \mod d$

$F^4 = 1$

**Sum Gate**

$CS_{(a,b)} |n, m\rangle_{(a,b)} = |n, m \oplus n\rangle_{(a,b)}$

**CPHASE**

$CZ^{(a,b)} = \sum_{s,r=0}^{d-1} \zeta^{-sr} Z^s_{(a)} \otimes Z^r_{(b)}$
General Circuit

\[
\begin{align*}
|\phi\rangle & \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow |0\rangle \\
|0\rangle & \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow |+\rangle \\
|+\rangle & \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow |0\rangle \\
|0\rangle & \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow F^{-1} \rightarrow |\phi\rangle
\end{align*}
\]

\[|+\rangle = \mathcal{F}|0\rangle\]
Continuous Variable Mirror Transport?

Qubits defined via position eigenstates of a Harmonic Oscillator

- S Lloyd & S Braunstein, PRL 82, 1784 (1999)
- S Braunstein & P van Loock, RMP, 77, 513 (2005)

\[ |0\rangle \quad |1\rangle \]

\[ \hat{X}(q) \equiv e^{-iq\hat{p}} \quad \hat{Z}(r) \equiv e^{+ir\hat{x}} \]

\[ \hat{X}(q')|q\rangle = |q + q'\rangle \quad \hat{Z}(r)|q\rangle = e^{irq}|q\rangle \]

**Generalised Pauli Ops**

**CV Control Not**

**Fourier Gate: 1/4 Rotation**

**Phase Gate: QND Interaction!**
CV Mirror Circuit

Same as before but with Continuous Variable gates!

$$\overline{CZ} \equiv \prod_{i=1}^{N-1} e^{i \hat{q}_i \otimes \hat{q}_{i+1}}$$

$$\overline{F} \equiv \prod_{i=1}^{N} F_i$$
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\[ CZ \equiv \prod_{i=1}^{N-1} e^{i\hat{q}_i \otimes \hat{q}_{i+1}} \]

\[ F \equiv \prod_{i=1}^{N} F_i \]
CV 3-Mode Mirror: Fock states!

3 Harmonic Oscillator Mirror: Numerical Wigner Functions

Mode 1: Displaced Vac
Mode 2: Fock State n=2
Mode 3: Displaced Vac

Before

After

\[
\overline{CZ} \equiv \prod_{i=1}^{N-1} e^{i\hat{q}_i \otimes \hat{q}_{i+1}}
\]

\[
\overline{F} \equiv \prod_{i=1}^{N} F_i
\]
Circuit QED Implementation?

- Fourier Gate: \( F^{-1} \) – 1/4 rotation in phase space: natural rotation of the Harmonic Oscillator:

- Most complex: CPHASE Gate:

\[
\hat{\Sigma}_{12} \equiv \exp[i\hat{X}_1 \otimes \hat{X}_2]
\]

QND Interaction

How to make \( \hat{\Sigma}_{12} \)?
Circuit-QED

- 1/4 wave resonators coupled via CPBs
- Require: fast tuning of CPW and CPBs

Tunable CPWs:
- Sandberg et al., Appl. Phys. Lett. (2008), 0801.2479
Circuit-QED

- 1/4 wave resonators coupled via CPBs
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Circuit-QED

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Hamiltonian for 2 CPW

\[
\hat{H}_\pm / \hbar = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \frac{\omega_0}{2} \hat{\sigma}_z - \left[ g_a (\hat{a} + \hat{a}^\dagger) \pm g_b (\hat{b} + \hat{b}^\dagger) \right] (\hat{\sigma}_+ + \hat{\sigma}_-)
\]

\[
\Sigma_{12} \equiv \exp[i\hat{X}_1 \otimes \hat{X}_2]
\]

Master Eqn

\[
\dot{\rho} = \frac{i}{\hbar} [\hat{H}_\pm, \rho] + \left( \frac{\gamma}{2} \mathcal{L}(\hat{\sigma}_-) + \frac{\kappa_a}{2} \mathcal{L}(\hat{a}) + \frac{\kappa_b}{2} \mathcal{L}(\hat{b}) \right) \rho
\]

• K. Moon & S. Girvin, PRL 95 140504 (2005)

Need ability to switch between sum and difference bias
Eliminate Fast CPB Dynamics

\[ \dot{\rho} = \mathcal{L}_3 \rho - \frac{1}{\hbar^2} \int_0^\infty d\tau \text{Tr}_{CPB} \left\{ e^{iH_1 \tau/\hbar} H_2 e^{-iH_1 \tau/\hbar}, [H_2, \rho^{th}_{CPB \otimes \rho}] \right\} , \]

\[ \dot{\rho} = -i/\hbar [\hat{H}_{eff}, \rho] + \sum_{j=a,b} \kappa_j \mathcal{L}(j) \rho + \eta \mathcal{L}(\hat{s}) \rho \]

\[ \hat{s} = \hat{X}_a \pm \hat{X}_b \]

\[ \hat{H}_{eff}/\hbar = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \chi \hat{s}^2 \]

To get QND from $H_{eff}$?: time dependent controls...

Effective Ham

$$\hat{H}_{eff} / \hbar = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \chi \hat{s}^2$$

Target Gate:

$$\hat{\Sigma}_{12} \equiv \exp[i \hat{X}_1 \otimes \hat{X}_2] = \exp(i \hat{X}^2_{a'}) \exp(-i \hat{X}^2_{b'})$$

$$\hat{T}(\theta, a, b) \equiv \exp(\theta [\hat{a}^\dagger \hat{b} - \hat{b}^\dagger \hat{a}]), \quad \theta = -\pi/4$$

To Build?:

$$\exp(-i \hat{X}^2_{b'})$$

Q Control:

$$\hat{H}'_{eff}(t) = \omega_+(t) (\hat{a'}^\dagger \hat{a'} + \hat{b'}^\dagger \hat{b'}) + 2\chi(t) \hat{X}^2_{a'}/b'$$

$$2\omega_+(t) = \omega_a(t) = \omega_b(t) \quad \chi(t) \sim \text{CPB freq}$$
To get UCPHASE from $H_{\text{eff}}$?

**Just the a-Part**

\[
\hat{H}_a'(\tau) = [1 + 0.02 \times C_1(\tau)]\hat{a}'\dagger\hat{a}' + [\epsilon + 0.02 \times C_2(\tau)]\hat{X}_a'^2, \quad \epsilon \sim 1 \times 10^{-3}
\]

**Gradient-Ascent Pulse Shaping (GRAPE)**

\[
\exp(i\hat{X}_a'^2) \text{ achieved with 99\% fidelity in 50 oscillator cycles}
\]

**May be able to move photon states around in a chain of coupled CPW resonators**

G Paz-Silva, S Rebic, J Twamley, T Duty, Phys Rev Lett. 102, 020503 (2009)
CV Mirror Circuit

Same as before but with Continuous Variable gates!

$$
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Can we do without local control in Q Devices?

Interestingly:

- e.g. Q Transport: using no local addressing?
  Perfect Q Transport of bosonic modes within a chain of CPW resonators

- e.g. Q Computing: Can we perform without ANY local addressing at all?
  Quick reference to two types of global QIP schemes: verified only in NMR to date

- e.g. Q Error Correction: without ANY local addressing?
  No Fault Tolerant QEC scheme using only global control
  New results on Unitary FT semi-global QEC
Globally controlled Fault Tolerant QEC?

- Semi-Global Scheme

- Uses **NO MEASUREMENTs** just global **RESETTING** of qubits

- Initial threshold estimates similar measured QEC schemes

- Huge technological advantage as no measurements required for FT QEC!!

G Paz-Silva, G Brennen, & J. Twamley arXiv:0911.XXXX
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Large optical nonlinearities

1875 John Kerr: refractive index of a media changed by electric field

$$P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} EE + \varepsilon_0 \chi^{(3)} EEE + \cdots$$

$$n = n_0 + n_2 I$$

Nonlinear coefficient for fused silica: $$n_2 \approx 10^{-16} \text{cm}^2/\text{W}$$

Need enormous electric fields to generate appreciable change in refractive index but large absorption on resonance

Quantum Mechanically: Kerr effect (single mode field):

$$\hat{H} \sim \hbar \omega (\hat{a}^\dagger \hat{a} + 1/2) + \hbar \frac{\chi}{2} (\hat{a}^\dagger)^2 \hat{a}^2$$
Large $\chi^{(3)}$?


**Light speed reduction to 17 metres per second in an ultracold atomic gas.** LV Hau, SE Harris, Z Dutton, CH Behroozi, Nature 397, 594 (1999).


Cavity Quantum Electrodynamics

Try using Jaynes-Cummings interaction between atomic and single mode photon:

\[ H = \hbar \omega_t \left( a^\dagger a + \frac{1}{2} \right) + \frac{\hbar \omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + \sigma^+ a) + H_\kappa + H_\gamma \]

- \( 2g \) = vacuum Rabi freq.
- \( \kappa \) = cavity decay rate
- \( \gamma \) = “transverse” decay rate
How to get large optical nonlinearities without strong absorption?
How to get large optical nonlinearities without strong absorption?
Electromagnetically Induced Transparency

Consider Electromagnetically Induced Transparency

Electromagnetically induced transparency: Optics in coherent media

EIT Absorption spectrum for:
\[ \gamma_{31} = 0, \ (a) \ \Omega_c = 0.3 \gamma_{21}, \ (b) \ \Omega_c = 2 \gamma_{21} \]
Schmidt & Imamoğlu

Giant Kerr nonlinearities obtained by electromagnetically induced transparency

\[ \mathcal{H}_{\text{eff}} = \hbar \eta (a^\dagger)^2 a^2 \]

Large Kerr nonlinearity with a single atom

Extra off-resonance transition causes Stark shifting of \( |3\rangle \) and leads to effective self-Kerr interaction for cavity photons

g_2, \omega_r

\[ \eta = \left( \frac{g_1}{\Omega_c} \right)^2 \left( \frac{g_2^2 \Delta}{\gamma_3^2 + \Delta^2} - \frac{g_1^2 \delta}{(\gamma_1 + \gamma_2)^2 + \delta^2} \right) \text{ for } g_1/\Omega_c \ll 1 \]
Take expts with “strong coupling” and estimate what self-Kerr coeff would be:

<table>
<thead>
<tr>
<th>Work</th>
<th>$g/2\pi$ (MHz)</th>
<th>$\kappa/2\pi$ (MHz)</th>
<th>$\gamma/2\pi$ (MHz)</th>
<th>$\eta/\kappa$</th>
</tr>
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<tbody>
<tr>
<td>D. Englund et al., Nature 450, 857 (2007)</td>
<td>8000</td>
<td>16000</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>K. M. Birnbaum et al., Nature 436, 87 (2005)</td>
<td>33</td>
<td>4.1</td>
<td>2.5</td>
<td>5.4</td>
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Larger self Kerr
How can we adapt Circuit-QED to generate Giant self-Kerr at microwave frequencies?
Using CPB qubits to build a CPB Molecule

Cap couple two CPB qubits to build a 4-level system

S. Rebic, J. Twamley, GJ Milburn, PRL 103, 150503 (2009)
Using CPB qubits to build a CPB Molecule

Cap couple two CPB qubits to build a 4-level system

\[ H = \sum_{j=1}^{2} \left( E_c^{(j)} (n^{(j)} - N_g^{(j)})^2 + E_j^{(j)} (1 - \cos \phi^{(j)}) \right) + E_m (n^{(1)} - N_g^{(1)}) (n^{(2)} - N_g^{(2)}) \]
Adjusting parameters can get 4-level system with arbitrary detunings:

\[ |1\rangle \quad |2\rangle \quad |3\rangle \quad |4\rangle \]

\[ g_1 \quad g_2 \quad \Omega_c \quad \omega_{12} \quad \omega_{34} \]

\[ \gamma_{21} \quad \gamma_{23} \quad \gamma_{31} \quad \gamma_{34} \quad \gamma_{41} \quad \gamma_{42} \]

No selection rules to suppress some decays...
Adjusting parameters can get 4-level system with arbitrary detunings:

Extra decays in the CPB molecule N-system!
Rough estimate of self-Kerr coefficient

If we ignore these extra decays and treat in the same limit as others:

\[ |4\rangle \quad \Delta \quad |2\rangle \]

\[ g_2, \omega_{34} \quad \gamma_{43} \quad \gamma_{23} \]

\[ \Omega_c \quad g_1 \quad \omega_{12} \]

\[ |3\rangle \quad \delta \quad |1\rangle \]

\[ \gamma_{41} \quad \gamma_{31} \quad \gamma_{21} \]

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<tr>
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<td>CPB Molecule</td>
<td>300</td>
<td>1</td>
<td>0.1</td>
<td>45,000</td>
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</table>
More exact calculation:

Model quantum Master equation:

\[ \dot{\rho} = \dot{\rho}_{sys} + \mathcal{L}\rho \]

\[
\dot{\rho}_{sys} = -i\Delta_{21} [\sigma_{22}, \rho] - i\Delta_{31} [\sigma_{33}, \rho] - i\Delta_{41} [\sigma_{44}, \rho] \\
+ g_1 \left[ a^\dagger \sigma_{12} - \sigma_{21} a, \rho \right] + g_2 \left[ a^\dagger \sigma_{34} - \sigma_{43} a, \rho \right] \\
+ \left[ \Omega_c^* \sigma_{32} - \sigma_{23} \Omega_c, \rho \right] + E_p \left[ a - a^\dagger, \rho \right],
\]

Hamiltonian

tiny driving of cavity

\[
\mathcal{L}\rho = \kappa \left( 2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a \right) \\
+ \sum_{(ij)} \frac{\gamma_{ij}}{2} \left( 2\sigma_{ji} \rho \sigma_{ij} - \sigma_{ij} \sigma_{ji} \rho - \rho \sigma_{ij} \sigma_{ji} \right).
\]

Cavity and internal decays

Get steady state:

\[ \rho(t \rightarrow \infty) \]
Second Order Correlation Function

\[ g^{(2)}(\tau) = \frac{\langle a^{\dagger}(t)a^{\dagger}(t+\tau)a(t+\tau)a(t) \rangle}{|\langle a^{\dagger}(t)a(t) \rangle|^2} \]
Lower $g^{(2)}(0)$ gives more antibunched and stronger blockade.
Effecting self-Kerr coefficient

Use known analytic formula for $g^{(2)}(0)$ in self-Kerr system to estimate $\eta$ in: $\mathcal{H}_{\text{eff}} = \hbar \eta (a^\dagger)^2 a^2$ along minimum curve

\[ \eta / \kappa \]

\[ E_p / \kappa \]

Can give huge optical nonlinearities

S. Rebic, J. Twamley, GJ Milburn, PRL 103, 150503 (2009)
HBT Experiment

To see photon antibunching: Hanbury-Brown-Twiss

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t + \tau) \hat{a}^\dagger(t) \hat{a}(t + \tau) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^2}$$

How to see this in Circuit-QED....do we need single photon detectors? Any other way?
To do HBT WITHOUT photon detectors

**Nice trick:**

\[
g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t+\tau)\hat{a}^\dagger(t)\hat{a}(t+\tau)\hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle^2}
\]

- Requires beamsplitter

- 50:50 Beamsplitter

- Homodyne

\[
\hat{b} = \sqrt{2}\hat{b} = \hat{a} + \hat{v}
\]

\[
\sqrt{2}\hat{c} = \hat{a} - \hat{v}
\]

**Correlation**

\[
g^{(2)}(\tau) = \frac{\langle \hat{b}^\dagger(t+\tau)\hat{b}(t+\tau)\hat{c}(t)\hat{c}(t) \rangle}{\langle \hat{b}^\dagger(t)\hat{b}(t) \rangle\langle \hat{c}^\dagger(t)\hat{c}(t) \rangle}
\]

**Quadratures**

\[
\hat{X}_+ = \hat{m} + \hat{m}^\dagger, \quad \hat{X}_- = -i(\hat{m} - \hat{m}^\dagger)
\]

**Second Order Correlation Function in terms of Quadratures**

\[
g^{(2)}(\tau) = \frac{\sum_{i,j} \langle \hat{X}_b^i(t+\tau)^2 \hat{X}_c^j(t)^2 \rangle - 2\sum_{i,k} \langle \hat{X}_k^i(t)^2 \rangle + 4}{\left(\sum_i \langle \hat{X}_b^i(t)^2 \rangle - 2\right)\left(\sum_i \langle \hat{X}_c^i(t)^2 \rangle - 2\right)}
\]

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- M. Mariantoni et al, arXiv: 0509737
- M. Mariantoni et al, PRB 78, 104508 (2008)
- N. Bergeal et al, arXiv: 0805.3451
Outline

Quantum control: Can we get more for less?
- Achieve universal q control using NO local addressing?
- Global Control: Transport/Computing
- Scheme to perform N-cavity multi-SWAP gate in Circuit-QED using global pulses
- Can we perform FT QEC with little local addressing or little/no measurement?

Nonlinear quantum optics
- Optical nonlinearities: single photon switch/blockade, deterministic q gates with single photons?
- Can we use the strong coupling to generate “huge” optical nonlinearities?
- Can we measure the photon anti-bunching: Hanbury-Brown-Twiss?

Coupling microwave photons to atomic ensembles
- Use atomic ensembles for memory etc, How to scale down the ensemble size but still retain large coupling strengths?
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People involved

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