Damping of high-Q violin modes in fused silica fibers and perspectives of application of electric charge velocity sensor to QND-measurements

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Introduction

- Fused silica suspension prototypes of the test masses were used in the interferometric gravitational wave detectors of the next generation (e.g., Advanced LIGO). The Q-factor of violin modes of fused silica fibers exceeds 10^15. This allows reducing of the test mass thermal noise associated with the suspension to the quantum noise level.
- High Q of the violin modes of the fused silica suspension may result in:
  - instability of the interferometer length control sensor;
  - interference associated with long ring-down times;
  - parametric instability due to the interaction between mechanical and optical modes.
- Violin mode damping allows one to solve this issues.
- We suggest a variant of a damping system for fused silica suspension fibers. An original sensor and actuator allowed the effective coupling of a thin, optically transparent, non-conducting fused silica fiber with an electric circuit.

Active damping system

- The optical sensor was based on the optical beam deflection in cylindrical fused silica fiber. The optical knife-edge technology for the spot displacement detection was used.
- In order to realize the force action on the fiber two parallel plain copper electrodes were glued to the silica block so that the fiber was in the gap between the electrodes. The distance between electrodes was about 1 mm. An electrical charge q was deposited on the segment of the fiber between the electrodes by touching the fiber with a bunch of 5 human hairs to minimize a mechanical impact on the fiber. Charge deposition was carried out in vacuum by means of a special manipulator. The decay time of charge deposited on fused silica in vacuum is more than 1 year. The conversion ratio of the sensor was \( C_0 = 4\Delta P/\Delta V \approx \omega_0 q \). Usually the magnitude of the deposited charge was about \( 10^{-12} \) C. If one chooses the phase of the force \( P_x \) exerted on the fiber by means of the phase shifter in the feedback loop so that the force was proportional to the speed of the fiber, e.g., \( P_x = \omega_0 C_0 \dot{Q}_x \), where \( C_0 \) is a dimensionless coefficient related to the total gain of the feedback loop then the feedback system only increases the damping rate of the oscillator \( \gamma \) by a factor \( (1 + C_0) \) and decreases the quality-factor \( Q_x = Q_x/(1 + C_0) \).

Cold damping in this system is realized if
\[
S_{\text{Th}}(\omega) < S_{\text{Th}}^{\text{opt}}(\omega),
\]
where \( S_{\text{Th}}^{\text{opt}}(\omega) \) is the thermal noise displacement power spectral density of the undamped fiber and \( S_{\text{Th}}(\omega) \) describes the optical sensor noise referred to sensor's input. The noise amplitude reduction coefficient is then determined by
\[
\frac{\sqrt{S_{\text{Th}}(\omega)/S_{\text{Th}}^{\text{opt}}(\omega)}}{1 + C_0} = 1 - Q_x(1 + C_0).
\]

Fused silica suspension prototype

The all-fused silica suspension prototype was designed with mind to exclude all additional losses that are not associated with the damping system. A fiber with a diameter of \( \leq 240 \) µm and a length of \( \leq 15 \) cm was welded to a fiber support structure that was cut from a \( 29 \times 6.5 \times 5.0 \) cm³ block of fused silica.

Experimental results

- Resonant frequencies of the fundamental violin mode and its first harmonic were found to be 677 Hz and 900 Hz. The Q-factors of the modes in vacuum with residual pressure of \( \leq 10^{-4} \) Torr were correspondingly \( 1.2 \times 10^9 \) and \( 3.3 \times 10^9 \).
- Displacement responsivity of the optical sensor \( R_0 = P_x \Delta P/\Delta V \) was found to be \( \sim 15 \) (µm)/(N·m) for the incident optical power \( P_L \approx 3 \) µW. The linear response of the optical sensor was in the range of about 1 µm. The conversion ratio of the sensor from the displacement of the fiber to the photodetector signal \( K_{s \text{opt}} = (1.6 \pm 0.1) \times 10^9 \text{m/V} / \text{m} \).
- Noise of the optical sensor was high because we did not use seismic isolation of the fiber support structure or stabilization of the laser intensity. \( S_{\text{n}}(\omega) \) was found to be \( 2.2 \times 10^{-9} \text{mV}^2 / \text{Hz} \) at the frequency range near the fundamental violin mode frequency. That was close to the thermal noise displacement power spectral density of \( S_{\text{Th}}^{\text{opt}}(\omega) = 1.5 \times 10^{-6} \text{mV}^2 / \text{Hz} \) that was calculated for the fundamental mode of the undamped fiber.

- The decay curves of the fundamental violin mode amplitude measured for different gains of the narrow-band amplifier are shown below in fig.(a). The insertion damping ratio \( Q_x^{\text{opt}} \) as a function of the narrow-band amplifier gain is shown in fig.(b).

Introduction II

In this column some other possible applications of the device that was used as an actuator in our violin mode damping system (see the left column) are discussed. The work on these applications is currently in progress, and there are no experimental results presented in this column.

Damping system with a combined sensor-actuator and an active resistor

An electrical charge moving in a gap between two electrodes connected through an active resistor induces electric current in the electrodes contour. The current \( i \) in this case according to generalized Ramo-Shockley theorem is described by equation
\[
i = R C \frac{dV}{dt} \frac{dV}{dt} + \frac{dV}{dt} = 0.
\]
Here \( C \) is the capacitance of the sensor's pair of electrodes. This equation combined with the equation of mechanical oscillator’s motion
\[
i = 2q \gamma + \frac{1}{2} q^2 - \frac{q^2}{2\Omega_{\text{opt}}} R
\]
allows one to calculate the insertion damping ratio
\[
Q_x^{\text{opt}} = \frac{\gamma R}{2\Omega_{\text{opt}} q^2 (\Omega_{\text{opt}} / q R)^2},
\]
that could be obtained at the optimal value of the resistance \( R_{\text{opt}} = \frac{q}{\gamma R} \). Here \( T_{\Omega} = T_{\text{opt}} \) and \( T_{\text{opt}} \) are correspondingly temperatures of the undamped fiber and the resistor. Note that if \( T_{\Omega} < T_{\text{opt}} \) the damping becomes cold.

For the fused silica suspension prototypes that we use the Q-factor can be lowered to \( 10^7 \) when the amount of the electrical charge deposited to the fiber is about \( 10^{-11} \) C. Although we did obtain this value via contact electrophoresis, this way is rather inconvenient. There are reasons to believe that using electron impact on the fiber is more prospective.

Active damping system with an electrical sensor and an electrical actuator

This damping system is similar to the one described in the left column, but the optical sensor is replaced with another charge-sensitive device. In this case the charge has to be deposited at least in two separate regions of the damped fiber. The advantage of this variant is that the insertion damping ratio can be varied over a wide range.

Limit of the sensor’s sensitivity

The described electrical device is in fact an electrical charge velocity sensor whose electrical current in the circuit is simply proportional to the velocity of the charge:
\[
I = \frac{dq}{dt} = \frac{q}{\Omega_{\text{opt}}}
\]
Despite this, the sensor’s sensitivity is constrained by the standard quantum limit even in the case of a free mass probe, because the back action of the sensor on the probe is coupled with the coordinate of the probe.

The possible scheme of velocity measurement is shown in the figure. The electrodes are connected to the inputs of a trans-impedance amplifier (current-to-voltage converter) with the conversion ratio \( \frac{R_{\text{in}}}{R_{\text{opt}}} \), for which the spectral densities of the noise fluctuations of the output voltage referred to the charge \( q_{\text{opt}} \), that has passed through the input probes \( Z_\text{in} \) and the fluctuations \( V_{\text{in}} \) of the input voltage \( V_\text{in} \) have to satisfy the equation
\[
S_{\text{opt}} - |S_{\text{in}}|^2 \geq \frac{R_{\text{opt}}}{R_{\text{in}}} \frac{R_{\text{opt}}}{R_{\text{in}}}
\]
where \( S_{\text{in}} \) is the cross-correlation spectral density of these noises.

It is easy to show that if we want to detect a classical force \( F_{\text{classical}} \) acting on the probe, its Hamiltonian can be written as
\[
H = \frac{p^2}{2m} + F_{\text{classical}} \dot{x},
\]
and the measured output voltage of the amplifier
\[
V_{\text{out}} = -R_{\text{in}} \frac{p^2}{2m} + R_{\text{in}} F_{\text{classical}} \dot{x}
\]
It is easy to show that the spectral density of the equivalent total noise force acting on the probe
\[
S_{\text{tot}}(\omega) = 2m \frac{1}{\Omega_{\text{opt}}} R_{\text{in}}
\]
which is the standard quantum limit for detection of a classical force acting on a free mass.