The Stability of an Optical Clock Laser Transferred to the Interrogation Oscillator for a Cs Fountain

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Abstract—We stabilize a microwave oscillator at 9.6 GHz to an optical clock laser at 344 THz by using a fiber-based femtosecond laser frequency comb as a transfer oscillator. With a second frequency comb, we independently measure the instability of the microwave source with respect to another optical clock laser frequency at 456 THz. The total fractional frequency instability of this optic-to-microwave and microwave-to-optic conversion resulted in an Allan deviation $\sigma_y$ of $1.2 \times 10^{-14}$ at 1 s averaging time (bandwidth of 50 kHz). The residual phase noise density is $-97$ dBc/Hz at a 10 Hz offset from the 9.6 GHz carrier. Replacing the existing quartz-based interrogation oscillator of the Physikalisch-Technische Bundesanstalt (PTB) caesium fountain CSF1 with this optically stabilized microwave source will reduce the instability contribution due to the Dick effect from the $10^{-13}$ level at 1 s averaging time to an insignificant level at the current status of CSF1. Therefore, this new microwave source can be an alternative to cryogenic sapphire-loaded cavity oscillators to overcome the limitations of state-of-the-art quartz oscillators.

Index Terms—Fountain clocks, frequency combs, microwave oscillators, optical frequency standards, phase noise measurement.

I. INTRODUCTION

IN THE EXISTING atomic caesium fountains used for the realization of the SI unit “second,” an intrinsic frequency instability contribution is caused by the phase noise of the employed microwave source, which is an effect that is often called the “Dick effect” [1], [2]. To overcome this limitation (typically at the low $10^{-13}$ level at 1 s averaging time), an ultrastable cryogenic sapphire-loaded cavity oscillator [3] has been used, and a quantum-projection noise-limited operation down to the low $10^{-14}$ level has been demonstrated [4], [5]. Here, we present an alternative approach using an optically stabilized microwave source that is designed to replace the quartz-based interrogation oscillator of Physikalisch-Technische Bundesanstalt (PTB) caesium fountain CSF1. The resulting instability is expected to only be dominated by quantum projection noise.

As ultrastable clock lasers have become new sources of superior stability [6], [7], and femtosecond combs can transfer this stability to the microwave domain [8], these two key elements allow a novel way of microwave generation based on optical clock lasers. While this combination is routinely used for absolute frequency measurements [9], [10], it also has an enormous potential to generate ultralow-jitter microwave signals in a novel way.

In 2005, Bartels et al. showed that in a synthesized 10 GHz signal, the femtosecond comb added an instability of $\sigma_y = 6.5 \times 10^{-16}$ at 1 s [11]. For averaging times longer than 1 s, the stability suffered from the $1/f$ frequency noise contribution of the comb. At present, the best reported residual phase noise from such a system is approximately $(-95 \text{ dBc/Hz} / f^2$ for Fourier frequencies $1 \text{ Hz} < f < 1 \text{ kHz}$ approaching a white noise level of $-140 \text{ dBc/Hz}$ at Fourier frequencies above $10 \text{ kHz}$ offset from the 10 GHz carrier [12]. The studies described earlier, using two femtosecond combs stabilized to a common optical reference, permit tests on the fidelity of the frequency division process from optical to microwave frequencies. Alternatively, using two independent combs stabilized to two independent optical references provides information on the absolute stability, reproducibility, and frequency accuracy. With this latter configuration, Bartels et al. [11] have demonstrated the synthesis of 10 GHz signals having a fractional frequency instability of less than $\sigma_y = 3.5 \times 10^{-15}$ and limited by the stability of the poorer of the two optical references. Similar results have been obtained by Kim et al. using a balanced optical–microwave phase detector for the extraction of microwave signals at 10 GHz from optical pulse trains [13].

These studies were all carried out using Ti:Sapphire-based femtosecond combs. However, for applications that require long-term continuous operation, fiber-based combs are a more attractive option, but there is little published work on the use of such systems for low-noise microwave synthesis.

In the only experiment published to date [14], the stability of a 10-GHz signal derived from a fiber comb was compared with an established high-stability frequency comb based on a Ti:Sapphire laser. Both combs were locked to a common optical reference with a stability of around $\sigma_y = 3 \times 10^{-15}$ at 1 s. For the 10 GHz signal, a short-term stability of $\sigma_y = 2 \times 10^{-14}$ at 0.1 s was achieved. These results already indicated that a microwave signal generated from an optical clock will be an excellent candidate to interrogate microwave fountain clocks.

In this paper, we present a novel way to synthesize an ultralow-noise microwave frequency using an ultrastable clock laser, a femtosecond comb that transfers this stability to the microwave domain, and a low-noise microwave source.

We use a frequency-measuring system that routinely compares the frequency of the Yb$^+$ frequency standard of PTB [15] with that of a Cs fountain clock CSF1 [16], [17]. The ultrastable Yb$^+$ clock laser is measured using a commercially available
clock laser
344 THz

fiber based femtosecond laser

control of: $f_{\text{rep}}, \nu_{\text{ceo}}$

H-maser

DRO 9.6 GHz
fast control

$f_{\text{reference}}$ from CSF1

Fig. 1. Setup for absolute frequency measurements and phase locking of a microwave source. DRO is the dielectric resonator oscillator; $a, b \in \mathbb{N}, c \in \mathbb{R}$.

fiber-based femtosecond comb [18]. Its 100 MHz repetition rate $f_{\text{rep}}$ is locked with a few 100 Hz bandwidth to a reference frequency from an H-maser. The self-referencing of the comb is arranged by the $f - 2f$ scheme—stabilizing the carrier offset frequency $\nu_{\text{ceo}}$ achieved with a bandwidth of several kilohertz.

As for femtosecond fiber combs, the white frequency noise extends to Fourier frequencies of up to 100 kHz, and the limited bandwidth of the control elements puts some constraints on ultraprecise measurements. We circumvent this problem by using the transfer oscillator concept derived by Telle et al. [19]. In this case, the femtosecond comb needs no fast servo control loops. More details are described in [20].

The optical frequency of a clock laser $\nu_L$ is derived from the simultaneous measurement of three radio frequencies and the known mode number $m$ according to

$$\nu_L(t) = m \cdot f_{\text{rep}}(t) + \nu_{\text{ceo}}(t) + \nu_x(t)$$

where $\nu_x$ is the beat signal of the Yb$^+$ clock laser $\nu_L$ with the comb line closest to it.

We upgraded our current frequency measurement system by adding a module that allows the simultaneous generation of a microwave signal. Additionally, the determination of the phase noise of highly stable microwave oscillators in real time is easily accomplished.

II. SETUP

In [11], a microwave signal was directly generated in a photodiode, detecting a high harmonic of the pulse repetition rate. In this paper, we start with a microwave oscillator, which is phase locked to an optical reference using the femtosecond comb.

The microwave source is a commercial 9.6 GHz dielectric resonator oscillator (DRO) with a state-of-the-art phase noise of $-115$ dBc/Hz at 10 kHz from the carrier (see Fig. 3). The frequency control input has a bandwidth of $>1$ MHz.

The setup for locking the DRO to an optical standard is shown in Fig. 1. The DRO’s output is mixed with the ninety-sixth harmonic of the comb’s repetition frequency ($a \times f_{\text{rep}}$), resulting in an intermediate beat signal $\nu_d$ at approximately 6 MHz. This signal is subsequently multiplied by a factor $b$ using a harmonic tracking oscillator.

The up-conversion of the microwave signal is accomplished by a down-conversion of the optical signal. As a first step, we take into account that the beat signal $\nu_x$ is derived from a beat with the frequency-doubled output of the comb; therefore, this signal has to be divided by 2 before it is mixed with the carrier offset frequency $\nu_{\text{ceo}}$. The subsequent division of the sum $(\nu_{\text{ceo}} + \nu_x/2)$ by a factor $c$ allows one to generate a beat signal $\nu_t$ between the up-converted microwave signal and the down-converted optical beats at an intermediate virtual frequency. The divisor $c$ is chosen according to

$$m = a \times b \times c.$$ 

This rational divider is realized by a direct digital synthesizer (DDS) with a resolution of 48 bits. With $\nu_d = \nu_{\text{DRO}} - a \times f_{\text{rep}}$ and $\nu_t = (\nu_{\text{ceo}} + \nu_x/2)/c - b \times \nu_d$, we obtain

$$\nu_t = \nu_L/c - b
\nu_{\text{DRO}}.$$ 

This signal corresponds to a virtual beat between the Yb$^+$ clock laser and the DRO. In our case, this signal is measured at an intermediate frequency of about 77 GHz.

Our choice of the divider ratios of $b$ and $c$ was guided by the following considerations: Increasing the factor $b$ decreases the demands on the phase noise of the DDS (divider $c$) and the subsequent electronics, whereas the noise issues are passed to the multiplier $b$. In our case, the phase delay of the multiplier limits the attainable electronic locking bandwidth of the harmonic tracking filter and leads to an optimum factor of $b = 8$.

Phase locking of the DRO is achieved by comparing the beat frequency $\nu_t$ with a reference frequency $f_{\text{reference}}$. To lock the DRO to a Cs fountain clock for long integration times, this reference frequency is steered by the atomic fountain clock.

This concept has two major advantages: It allows a comfortable real-time measurement of the phase noise of the DRO and a locking of the DRO to an optical clock with a bandwidth of up to several megahertz.

For a verification of the DRO’s performance with respect to phase noise and instability, a second independent measurement system is required. For this purpose, we use a second frequency comb in combination with the clock laser of the Ca optical frequency standard [21].

Both of the clock laser systems with their accompanying femtosecond combs are located in different buildings and are linked by 300 m of coaxial cable (type: FSJ1). To avoid the degradation of the signal-to-noise ratio (SNR), we transmit the frequency of the DRO divided by a factor of 8.

At the Ca laser setup, the transmitted frequency of 1.2 GHz is frequency doubled and compared with the twenty-fourth harmonic of the Ca–femtosecond comb in the same way as previously described. The difference in the nearly identical setups is that the optical frequency of the Ca system is at 456 THz, and the transfer signal $\nu_d$ (Ca) is only used for the analysis.

To achieve the highest possible resolution, the factor $b$ here is set to 1024, and the divider $c$ is accordingly adjusted. The phase noise of the DRO is thus analyzed at a virtual frequency of 2.4 THz. Multiplied by such a huge factor, the noise of the DRO is measurable with a conventional spectrum analyzer.

For further data analysis and processing in the time domain, we use a multichannel accumulating counter with synchronous
readout and zero dead time referenced to an H-maser that is controlled by the caesium fountain clock CSF1.

III. RESULTS

The Yb\(^+\) clock laser provides the stability in the optical domain (triangles in Fig. 2). The curve shown is the result of an optical-to-optical frequency comparison between the Ca clock laser and the Yb\(^+\) clock laser. For very short times (\(\tau < 0.1\) s), the Allan deviation \(\sigma_y(\tau)\) is dominated by the linewidth of the clock lasers. For intermediate times (0.2 s < \(\tau < 50\) s), the stability is limited by the thermal noise of the optical reference resonators, whereas for \(\tau > 50\) s, \(\sigma_y(\tau)\) increases due to the relative drift of the resonators [22].

For an ideal down-conversion of this stability by means of a femtosecond comb, we expect to achieve the same low instability for our microwave source. Using our frequency counting system, we measured the Allan deviation of the stabilized microwave signal. As shown in Fig. 2 (dots), the instability (measured with a high frequency cutoff of 50 kHz) exhibits a \(1/\tau\) behavior for \(\tau < 10\) s, which results in \(\sigma_y(1\) s) = \(1.2 \times 10^{-14}\) at 1 s averaging time. At \(\tau \sim 10\) s, the stability reaches that of the optical frequency standard. For longer averaging times (\(\tau > 50\) s), the residual drift between the two optical resonators leads to an increase of \(\sigma_y\). For comparison, we show the stability of the in-loop signal (squares), the stability of the 5 MHz quartz (open grey circles) that is currently used in a synthesis chain for the CSF1 fountain clock [23], and the typical data of an ultrastable cryogenic sapphire-loaded cavity oscillator (asterisks).

In the following, we discuss in the phase noise domain some of the technical limitations that hamper the ideal performance.

As we detect a high harmonic of the pulse repetition frequency with an ultrafast photo detector, the achievable SNR is limited by the shot noise and the dynamic range of the photodiode. In our case, this causes a white phase noise level of \(-134\) dBc/Hz, which is indicated by the constant line in Fig. 3.

Another technical restriction is due to the fact that the generated microwave signal is transmitted over a 300 m coaxial cable to the analyzing system in another building. At 9.6 GHz, the attenuation of the cable is significant and would degrade the SNR. To avoid this, we divide the signal by 8. At 1.2 GHz, we achieve a similar SNR for the transmitted signal and the signal derived from the photodiode of the analyzing femtosecond comb. The price to be paid is an additional phase noise of the divider. The specification of the divider’s phase noise is shown in Fig. 3 as diamonds. For Fourier frequencies \(f < 10\) kHz, the noise of the divider is above the level due to the photodiode and is dominated by \(1/f\) phase noise, which results in \(-110\) dBc/Hz at an offset of \(f = 10\) Hz. While the free-running DRO has an excellent phase noise at Fourier frequencies above several megahertz, its low-frequency noise is dominated by the flicker of the frequency noise. This reaches \(-56\) dBc/Hz at 100 Hz offset from the 9.6 GHz carrier.

The crossing point at 50 kHz of the DRO’s phase noise with that of the photodiode’s shot noise determines the ideal locking bandwidth. The phase-locked loop (PLL) for phase locking the DRO frequency to the optical frequency standard uses an...
additional second integrator at 12 kHz. This results in a significant reduction of the DRO’s phase noise (red curve in Fig. 3). The total phase noise exhibits a white phase-noise level of $-123 \text{ dBc/Hz}$ for $3 \text{ kHz} < f < 50 \text{ kHz}$. (Data beyond 50 kHz reflect a rolloff with $1/f^3$ due to the tracking oscillator of the analyzing system.) This white phase-noise level can be changed by adjusting the optical power at the photodiode. observations have been reported by Newbury et al. [24] and point to a possible AM/FM conversion within the photo detector. This limitation will be studied in more detail in the future.

For Fourier frequencies $f < 3 \text{ kHz}$, the noise is dominated by the flicker of the phase noise. This level of $1/f$ noise is about 10 dB above the noise expected from the specifications of the frequency divider. At $f = 100 \text{ Hz}$, the phase noise is suppressed by 52 dB with respect to the noise of the free-running DRO and reaches $L(f) = -95 \text{ dBc/Hz}$ at 10 Hz offset from the 9.6 GHz carrier.

The integrated phase noise up to a high-frequency cutoff of 50 kHz leads to an Allan deviation of $\sigma_\alpha(T) = 9 \times 10^{-15}/\tau$. This is in excellent agreement with the data derived from the time-domain measurements (see Fig. 2).

The corresponding in-loop signal of the stabilized DRO, as analyzed by the phase noise of the transfer beat $\nu_i$ (green dots in Fig. 3), is well below $-140 \text{ dBc/Hz}$ for $f > 40 \text{ Hz}$, showing a slight increase for $f < 40 \text{ Hz}$, which is in good agreement with the Allan standard deviation of $\sigma_\alpha(T) = 2 \times 10^{-15}/\tau$ of the in-loop signal (squares) shown in Fig. 2. Again, for comparison, we additionally show the phase noise of the aforementioned 5 MHz quartz oscillator.

We have achieved a continuous operation of the complete setup over several days. We have thus obtained a reliable self-contained module for synthesizing an ultralow-noise microwave frequency.

IV. PROSPECTS: AN INTERROGATION OSCILLATOR FOR A FOUNTAIN CLOCK

The PTB caesium fountain CSF1 currently uses a recently developed new 9 GHz synthesis chain [23]. In this synthesis, a 9.6 GHz YIG oscillator is locked to a 5 MHz quartz oscillator via a divider chain. The signal from the atoms is then used to steer the frequency of the quartz oscillator, whose instability specification is shown in Fig. 2.

Mainly caused by the time needed to load and detect the atoms, the pulsed operation mode of a caesium fountain comprises a significant amount of dead time, during which the quartz oscillator frequency is not controlled by the atomic resonance signal. Such a dead time results in a degradation of the fountain frequency instability that is caused by the frequency noise of the interrogation oscillator (Dick effect) [2].

For CSF1, the long loading times of the magnetooptical trap are used to increase the atom number and evaluate the collisional frequency shift [16], [17]. In Fig. 4, the solid line shows the calculated instability contribution due to the Dick effect for CSF1 when the loading time and, thus, the dead time are varied. The calculation is based on the phase-noise data from the data sheet of the employed quartz crystal oscillator and the sensitivity function calculated for CSF1 [2]. With an increasing dead time, an increasing number of oscillator phase-noise components at small Fourier frequencies contributes and, thus, degrades the stability. The black data points depict a set of measured frequency instabilities using a hydrogen maser as a reference for CSF1. For loading times longer than half a second, the measured instability is clearly dominated by the local oscillator noise via the Dick effect.

If the instability contribution due to the Dick effect is calculated based on the measured phase noise (Fig. 3) of the optically stabilized microwave source, the data shown by the dashed line in Fig. 4 are obtained. At this level, which is at or even below $\sigma_\alpha(1 \text{ s}) = 1 \times 10^{-14}$, the instability contribution due to the Dick effect would have a negligible effect on the overall CSF1 stability, which would then be quantum projection noise limited by the currently accessible numbers of detected atoms for different loading times. From these numbers, it can be expected that for normal operation at a short loading time, the overall instability is reduced by a factor of 2 by employing the optically stabilized microwave source instead of the quartz-based source. For the long loading times used in collisional shift evaluations, even larger improvements up to a factor of 4 can be achieved.

V. CONCLUSION

We have realized a highly stable microwave source at 9.6 GHz by using a fiber-based femtosecond frequency comb and an optical reference frequency. We have demonstrated that the stability of the optical clock laser can be transferred to the microwave domain without tight locking the frequency comb, generating a stabilized microwave signal with a stability superior to common 9 GHz synthesizers based on ultrastable 5 MHz quartz oscillators.

With an achieved short-term stability of $1 \times 10^{-14}$ at 1 s of the optically stabilized DRO, the Dick effect would give a negligible contribution to the overall CSF1 instability, which would then be quantum projection noise limited. Further improvements can be achieved by increasing the number of atoms through loading from an atomic beam [5].

REFERENCES


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Photograph and biography not available at the time of publication.

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